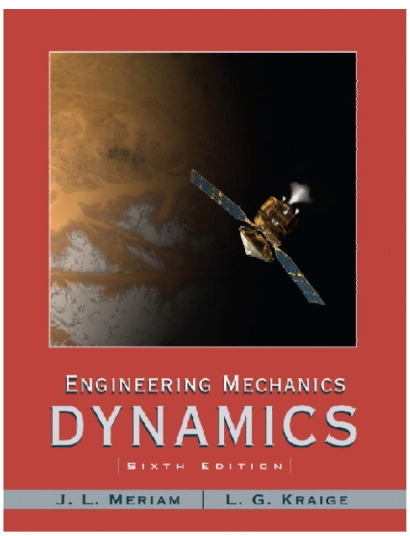
Thi-Qar University
College of Engineering
Mechanical Department
Second Stage

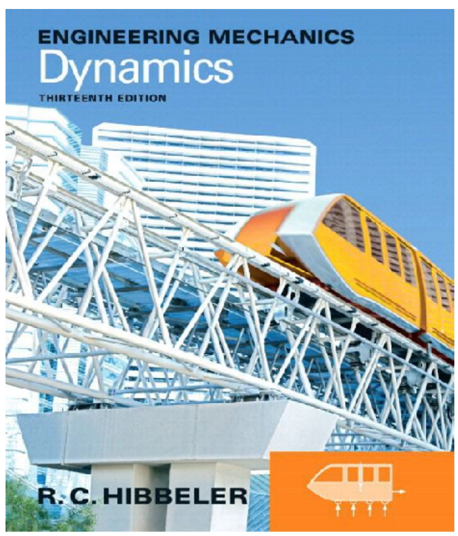
Engineering Mechanics – Dynamics

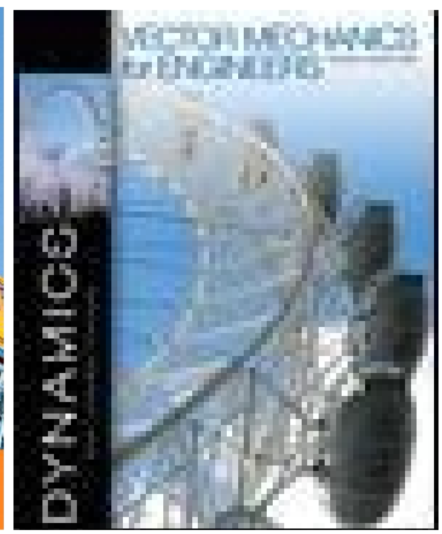
Lecturer: JAAFAR Mohammed Hamzah
M.Sc. Applied Mechanical Engineering

References

i. Engineering Mechanics Dynamics, J. L. Meriam, (6th, and 5th Edition).
ii. Engineering Mechanics Dynamics, R. C. Hibbeler, (13th Edition).
iii. Vector Mechanics for Engineers: Dynamics, F. P. Beer , E. R., Johnston,P. J. Cornwel.
iv. Schaum's solved problems series Vol. 2: Dynamics, Joseph F. Shelley.
v. Lecture notes & slides.







Grade

i.	Attendance & Homework	3 %
ii.	Two Quizzes	2 %
iii.	Semester1	17.5%
iv.	Semester 2	17.5%
v.	Final	60%

Contains of

Dynamics

PART I: First Semester

DYNAMICS OF PARTICLES

Chapter 1: INTRODUCTION TO DYNAMICS

- Definition of Dynamics
- Basic Concepts
- Newton's Laws
- Units

Chapter 2: KINEMATICS OF PARTICLES

- Introduction
- Rectilinear Motion
- Plane Curvilinear Motion
- Rectangular Coordinates (x-y)
- Normal and Tangential Coordinates (n-t)
- Polar Coordinates (r-θ)
- Space Curvilinear Motion
- Relative Motion (Translating Axes)
- Constrained Motion of Connected Particles

Chapter 3: KINETICS OF PARTICLES

- Introduction

FORCE, MASS AND ACCELERATION

- Newton's Second Law – Equation of Motion
- Rectilinear Motion
- Curvilinear Motion

WORK AND ENERGY

- Work and Kinetic Energy
- Potential Energy

IMPULSE AND MOMENTUM

- Linear Impulse and Linear Momentum
- Angular Impulse and Angular Momentum
- Impact

PART II: Second Semester
DYNAMICS OF RIGID BODIES

Chapter 5: PLANE KINEMATICS OF RIGID BODIES

- Introduction
- Rotation
- Absolute Motion
- Relative Velocity
- Instantaneous Center of Zero Velocity
- Relative Acceleration
- Motion Relative to Rotating Axes

Chapter 6: PLANE KINETICS OF RIGID BODIES

- Introduction

FORCE, MASS AND ACCELERATION

- General Equations of Motion
- Translation
- Fixed Axis Rotation
- General Plane Motion

WORK AND ENERGY

- Work –Energy Relations
- Potential Energy

IMPULSE AND MOMENTUM

- Impulse – Momentum Equations

- **References**
  - Engineering Mechanics Dynamics, J. L. Meriam, (6th, and 5th Edition)
  - Engineering Mechanics Dynamics, R. C. Hibbeler, (13th Edition)
  - Bedford and Fowler Engineering Mechanics – Dynamics SI edition, Addison-Wesley, 1996.

    .

    .
  - Lecture notes & slides.

- **Exam**
  - Quiz #1: 07 / 11 / 2013

    Group A: 10:30 AM to 11:00 AM

    Group B: 12:30 AM to 1:00 AM
  - Season1: **29 / January2014 /**
  - Quiz #2: 11 / 03 / 2014

    Group A: 10:30 AM to 11:00 AM

    Group B: 10:30 AM to 11:00 AM
  - Season2: **20 / April / 2014**
  - Final: **June – July / 2014**

Chapter 1

Introduction to Dynamics

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

What is Mechanics?

- A branch of physical science which deals with the effects of forces on objects

- Two parts: *Statics* (equilibrium of bodies) and
  Dynamics (motion of bodies)

- Applications:
  - Strength of structures and machines (houses, robots, cars, airplanes)
  - Vibrations (engine vibrations, bridges, wheels)
  - Fluid mechanics (airplanes, fluid machinery)
  - Electrical machines and apparatus (motors, transducers)

Mechanics Fields of Study

- *Statics*
  - **Rigid** bodies in **equilibrium** → Forces
- *Dynamics*
  - **Rigid** bodies in **motions** → Forces and motions
- *Strength of Materials (Mechanics of Materials)*
  - **Deformable** bodies in **equilibrium** → Strength and deformation
- *Fluid Mechanics*
  - **Deformable** bodies in **motions** → Pressure and flow
- *Mechanics of Machinery*
  - Dynamics of mechanism including linkages
- *Vibration*
  - **Rigid** and **deformation** bodies in **repetitive motions**

Dynamics – Kinematics of particles

Analysis of bodies in motion

1) Kinematics

- study of the geometry of motion
- relate displacement, velocity, acceleration, and time without reference to the source/cause of motion

2) Kinetics

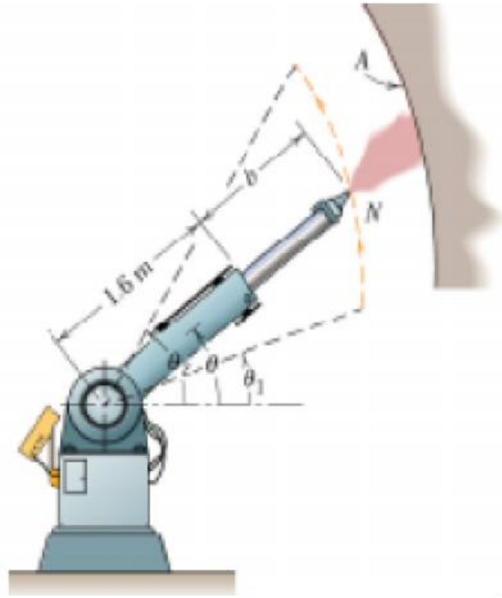
- study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body
- To predict the motion caused by given forces or to determine the forces required to produce a given motion.

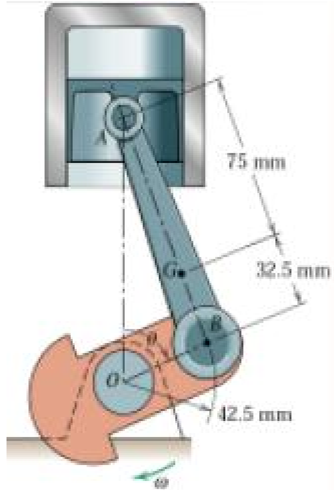
Rectilinear motion: position, velocity, and acceleration of a particle as it moves along a straight line

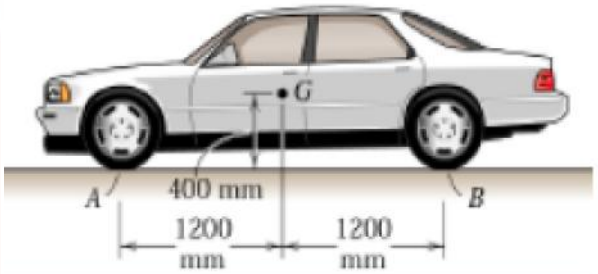
Curvilinear motion: position, velocity, and acceleration of a particle as it moves along a curved line

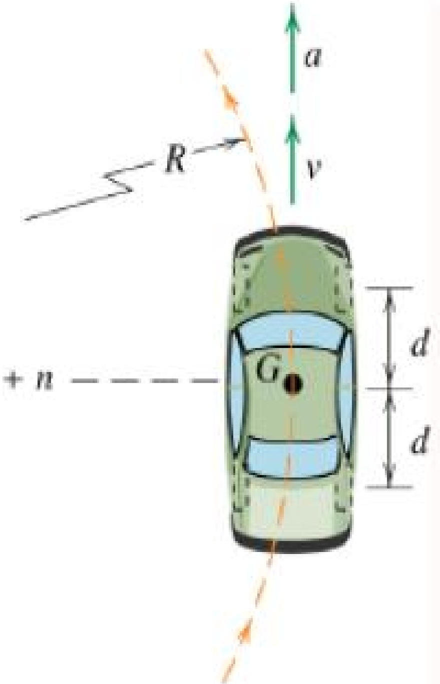
Applications of Dynamics

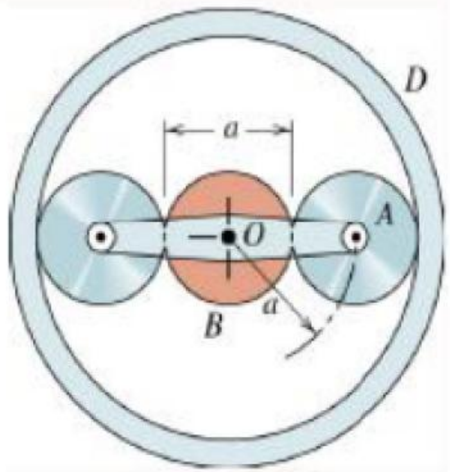
- Robot Arm
- Car Engine
- Vehicle Dynamics
  - braking /accelerating
  - cornering
- Planetary Gear











Learning Strategies

Recommendation:

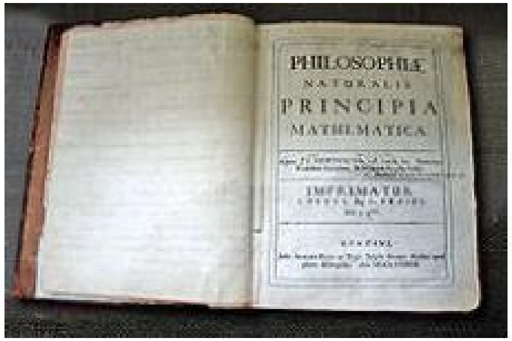
- If possible, read ahead
  - read ahead (+20% understanding), class (+30%), exercise (+40%)
- Two notebooks: for notes and exercises
- Exercise:
  - do exercise before looking at solutions
  - do in steps and make it easy to read
  - in case of getting stuck, ask or look at solutions

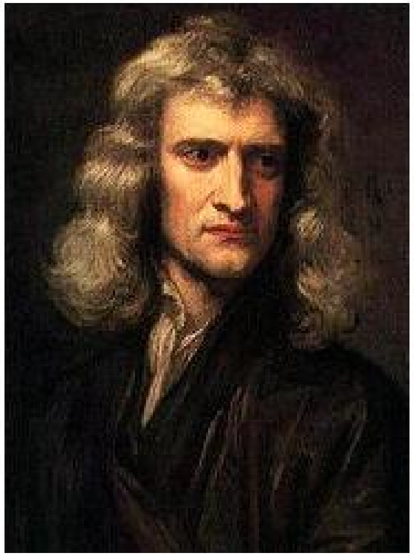
Exam Strategies

- Do step by step
- Write the laws to be used: 2nd law…
- Draw Free Body Diagram
- Show coordinates: x, y…
- Define variables
- Show calculations
- State directions of vectors: vel, acc, force…
- Show units at numerical answers: N, m/s…
- Use common sense to check the answer
- Make it clean

Who Is Newton?

- Born: 1643 in England
- Physicist, Mathematician, Astronomer, Philosopher etc.
- "Mathematical Principles of National Philosophy" known as "Principia" (1687)
  - Classical mechanics: Laws of Gravitation, Laws of Motion
- Calculus, Reflecting telescope, law of cooling, speed of sound, Newton's method for finding roots of a function, power series etc.



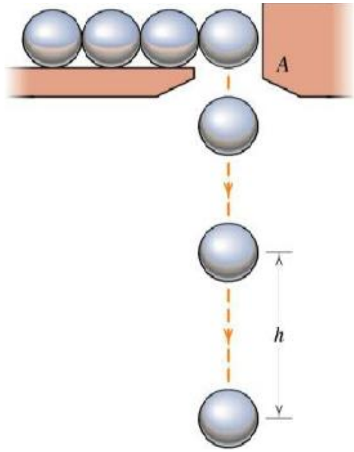


Chapter 2
Kinematics of Particles

BY: JAAFAR MOHAMMED HAMZAH

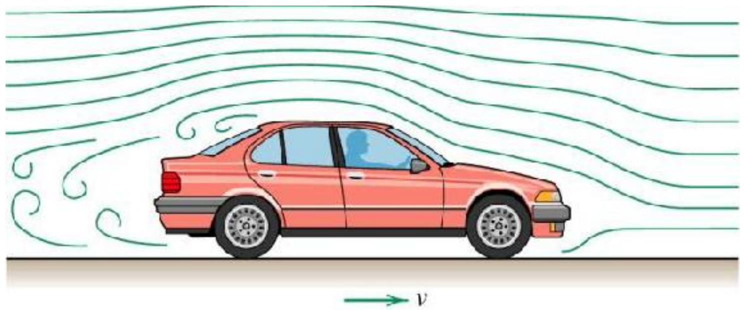
M.Sc. Mechanical Engineering

Kinematics of Particles



What is Kinematics of Particles?

- Study of **motion** of bodies (assumed as particles) **without** reference to **forces**
- Kinematics of Particles "describes" motion of particle, generally, the **relations between**
  - Position/displacement
  - velocity
  - Acceleration



- Easy example: A car
  - Given the velocity as a function of time, how far did the car moved for a given period of time? What is the acceleration at each point in time?

Kinematics of Particles

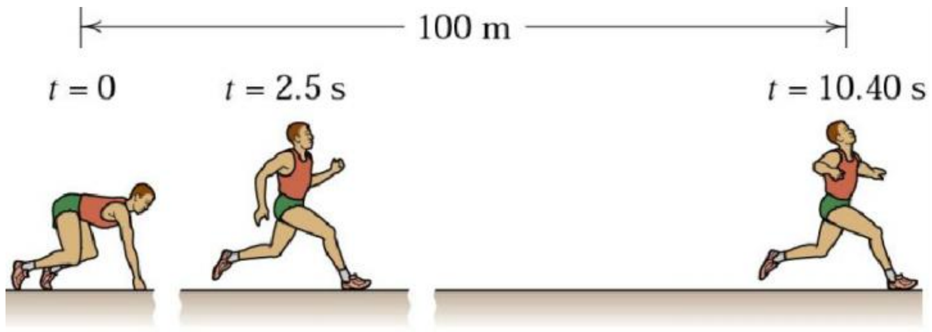
Topics

- One dimension
  - Rectilinear
- Two dimensions
  - Rectangular Coordinates (*x*-*y*)
  - Normal and Tangential Coordinates (*n*-*t*)
  - Polar Coordinates (r-θ)
- Relative Motion

2/2 Rectilinear Motion

Rectilinear Motion

= Motion in a straight line

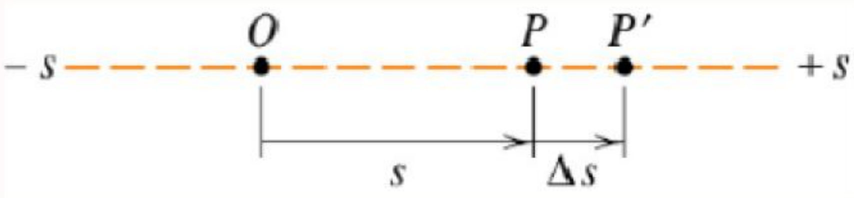


1. Displacement and Instantaneous Velocity
2. Instantaneous Acceleration
3. Graphical Interpretation
4. Special Case: Constant Acceleration
5. Examples

Rectilinear Motion

1. Displacement and Instantaneous Velocity

- For a straight motion of a particle;



- Position of P is specified by the displacement *s* (scalar) measured from some fixed point *O*.
- During Δt sec, P moved Δs m
- Average speed, $v_{av} = \Delta s / \Delta t$ m/s
- $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$

Instantaneous Velocity

$$v = \frac{ds}{dt} = \dot{s}$$

Rectilinear Motion

2. Instantaneous Acceleration

- Similarly, we can define instantaneous acceleration
- At time t_1 the velocity is v_1, at time t_2 the velocity is v_2
- So the average acceleration is

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

- Again, taking the limit as $\Delta t \to 0$ or $t_2 \to t_1$,

Instantaneous Acceleration

$$a = \frac{dv}{dt} = \dot{v} \qquad \text{or} \qquad a = \frac{d^2 s}{dt^2} = \ddot{s}$$

- Using the equations, we have $vdt = ds$ and $adt = dv$
- Eliminating dt, we have

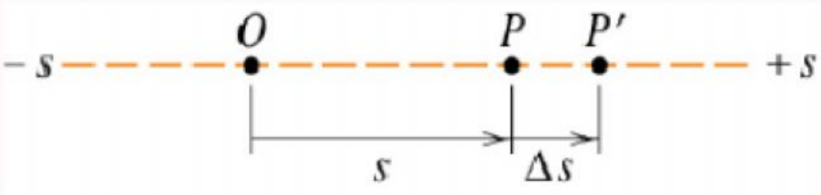
Instantaneous Acceleration

$$vdv = ads$$

Rectilinear Motion

Notes on directions

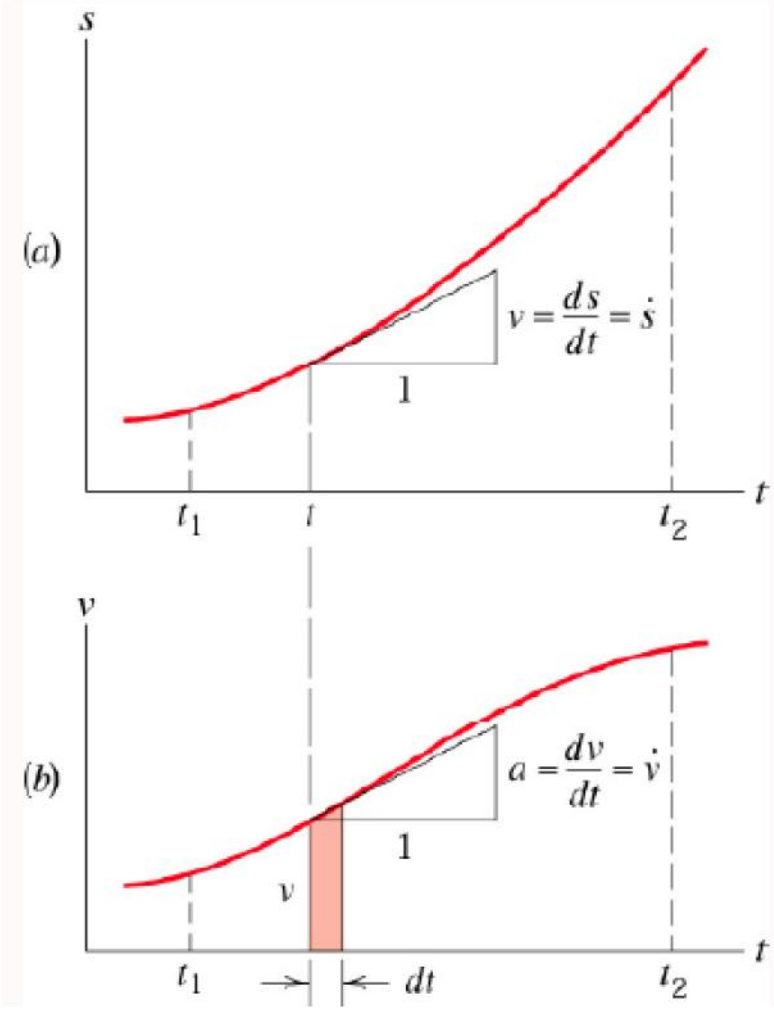
- Positive direction of a, v, and s must be the same!



- If we defind $+s$ to the right
- $\vec{v}$ and $\vec{a}$ pointing to the right are positive.
- Positve v means s is increasing (since ds is positive).
- Similarly, positive a means v is increasing.

Rectilinear Motion

3. Graphical Interpretation

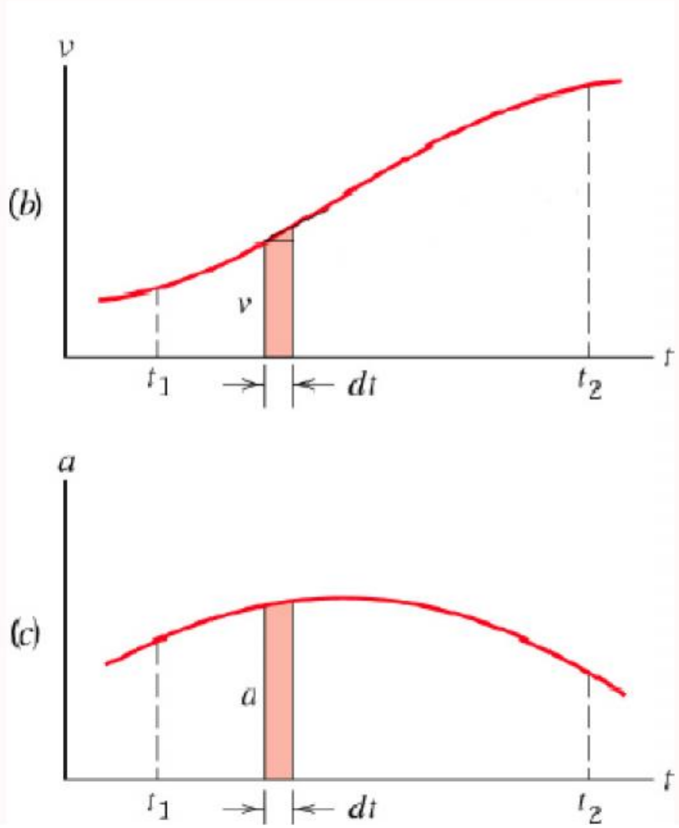


- Slope of *s*-*t* curve = velocity
- Slope of *v*-*t* curve = acceleration

Rectilinear Motion

3. Graphical Interpretation

- The usual interpretations: Area under curves



- Area under *v*-*t* curve = (changes in) displacement

$$\int_{t_1}^{t_2} v\,dt = \int_{s_1}^{s_2} ds = s_2 - s_1$$

- Area under *a*-*t* curve = (changes in) velocity

$$\int_{t_1}^{t_2} a\,dt = \int_{v_1}^{v_2} dv = v_2 - v_1$$

Rectilinear Motion

4. Special Case: Constant Acceleration

Constant Acceleration: $v(t)$

$$v(t) = v_1 + a(t - t_1)$$

Constant Acceleration: $v(s)$

$$v^2(s) = v_1^2 + 2a(s - s_1)$$

Constant Acceleration: $s(t)$

$$s = s_1 + v_1(t - t_1) + \frac{a}{2}(t - t_1)^2$$

Rectilinear Motion

Example 1:

The velocity of a particle which moves along the s-axis is given by $v = 2 - 4t + 5t^{3/2}$, where t is in seconds and v is in meters per second. Evaluate the position s, velocity v, and acceleration a when $t = 3\ s$. The particle is at the position $s_o = 3\ m$ when $t = 0$.

<u>Solution:</u>

$$v = 2 - 4t + 5t^{3/2}$$

$$a = \frac{dv}{dt} = -4 + \frac{15}{2} t^{1/2}$$

$$\frac{ds}{dt} = 2 - 4t + 5t^{3/2}$$

$$\int_{s_o=3}^{s} ds = \int_{0}^{t} (2 - 4t + 5t^{3/2})\, dt$$

$$s = 3 + 2t - 2t^2 + 2t^{5/2}$$

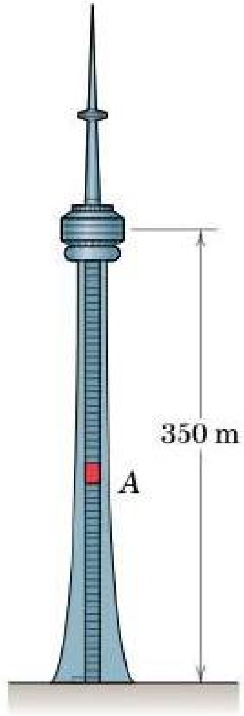
At $t = 3s$:

$$\begin{cases} s = 22.2\ m \\ v = 15.98\ m/s \\ a = 8.99\ m/s^2 \end{cases}$$

Rectilinear Motion

Example 2:

The main elevator A of the CN Tower in Toronto rises about 350 m and for most of its run has a constant speed of 22 km/h. Assume that both the acceleration and deceleration have a constant magnitude of $\frac{1}{4}g$ and determine the time duration t of the elevator run.



Solution:

Acceleration period:

$v = v_0 + at$: $\frac{22}{3.6} = 0 + \frac{9.81}{4} t_a$, $t_a = 2.49\ s$

Note that the deceleration time $t_d = t_a$

$v^2 = v_0^2 + 2a\Delta s$: $\left(\frac{22}{3.6}\right)^2 = 0^2 + 2\,\frac{9.81}{4}\,\Delta s_a$

$\Delta s_a = 7.61\ m = \Delta s_d$

Cruise period: $\Delta s_c = 350 - \Delta s_a - \Delta s_d = 335\ m$

$\Delta s = v_c t_c$: $335 = \frac{22}{3.6} t_c$, $t_c = 54.8\ s$

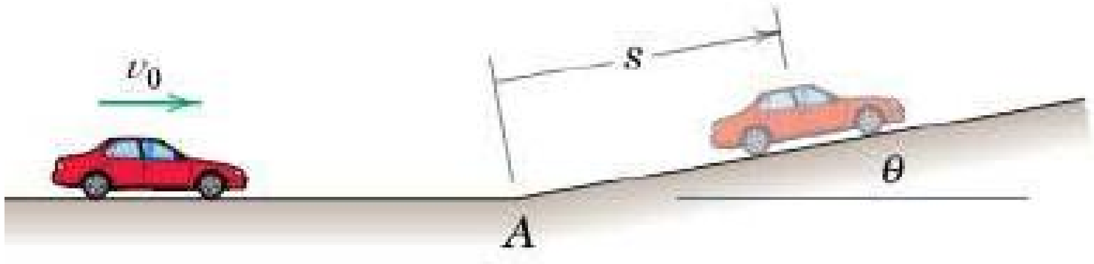
Total run time $t = t_c + t_a + t_d = \underline{59.8\ s}$

Rectilinear Motion

H.W 1:

The car traveling at a constant speed $v_o = 100\,\text{km/h}$ on the level portion of the road. When the 6-percent ($\tan\theta = 6/100$) incline is encountered, the driver does not change the throttle setting and consequently the car decoration at the constant rate g $\sin\theta$. Determine the speed of the car (a) 10 seconds after passing point A and (b) when $s = 100$m.

Ans. (*a*) $v = 21.9$ m/s, (*b*) $v = 25.6$ m/s



H.W 2: Solve problems: 2/5, 2/10, 2/44 in the Book (Meriam 6th Edition).

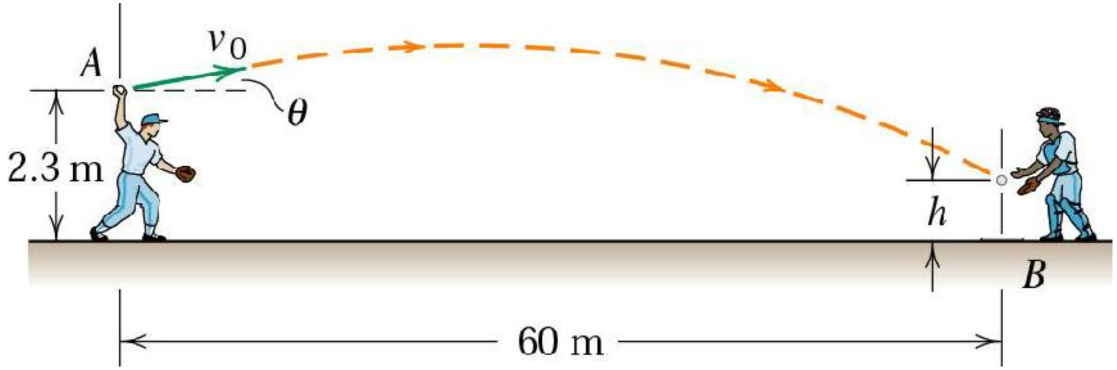
2/3-6 Plane Curvilinear Motion

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

Plane Curvilinear Motion

= Motion in a plane (2 dimensions)



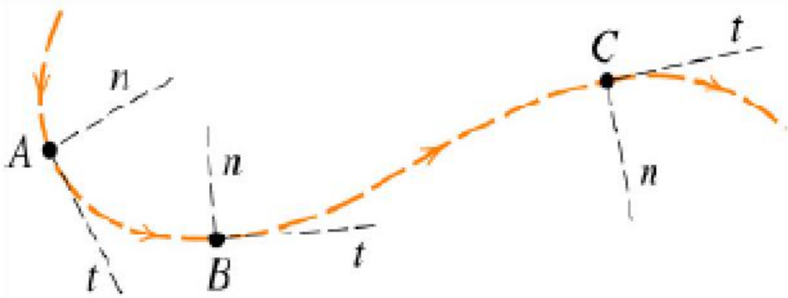
1. Rectangular Coordinates (*x*-*y*)
2. Normal and Tangential Coordinates (*n*-*t*)
3. Polar Coordinates (*r*-*θ*)

Plane Curvilinear Motion

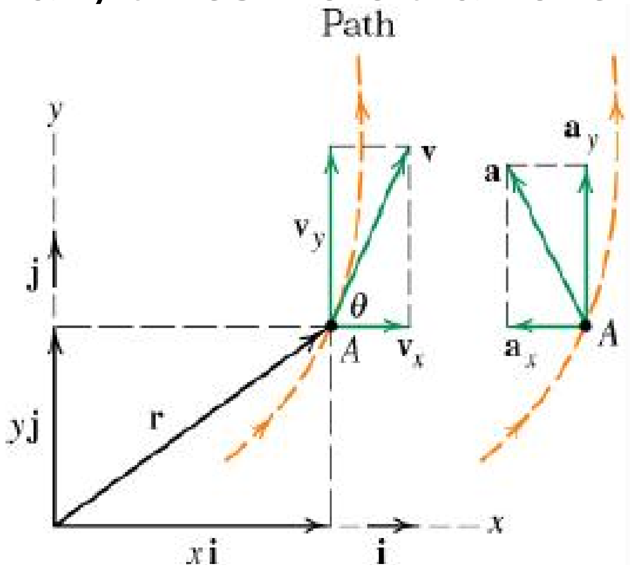
1. Rectangular Coordinates (*x*-*y*)

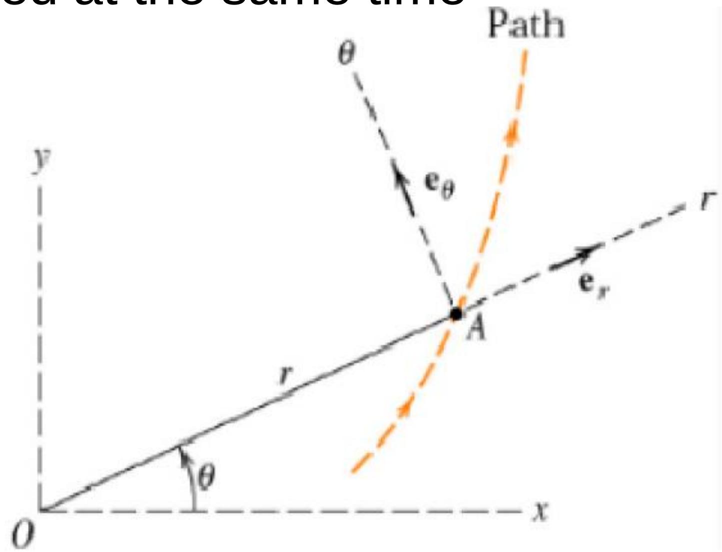
2. Normal and Tangential Coordinates (*n*-*t*)

3. Polar Coordinates (*r*-*θ*)



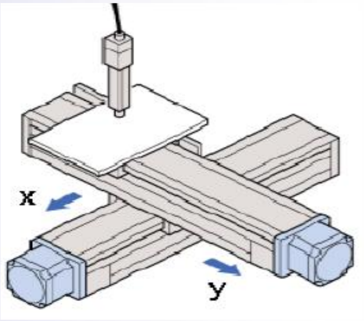
Notes: Usage will depend on the situation.
Usually, more than one system can be used.
Many times more than one system is needed at the same time



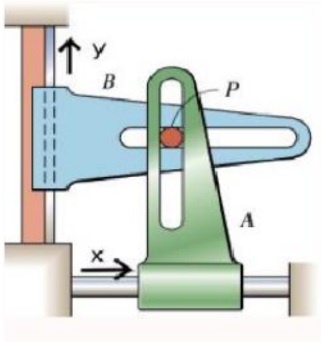


2/4 Rectangular Coordinate (*x*-*y*)

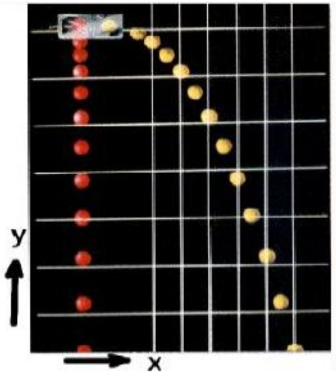
Applications:



Motorized x-y table

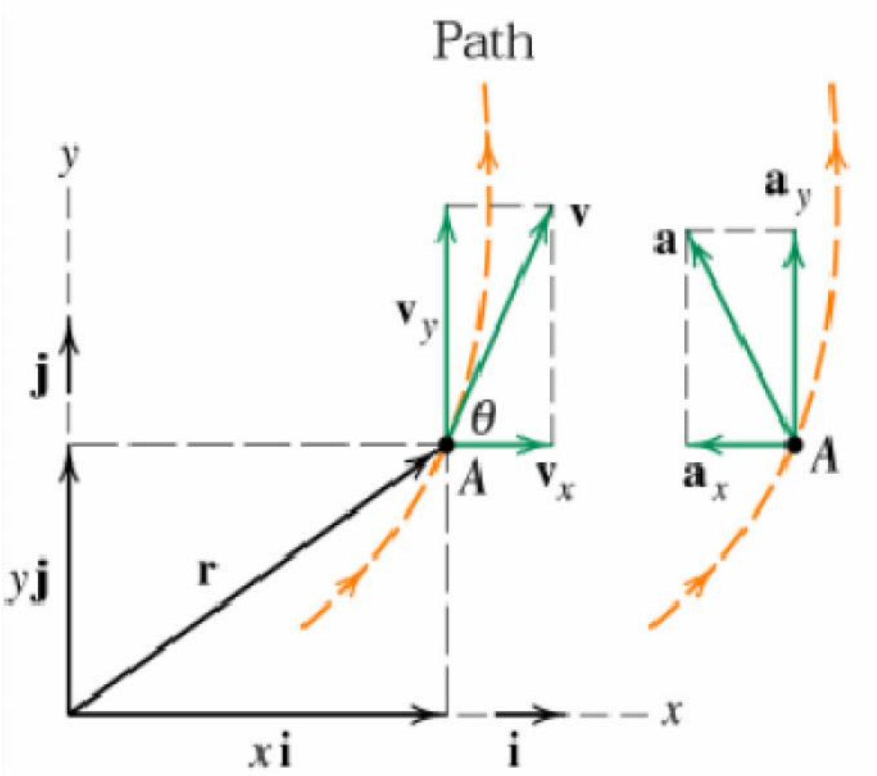


A model of the x-y table



Projectile Motion

1. Rectangular Coordinates (x-y)



Position vector: $\vec{r} = x\hat{i} + y\hat{j}$

Velocity vector: $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} = v_x\hat{i} + v_y\hat{j}$

Acceleration vector: $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} = \dot{v}_x\hat{i} + \dot{v}_y\hat{j}$

Magnitude & Direction

- Pythagoras

$$r = \sqrt{x^2 + y^2}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$a = \sqrt{a_x^2 + a_y^2}$$

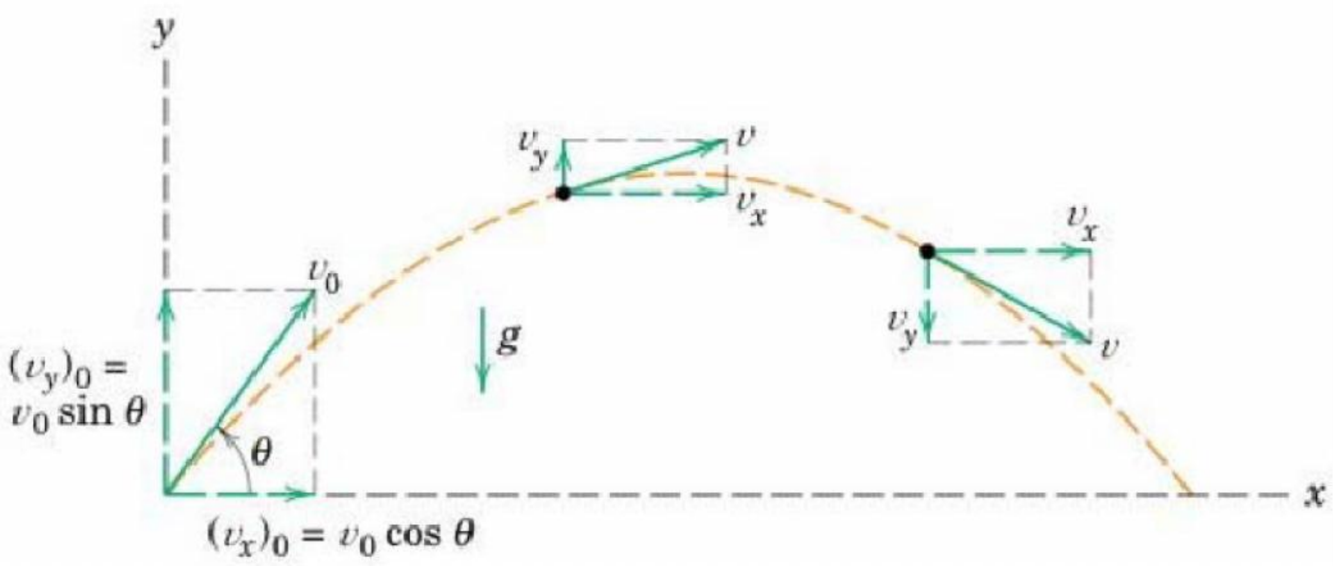
- Trigonometry (sine and cosine laws, etc.)

eg. $$\tan\theta = \frac{v_y}{v_x}$$

1. Rectangular Coordinates (x-y)

Projectile Motion

For the shown axis, it can use the laws of motion with constant acceleration in the projectile motion application as follows:



$a_x = 0 \qquad a_y = -g$

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - gt$$

$$x = x_0 + (v_x)_0 t \qquad y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

$$v_y^2 = (v_y)_0^2 - 2g(y - y_0)$$

Note1: if the projectile is directed upward then:

$$a_y = -g$$

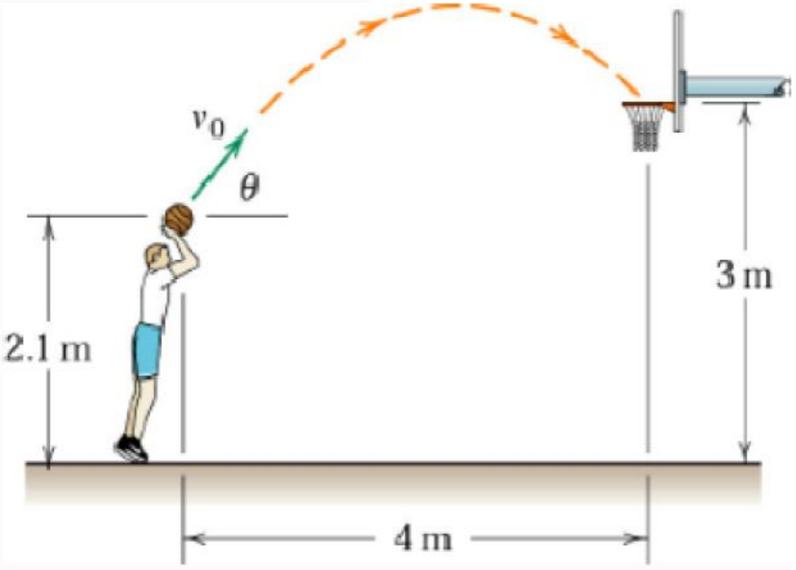
Else if it directed downward then: $a_y = g$

Note2: $v_x = (v_x)_0$ always const. then: $a_x = 0$

1. Rectangular Coordinates (x-y)

Example 1:

The basketball player likes to release his foul shots at an angle $\theta = 50°$ to the horizontal as shown. What initial speed v_0 will cause the ball to pass through the center of the rim?



Solution:

Use x–y coordinates with origin at the release point : y, x

$x = x_0 + v_{x_0} t$ @ hoop :

$$4 = 0 + (v_0 \cos 50°) t_f$$

$$t_f = 6.22 / v_0$$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ hoop :

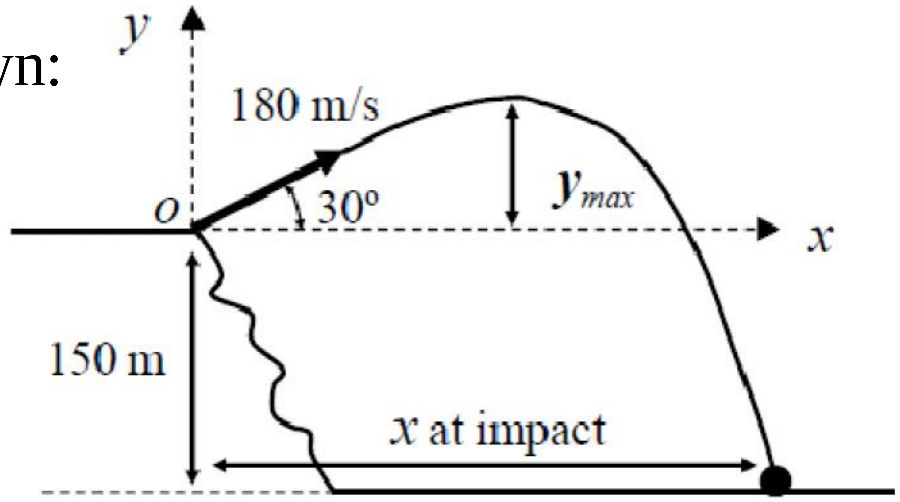
$$3 = 2.1 + v_0 \sin 50° \left(\frac{6.22}{v_0}\right) - \frac{1}{2} g \left(\frac{6.22}{v_0}\right)^2$$

$v_0 = 7$ m/s

1. Rectangular Coordinates (x-y)

Example 2:

Given: Projectile fired off a cliff as shown:



Find: x at impact and y_{max}

Solution:

Time of flight from y motion:

$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$

$-150 = 0 + (180 \sin 30) t - \frac{1}{2}(9.81) t^2$

$t = 19.91\ s$

Now solve for x_{impact}:

$x = x_0 + (v_x)_0 t$

$x_{impact} = 0 + (180 \cos 30)(19.91)$

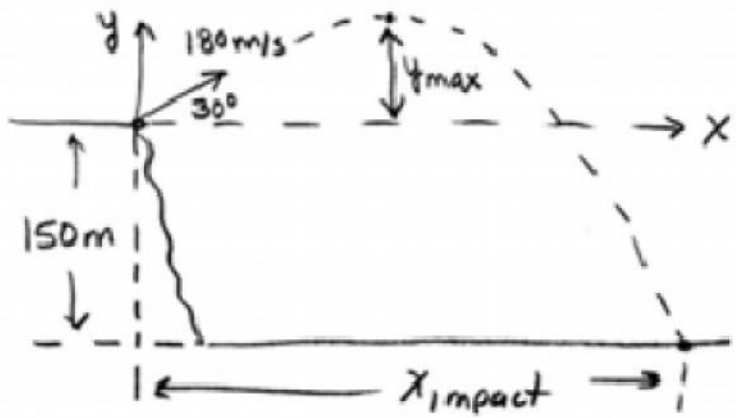
$\boxed{x_{impact} = 3100\ m}$

y_{max} occurs where $v_y = 0$

$v_y^2 = (v_y)_0^2 - 2g(y - y_0)$

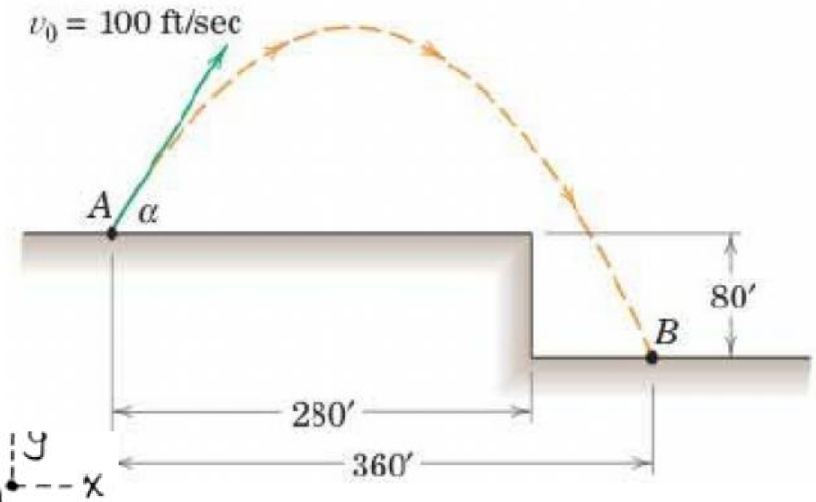
$0^2 = (180 \sin 30)^2 - 2(9.81)(y_{max} - 0)$

$\boxed{y_{max} = 413\ m}$



1. Rectangular Coordinates (x-y)

Example 3:



A projectile is launched from point A with an initial speed shown. Determine the minimum value of the launch angle α for which the projectile will land at point B.

Solution:

With x-y coordinates, origin at A: y, A, x

$x = x_0 + v_{x_0} t$ @ B: $\quad 360 = 0 + (100 \cos\alpha) t_f \quad (1)$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $\quad -80 = 0 + (100 \sin\alpha) t_f - \frac{1}{2}(32.2) t_f^2 \quad (2)$

Simultaneous solutions of (1) & (2):

$$\begin{cases} t_f = 4.03 \text{ sec}, & \alpha = 26.8^\circ \quad (a) \\ t_f = 5.68 \text{ sec}, & \alpha = 50.7^\circ \quad (b) \end{cases}$$

Check at corner $[(x, y) = (280', 0)]$:

(a) $t_c = \dfrac{280}{100 \cos 26.8^\circ} = 3.14$ sec

$y_c = 100 \sin 26.8^\circ (3.14) - \dfrac{32.2}{2}(3.14)^2 = -16.94$ ft

So conditions (a) are not possible.

(b) $t_c = \dfrac{280}{100 \cos 50.7^\circ} = 4.42$ sec

$y_c = 100 \sin 50.7^\circ (4.42) - \dfrac{32.2}{2}(4.42)^2 = 27.5$ ft

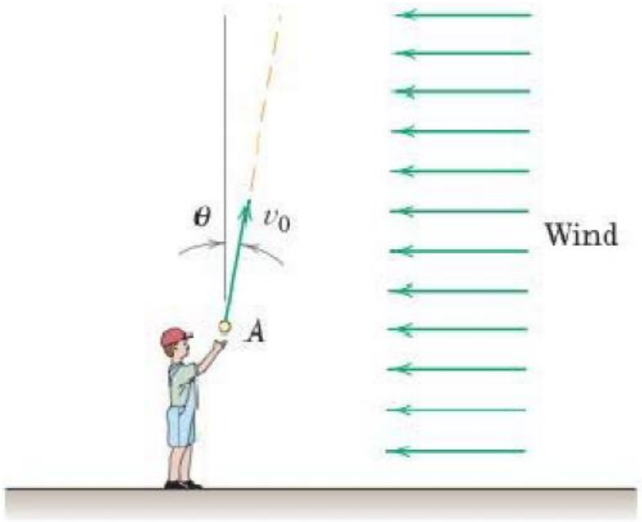
Conditions (b) result in clearance at corner

Ans. $\underline{\alpha = 50.7^\circ}$

1. Rectangular Coordinates (x-y)

Example 4:

A boy throws a ball upward with a speed $v_0 = 12$ m/s. The wind imparts a horizontal acceleration of 0.4 m/s^2 to the left. At what angle θ must the ball be thrown so that it returns to the point of release? Assume that the wind does not affect the vertical motion.



Solution:

$v_{x_0} = v_0 \sin\theta = 12 \sin\theta$

$v_{y_0} = v_0 \cos\theta = 12 \cos\theta$

$v_y = v_{y_0} - gt$ applied at end of flight:

$-12\cos\theta = 12\cos\theta - 9.81 t_f \;, \quad t_f = 2.45\cos\theta$

$v_x = v_{x_0} - 0.4t$ applied at end of flight:

$-12\sin\theta = 12\sin\theta - 0.4(2.45\cos\theta)$

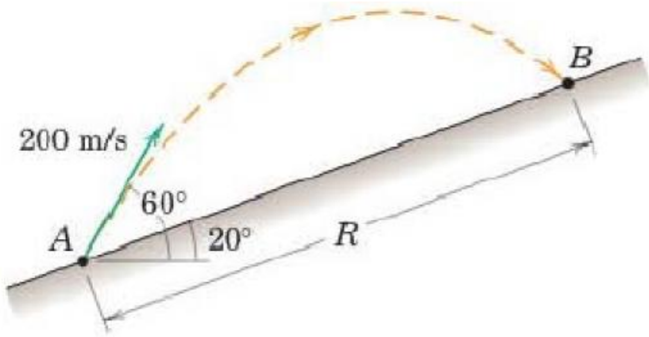
$24\sin\theta = 0.979\cos\theta \;, \quad \tan\theta = 0.041$

$\underline{\theta = 2.33^\circ}$

H.W1:

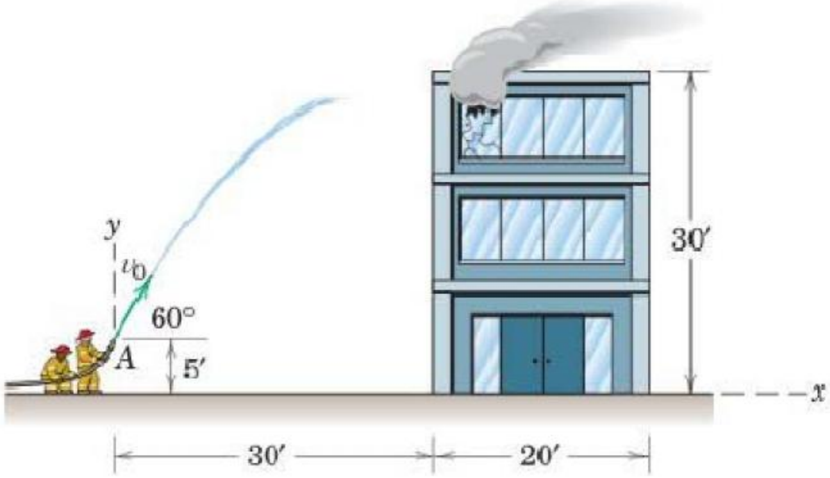
A projectile is launched with an initial speed of 200 m/s at an angle of 60° with respect to the horizontal. Compute the range R as measured up the incline.

Ans. $R = 2970$ m



H.W 2:

Water issues from the nozzle at A, which is 5 ft above the ground. Determine the coordinates of the point of impact of the stream if the initial water speed is (a) $v_0 = 45$ ft/sec and (b) $v_0 = 60$ ft/sec.



Ans. (a): *(x,y)=(30 ft,28.3 ft)*

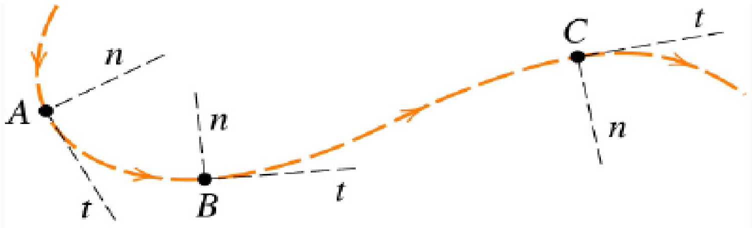
(b): *(x,y)=(99.6 ft,0)*

2/5 Normal and Tangential Coordinate (*n*-*t*)

2. Normal And Tangential Coordinate (*n*-*t*)

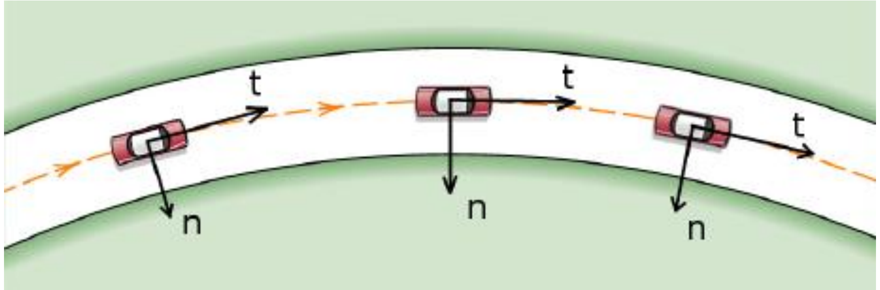
- Introduction
- Velocity
- Acceleration
- Special Case: Circular Motion
- Examples

2. Normal And Tangential Coordinate (*n*-*t*)



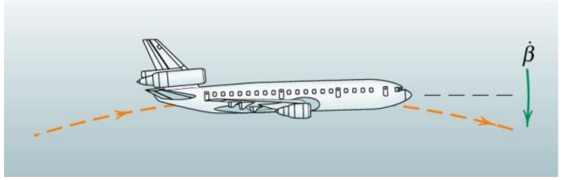
- Most convenient when position, velocity, and acceleration are described relative to the path of the particle itself
- Origin of this coordinate moves with the particle (Position vector is zero)
- The coordinate axes rotate along the path
  - *t* coordinate axis is **tangential** to the path and points to the direction of **positive** velocity.
  - *n* coordinate axis is **normal** to the path and points **toward center** of curvature of the path.

2. Normal And Tangential Coordinate (*n*-*t*)



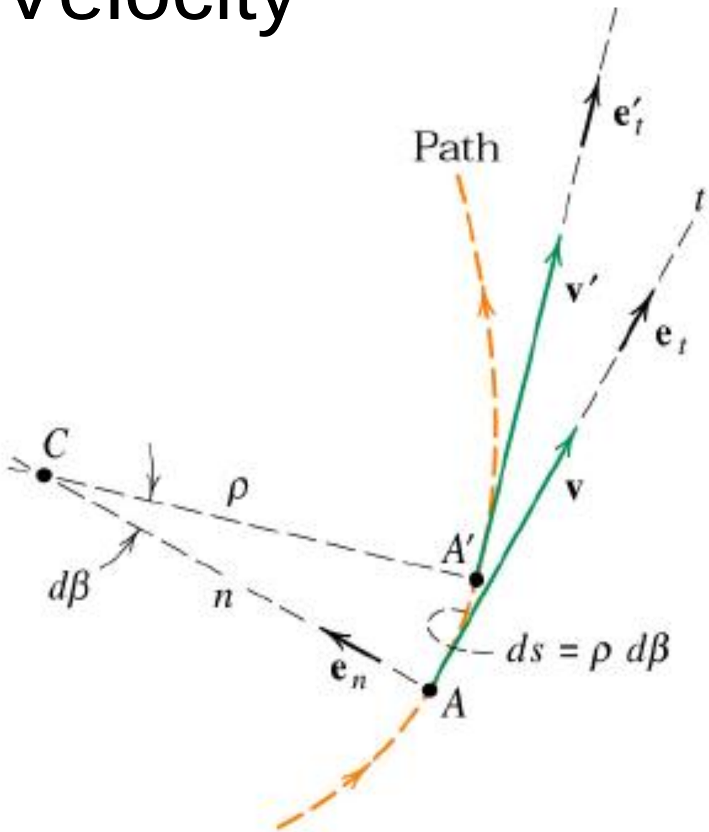
Applications

- Moving car
  - Forward/backward velocity and forward/backward/lateral acceleration make more sense to the driver.
  - Brake and acceleration forces are often more convenient to describe relative to the car (in the *t* direction)
  - Turning (side) force also easier to describe relative to the car (in the *n* direction)



2. Normal And Tangential Coordinate (*n*-*t*)

Velocity



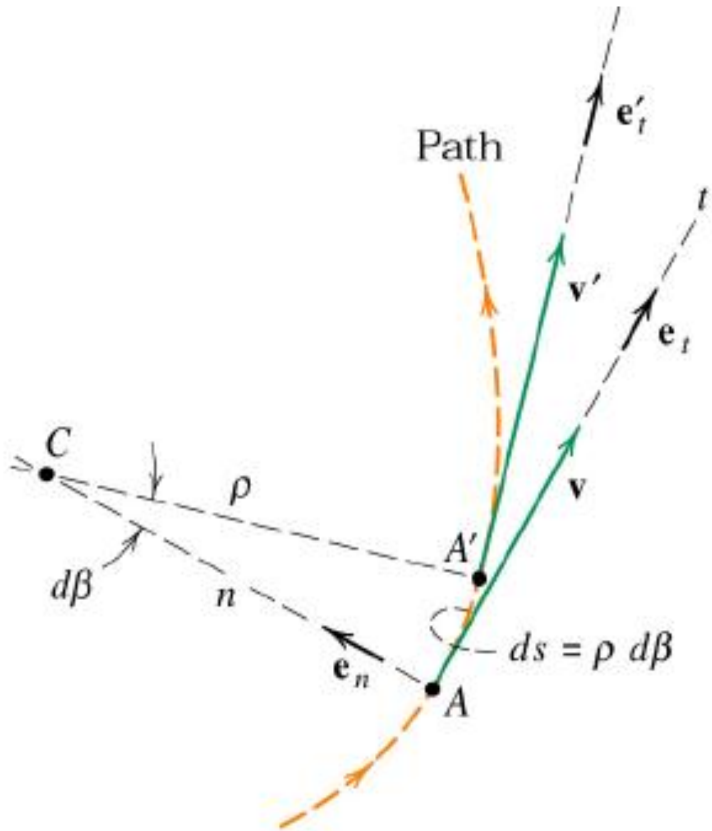
- For a short period of time, *dt*
- Path from *A* to *A′* can be approximated as an arc of a circle
- The center of the circle is at *C*, the **center of curvature**.
- The radius of this circle is call the **radius of curvature**, ρ

Notes:

- The center of curvature *C* can move
- Radius of curvature ρ is not constant

2. Normal And Tangential Coordinate (*n*-*t*)

Velocity



- During dt, $\hat{e}_n$ rotated $d\beta$
- Distance travelled is $ds = \rho d\beta$
- Recall that $\vec{v}$ is tangent to the path and that $v = ds/dt$

Velocity (*n*-*t*)

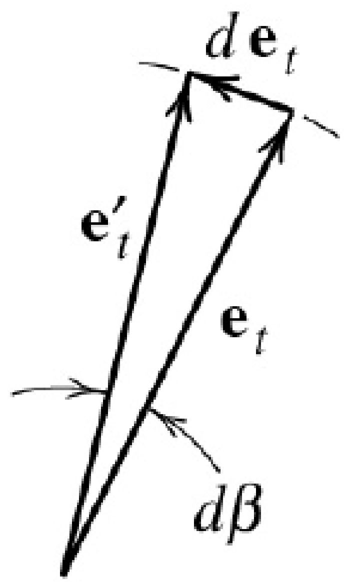
$$\vec{v} = v\,\hat{e}_t = \rho\dot{\beta}\,\hat{e}_t$$

2. Normal And Tangential Coordinate (*n*-*t*)

Acceleration

- $\vec{a} = d\vec{v}/dt \ = v\dfrac{d\,\hat{e}_t}{dt} + \dot{v}\,\hat{e}_t$
- Now we need $\frac{d\,\hat{e}_t}{dt}$
- From the figure, $\hat{e}_t$ changes $d\beta$ in dt

$$d\,\hat{e}_t = |\hat{e}_t| \times d\beta\hat{e}_n$$



Derivative of $\hat{e}_t$

$$\frac{d\hat{e}_t}{dt} = \dot{\beta}\,\hat{e}_n$$

Acceleration (*n*-*t*)

$$\vec{a} = \frac{v^2}{\rho}\,\hat{e}_n + \dot{v}\,\hat{e}_t$$

$$a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta} > 0$$

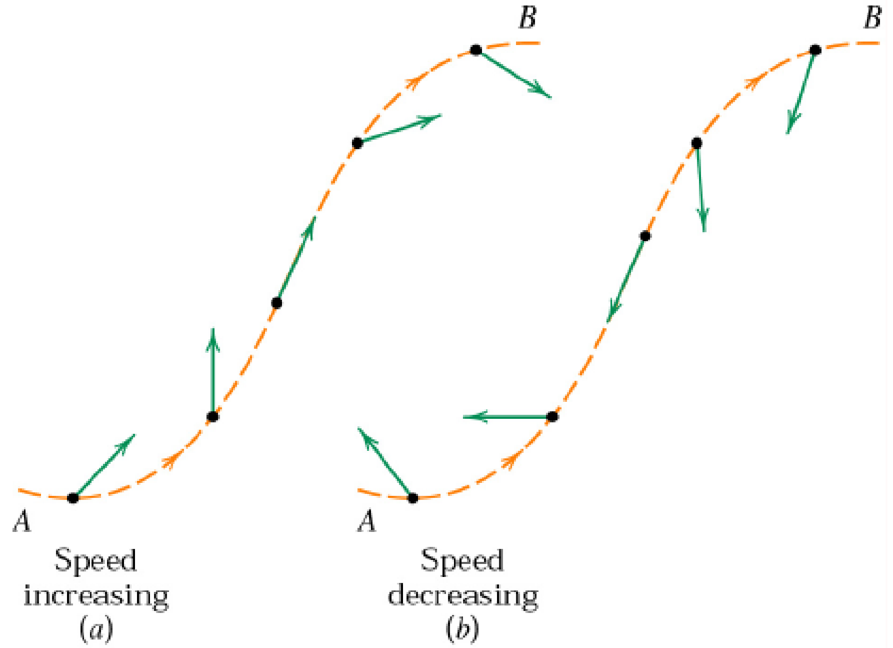
$$a_t = \dot{v} = \ddot{s}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

2. Normal And Tangential Coordinate (*n*-*t*)

Acceleration

Directions of a_t and a_n



The arrows show the **acceleration** of a particle is moving from A to B

If speed is increasing a_t // **v** // $\mathbf{e}_t$

If speed is decreasing a_t // - **v** // - $\mathbf{e}_t$

a_n is always directed **toward** the center of curvature

2. Normal And Tangential Coordinate (*n-t*)

v and a_t

The formula for the velocity/acceleration in the t direction is the same as those of rectilinear motion.

$$a_t = \dot{v} = \ddot{s}$$ = change in speed

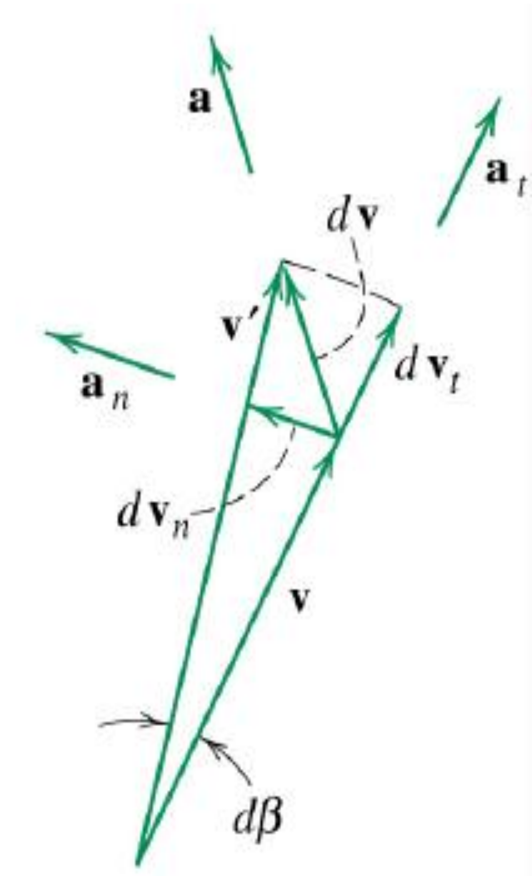
$$v = \frac{ds}{dt}$$

$$a_t = \frac{dv}{dt}$$

$$a_t \, ds = v \, dv$$

2. Normal And Tangential Coordinate (*n*-*t*)

Geometric representation

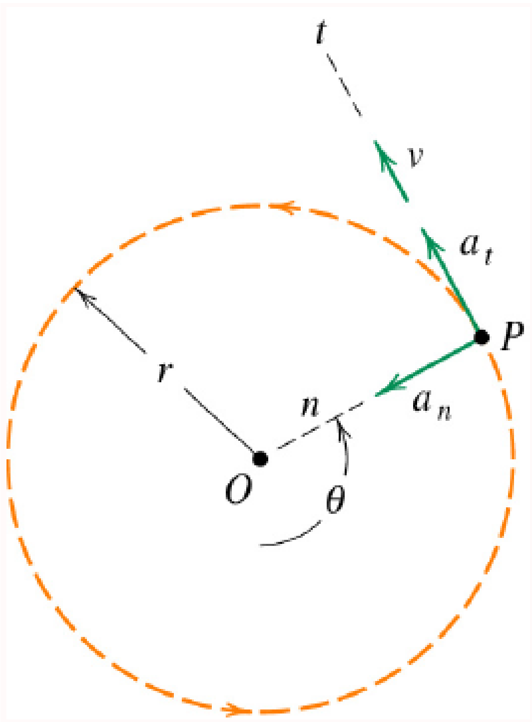


- a_n is a result of change in the magnitude of $\vec{v}$

- a_t is a result of change in the direction of $\vec{v}$

2. Normal And Tangential Coordinate (*n*-*t*)

Special Case: Circular motion



- Radius of curvature ρ becomes constant radius r
- β is an angle θ from any reference to $\overline{OP}$

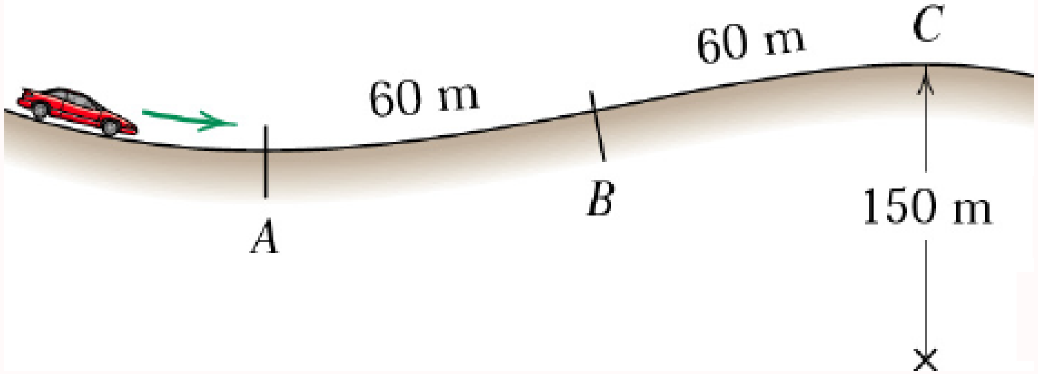
Circular Motion (*n*-*t*)

$$
\begin{aligned}
v &= r\dot{\theta} \\
a_n &= v^2/r = r\dot{\theta}^2 = v\dot{\theta} \\
a_t &= \dot{v} = r\ddot{\theta}
\end{aligned}
$$

2. Normal And Tangential Coordinate (*n*-*t*)

Example 1: Car on a hill

To anticipate the dip and hump in the road, the driver of a car applies her brakes to produces a uniform deceleration. Her speed is 100 km/h at the bottom A of the dip and 50 km/h at the top C of the hump, which is 120 m along the road from A. If the passengers experience a total acceleration of 3 m/s^2 at A and if the radius of curvature of the hump at C is 150 m, calculate (a) the radius of curvature ρ at A, (b) the acceleration at the inflection point B, and (c) the total acceleration at C.



Ans: $\rho_A = 432\,\mathrm{m}$

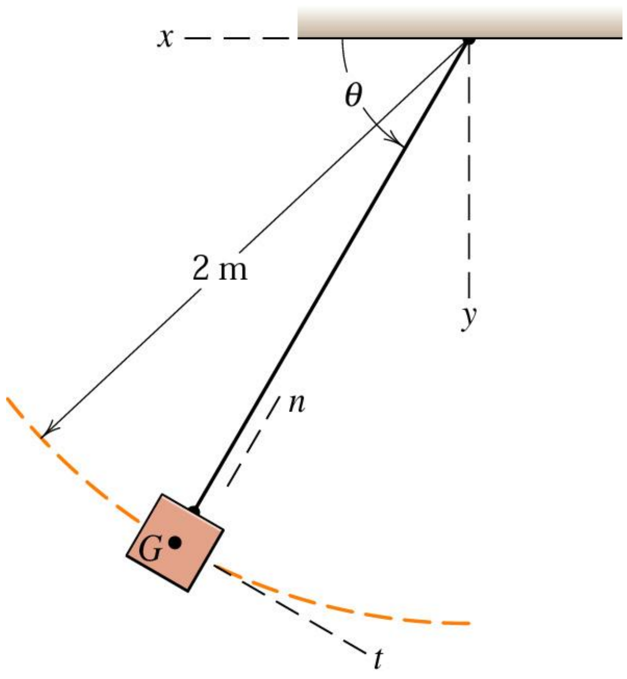
$a_B = 2.41\,\mathrm{m/s^2}$ in -t direction

$\vec{a} = 1.286\,\hat{e}_n - 2.41\,\hat{e}_t\ \mathrm{m/s^2}$

2. Normal And Tangential Coordinate (*n-t*)

Example 2: Pendulum

Write the vector expression of the acceleration **a** of the mass center G of the simple pendulum in both *n-t* and *x-y* when $\theta = 60^\circ, \dot{\theta} = 2 \text{ rad/s and } \ddot{\theta} = 2.45 \text{ rad/s}^2$

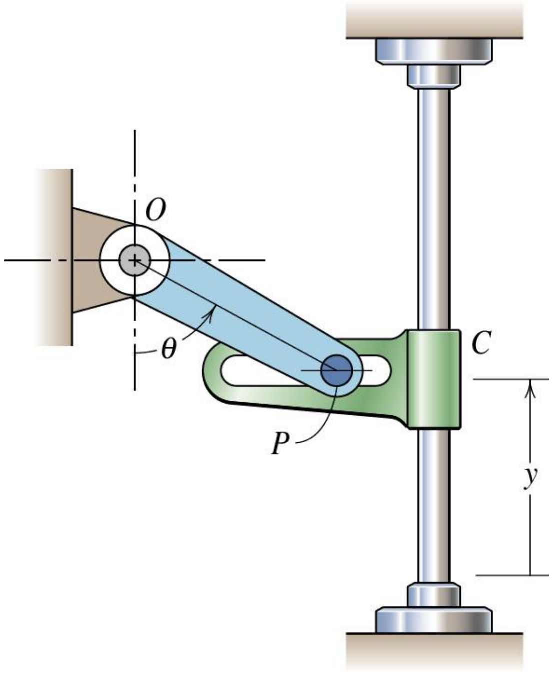


2. Normal And Tangential Coordinate (*n-t*)

Example 3: Crank and Slot

Pin P in the crank PO engages the horizontal slot in the guide C and controls its motion on the fixed vertical rod. Determine the velocity and the acceleration of the guide C if a) $\dot{\theta} = \omega \quad \ddot{\theta} = 0$

b) $\dot{\theta} = 0 \quad \ddot{\theta} = \alpha$



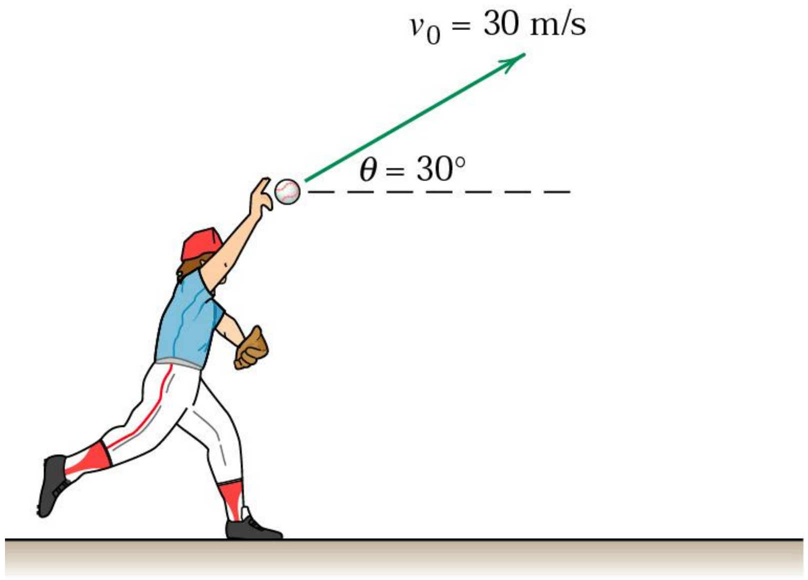
Ans: a) $\dot{y} = r\omega \sin\theta \qquad \ddot{y} = r\omega^2 \cos\theta$

b) $\dot{y} = 0 \qquad \ddot{y} = r\alpha \sin\theta$

2. Normal And Tangential Coordinate (*n*-*t*)

Example 4: Baseball

A baseball player releases a ball with the initial conditions shown. Determine the radius of curvature of the trajectory a) just after release and b) at the apex. For each case, compute the time rate of change of the speed.



Ans: a) 105.9 m, -4.91 m/s^2

b) 68.8 m, 0 m/s^2

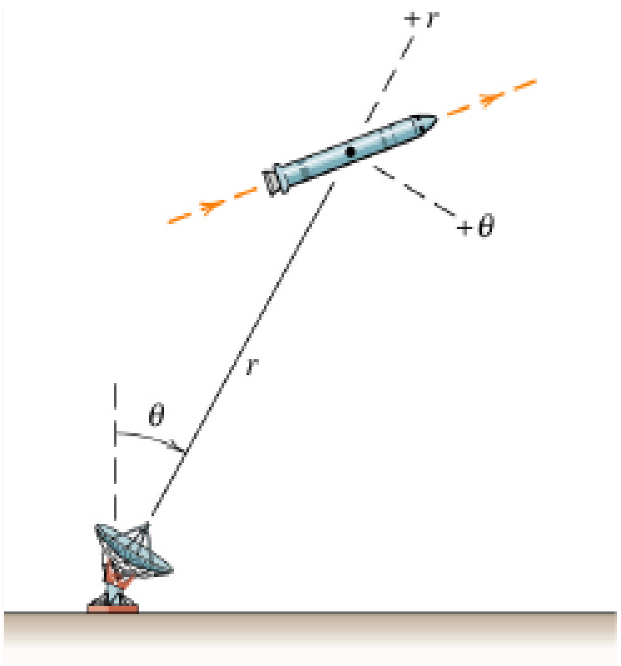
2/6 Polar Coordinates (r-θ)

3. Polar Coordinates (r-θ)

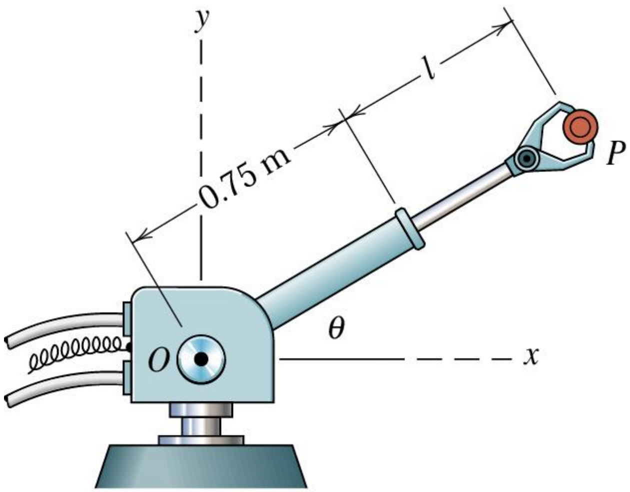
- Position
- Time derivative of unit vectors: $\frac{d\hat{e}_r}{dt}$ and $\frac{d\hat{e}_\theta}{dt}$
- Velocity
- Acceleration
- Special Case: Circular Motion
- Examples

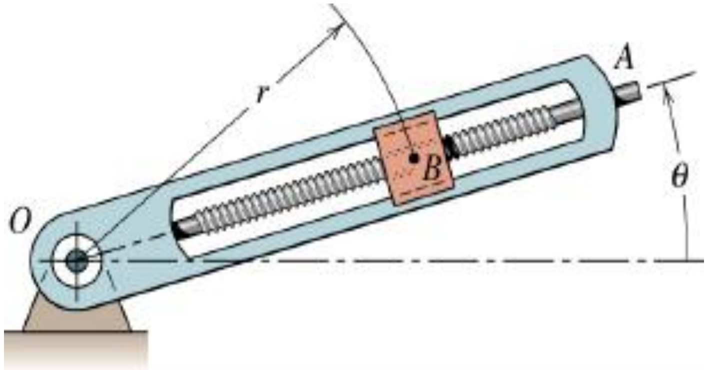
3. Polar Coordinates (*r*-*θ*)

Applications



Radar tracking

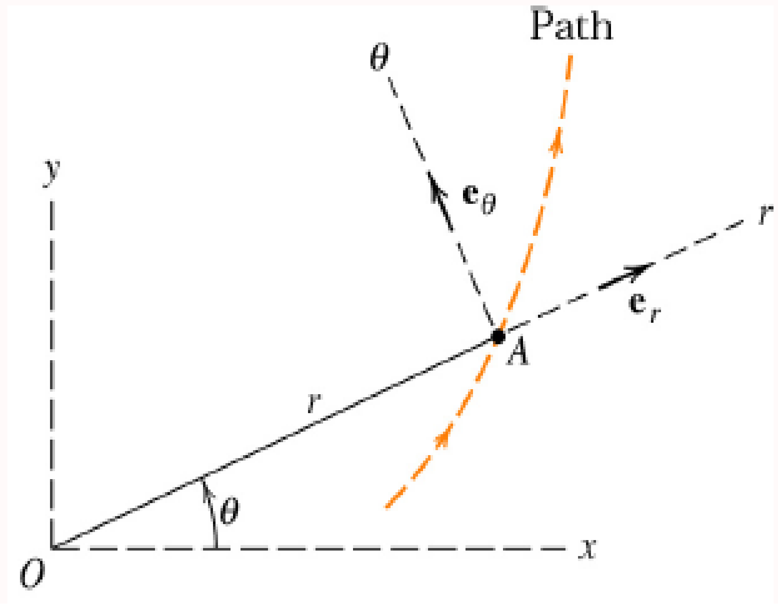




Robot Manipulator

3. Polar Coordinates (r-θ)

Position Vector



- Define a reference frame then the define r and θ
- The position vector

$$\vec{r} = r\,\hat{e}_r$$

- $\hat{e}_r$ is the unit vector in the direction of $\vec{r}$

$$\hat{e}_r = \cos(\theta)\,\hat{i} + \sin(\theta)\,\hat{j}$$

- $\hat{e}_\theta$ is the unit vector in the θ direction

$$\hat{e}_\theta = -\sin(\theta)\,\hat{i} + \cos(\theta)\,\hat{j}$$

3. Polar Coordinates (*r*-*θ*)

Velocity

$$\vec{v} = \frac{d(r\hat{e}_r)}{dt} = \dot{r}\,\hat{e}_r + r\frac{d\hat{e}_r}{dt}$$

Time derivative of unit vectors

$$\hat{e}_r = \cos(\theta)\,\hat{i} + \sin(\theta)\,\hat{j}$$

$$\begin{aligned} \frac{d\hat{e}_r}{dt} &= \frac{d}{dt}(\cos(\theta)\,\hat{i} + \sin(\theta)\,\hat{j}) \\ &= -\sin(\theta)\dot{\theta}\,\hat{i} + \cos(\theta)\dot{\theta}\,\hat{j} \end{aligned}$$

$$\boxed{\frac{d}{dt}\hat{e}_r = \dot{\theta}\hat{e}_\theta}$$

note that $\frac{d\hat{i}}{dt} = 0$ and $\frac{d\hat{j}}{dt} = 0$

$$\hat{e}_\theta = -\sin(\theta)\,\hat{i} + \cos(\theta)\,\hat{j}$$

3. Polar Coordinates (r-θ)

Time derivative of unit vectors

$$\hat{e}_\theta = -\sin(\theta)\,\hat{i} + \cos(\theta)\,\hat{j}$$

$$\begin{aligned} \frac{d\hat{e}_\theta}{dt} &= \frac{d}{dt}(-\sin(\theta)\,\hat{i} + \cos(\theta)\,\hat{j}) \\ &= -\cos(\theta)\dot{\theta}\,\hat{i} - \sin(\theta)\dot{\theta}\,\hat{j} \end{aligned}$$

$$\frac{d}{dt}\hat{e}_\theta = -\dot{\theta}\hat{e}_r$$

note that $\frac{d\hat{i}}{dt} = 0$ and $\frac{d\hat{j}}{dt} = 0$

$$\hat{e}_r = \cos(\theta)\,\hat{i} + \sin(\theta)\,\hat{j}$$

3. Polar Coordinates (r-θ)

Velocity

$$\vec{v} = \frac{d(r\hat{e}_r)}{dt} = \dot{r}\,\hat{e}_r + r\frac{d\hat{e}_r}{dt}$$

Velocity (Polar)

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$v_r = \dot{r} \qquad v_\theta = r\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2}$$

Acceleration

$$\begin{aligned}
\vec{a} &= \frac{d}{dt}(\dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta) \\
&= \ddot{r}\,\hat{e}_r + \dot{r}\frac{d}{dt}\hat{e}_r + \dot{r}\dot{\theta}\,\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta + r\dot{\theta}\frac{d}{dt}\hat{e}_\theta \\
&= \ddot{r}\,\hat{e}_r + \dot{r}\dot{\theta}\,\hat{e}_\theta + \dot{r}\dot{\theta}\,\hat{e}_\theta + r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r
\end{aligned}$$

3. Polar Coordinates (r-θ)

Acceleration

Acceleration (Polar)

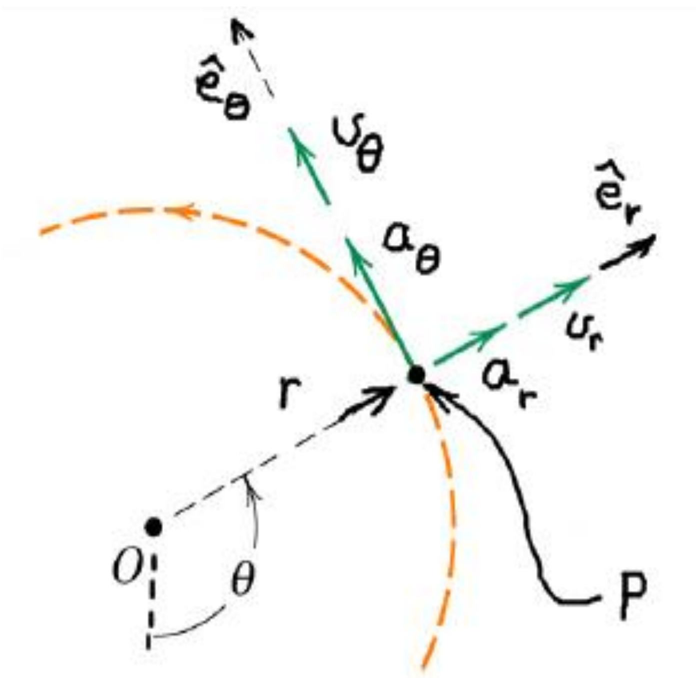
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$a = \sqrt{a_r^2 + a_\theta^2}$$

3. Polar Coordinates (r-θ)

Circular Motion



- $\hat{e}_r$ points from O toward P
- $\hat{e}_\theta$ perpendicular to $\hat{e}_r$ and toward positive θ
- Velocity

$$\begin{aligned} v_r &= \dot{r} = 0 \\ v_\theta &= r\dot{\theta} \end{aligned}$$

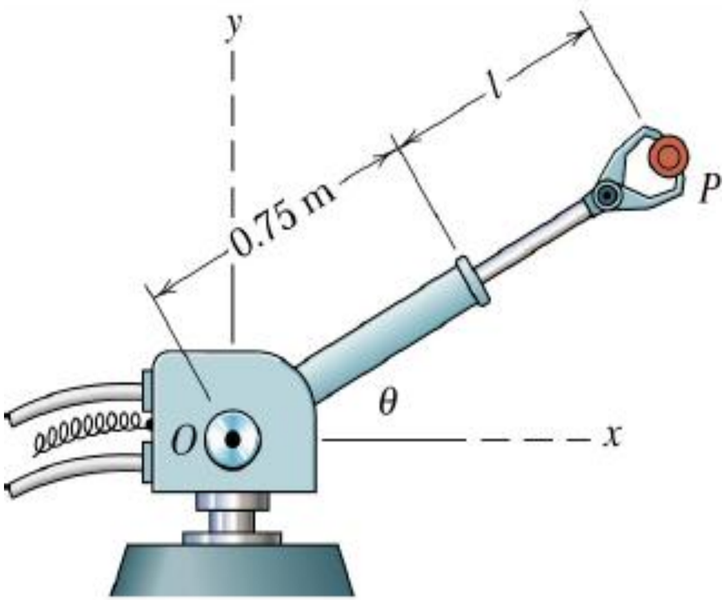
- Acceleration

$$\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = r\ddot{\theta} \end{aligned}$$

3. Polar Coordinates (r-θ)

Example 1: Robot Arm

The robot arm is elevating and extending simultaneously. At a given instant, $\theta = 30^\circ$, $\dot{\theta} = 10$ deg/s constant, $l = 0.5$ m, $\dot{l} = 0.2$ m/s, and $\ddot{l} = -0.3$ m/s^2. Compute the magnitude of the velocity, $\vec{v}$, and acceleration, $\vec{a}$, of the gripped part P. In addition, express $\vec{v}$ in terms of the unit vectors $\hat{\text{i}}$ and $\hat{\text{j}}$.



Ans:

$v_r = 0.2$ m/s

$v_\theta = 0.218$ m/s

$\vec{v} = 0.064\hat{\text{i}} + 0.289\hat{\text{j}}$m/s

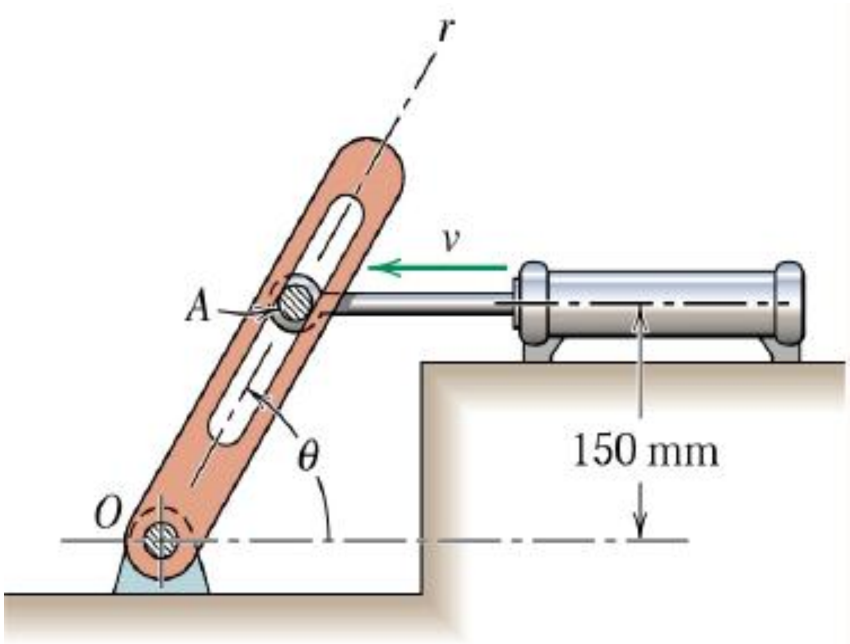
$a_r = -0.338$

$a_\theta = 0.07$

3. Polar Coordinates (*r*-*θ*)

Example 2: Hydraulic Cylinder

The piston of the hydraulic cylinder gives pin A a constant velocity v = 1.5 m/s in the direction shown for an interval of its motion. For the instant when θ = 60°, determine $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$, where $r = \overline{OA}$

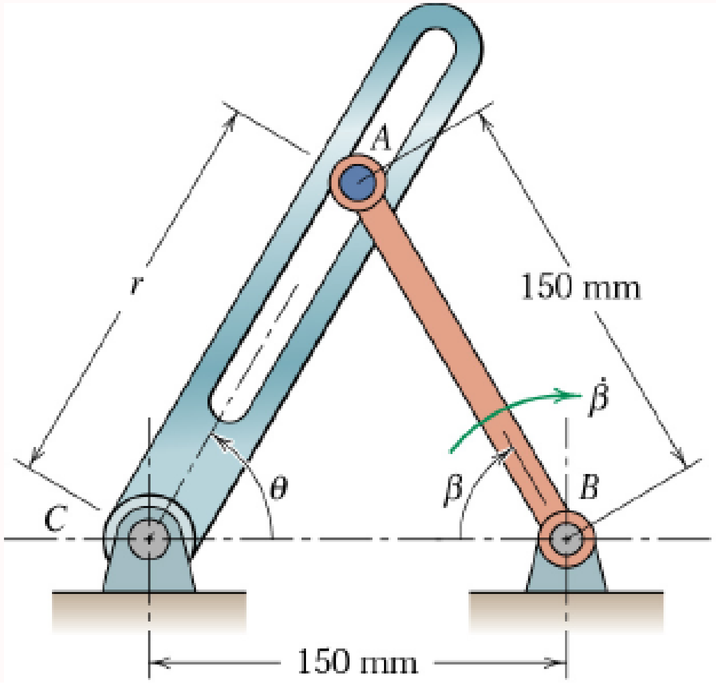


Ans: -0.75 m/s, 7.5 rad/s, 9.74 m/s^2, 65 rad/s^2

3. Polar Coordinates (r-θ)

Example 3: Two links

Link AB rotates through a limited range of the angle β, and its end A causes the slotted link AC to rotate also. For the instant represented where β = 60° and $\dot{\beta}$ = 0.6 rad/s constant, determine the corresponding values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$.



Ans: $\dot{r} = 0.078$ m/s $\dot{\theta} = -0.3$ rad/s

$\ddot{r}$ = -0.0135 m/s^2 and $\ddot{\theta}$ = 0

Three Coordinates (*x-y*, *n-t*, *r-θ*)

	x-y	***n-t***	***r-θ***
Origin	fixed	moving with particle	fixed
Unit vectors	fixed	moving with particle (tangent and normal to the path/velocity)	moving with particle (along and normal to the radius)
Applications	Projectile motion	Airplane, Car, Rocket	Radar, Satellite Dish, Slotted link, Robot arm, Cable
Keywords	Horizontal, vertical	Path, Radius of curvature, normal, tangential, change in speed	Radial, transverse

2/8 Relative Motion (translating axes)

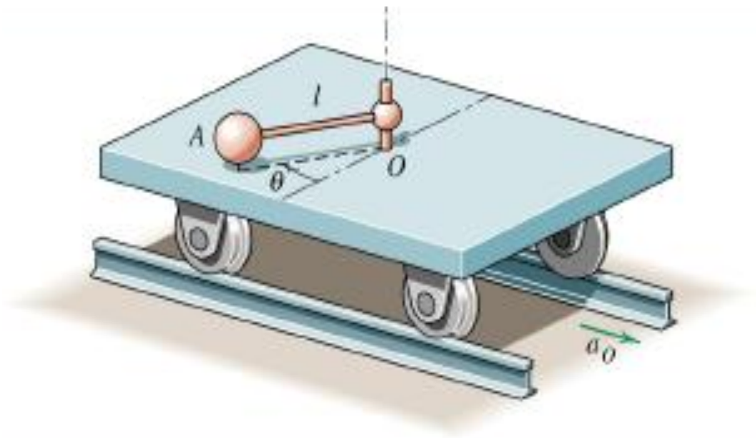
Relative Motion

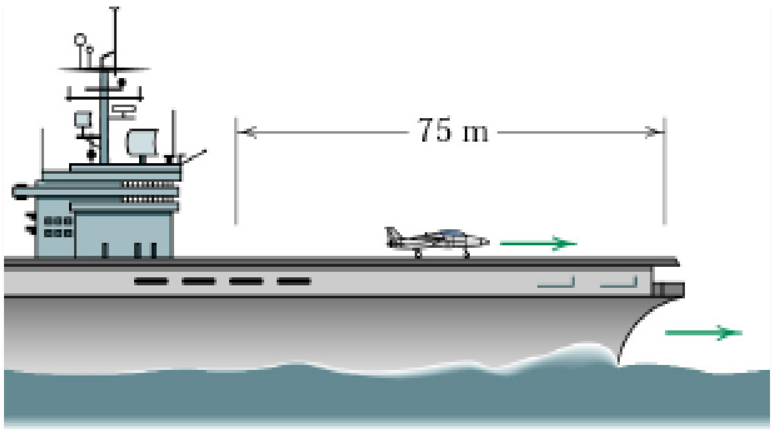
- Introduction
- Velocity and acceleration relation
- Choices of coordinates
- Examples

Relative Motion

- Absolute Motion
  - Motions relative to a **non-moving and non-rotating reference frame** is called **absolute motion**.
  - For engineering problems on earth, a reference frame **fixed on earth** is considered fixed (or not moving and not rotating).
  - A fixed **observer** on earth that is not moving and not rotating can be used to observe absolute motions of bodies.
- Relative Motion
  - However, many times motions are often easier to describe **relative** to a **moving reference frame** or **moving observer**.
  - Here we will look at motions relative to **translating reference frame**; i.e., moving but not rotating.

Relative Motion

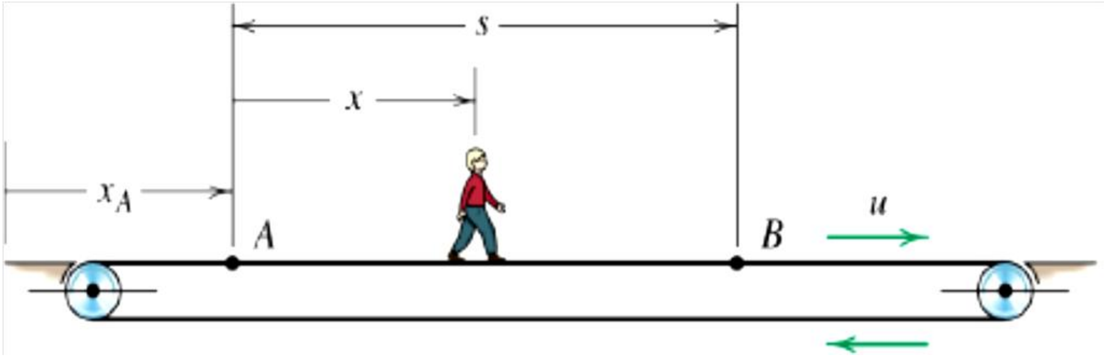




- Motion of A is easier to describe using a reference frame fixed to the carriage
- A moves in a circle relative to the carriage.

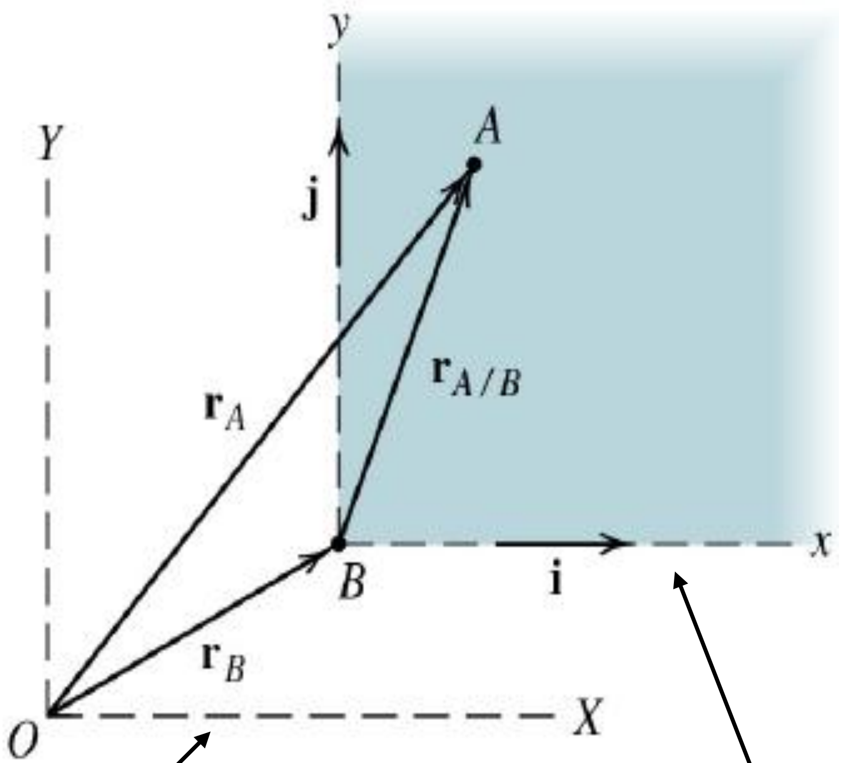
- Absolute velocity of the plane = the velocity of the plane relative to the ship + absolute velocity of the ship

Relative Motion



Suppose a man is walking on a really long moving walkway shown in the picture. The moving walkway as a constant speed of 0.5 m/s. The man is walking at a constant speed of 1 m/s relative to the walkway. Suppose he dropped his hat but did not notice. So, he kept walking. After 30 second, he found out that he dropped his hat. So, he turned back and started walking back to get this hat at the same speed as before. How long will it take for the man to walk back to his hat?

Relative Motion



- $\vec{r}_A$ is the absolute position vector of A
- $\vec{r}_B$ is the absolute position vector of B
- Let attach an observer at B fixed on yBx
- $\vec{r}_{A/B}$ is the relative position vector of A relative to yBx; i.e., relative to the observer at B.

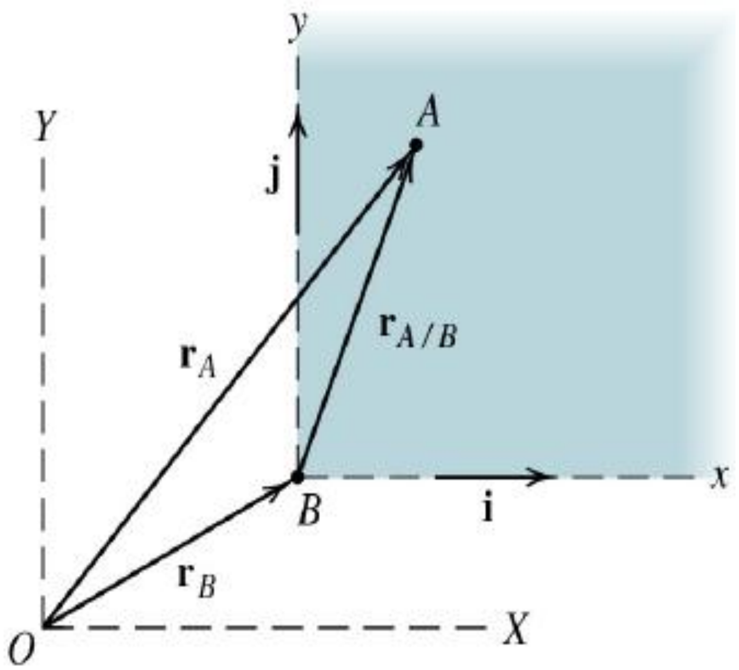
Fixed reference frame

Moving reference frame (translating only)

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

Relative Motion

v and **a** relationship



$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

Relative Velocity

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

Relative Acceleration

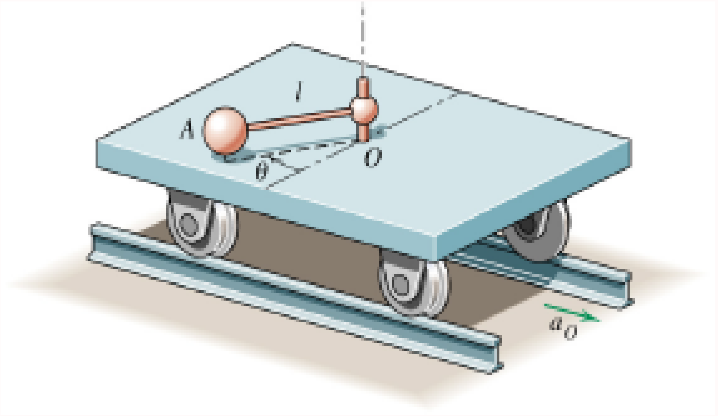
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

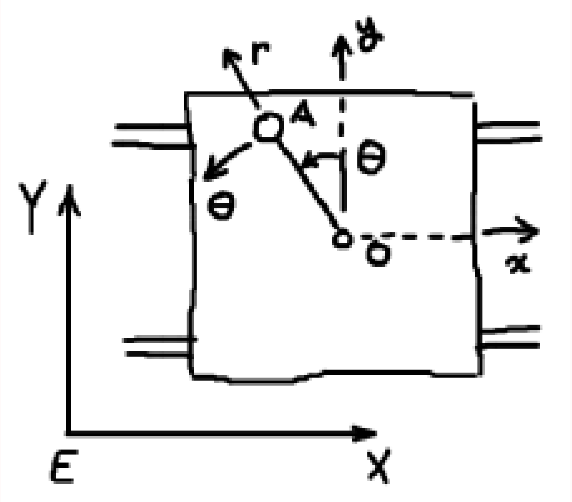
- $\vec{v}_{A/B}$ is the velocity of A as observed by B
- A/B will be used only when B is not rotating.

Relative Motion

Choices of coordinates

Any of the three coordinates can be used for the fixed frame or the translating frame (moving but not rotating).





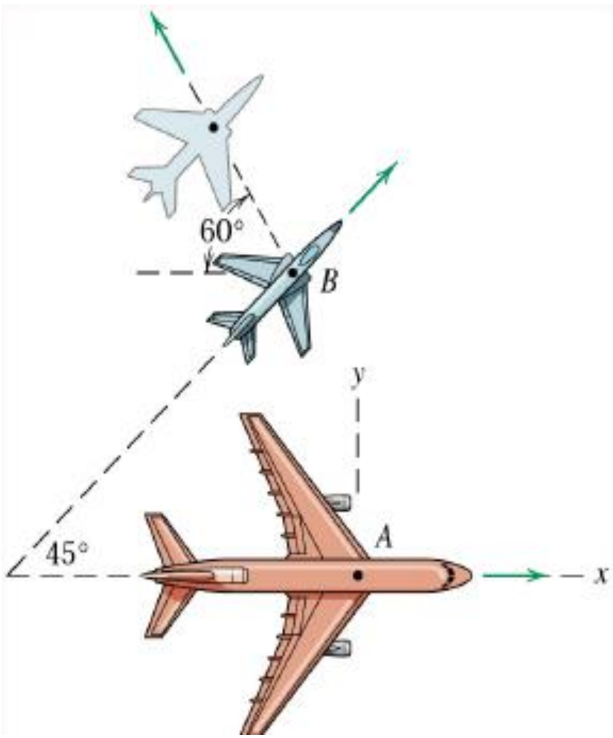
- *n*-*t* or *r*-*θ* for motion of A relative to O
- *x*-*y* for motion of the carriage

- Convenient to use *X*-*Y* to denote the fixed frame
- And, *x*-*y* for the moving frame

Relative Motion

Example 1: Two planes

Passengers in the jet transport A flying east at a speed of 800 km/h observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the 45° northeast direction, plane B appears to the passengers in A to be moving away from the transport at the 60° angle as shown. Determine the true velocity of B.

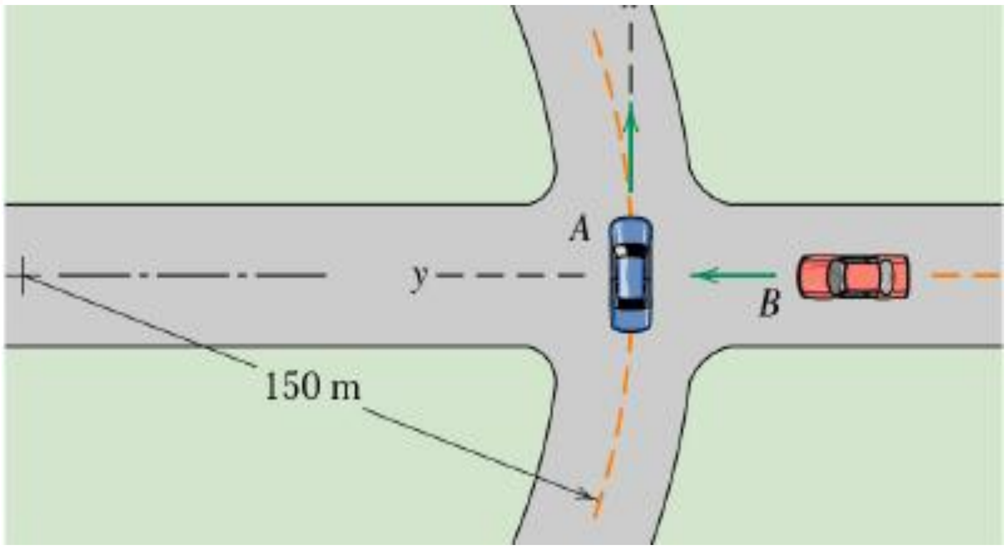


Ans: 717 km/h

Relative Motion

Example 2: Two cars

Car A rounds a curve of 150-m radius at a constant speed of 54 km/h. At the instant represented, car B is moving at 81 km/h but is slowing down at the rate of 3 m/s^2. Determine the velocity and acceleration of car A as observed from car B. [Extra] Find the curvature of A as observed from B.

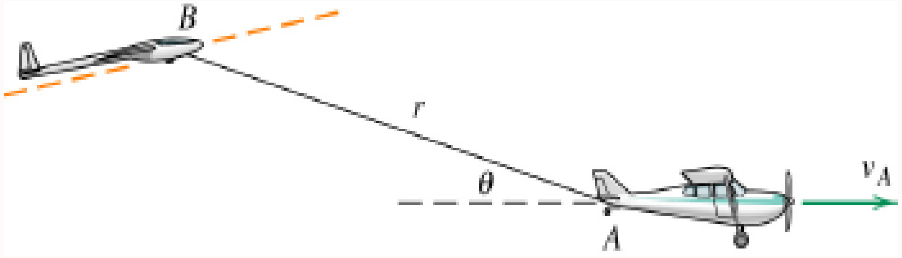


Ans: $v_{A/B} = 15i - 22.5j$ m/s, $a_{A/B} = 4.5j$ m/s^2

Relative Motion

Example 3: Two planes

Airplane A is flying horizontally with a constant speed of 200 km/h and is towing the glider B, which is gaining altitiude. If the tow cable has a length r = 60 m and θ is increasing at the constant rate of 5 degree per second, determine the magnitudes of the velocity $\vec{v}$ and acceleration $\vec{a}$ of the glider for the instant when θ = 15°.



Ans: v = 206 km/h, a = 0.457 m/s^2

2/9 Constrained Motion of Connected Particles

BY: JAAFAR M. HAMZAH

M.Sc. Mechanical Engineering

Constrained Motion

1. **One Degree of Freedom**

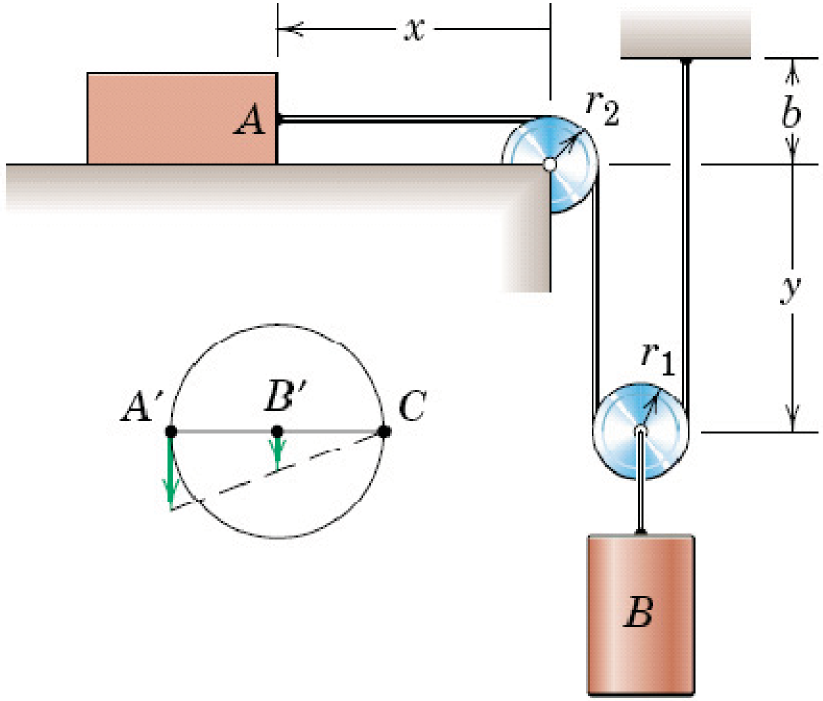
2. **Two Degree of Freedom**

3. **Examples**

Constrained Motion

1. One Degree of Freedom

- Simple system of **two interconnected particles.**
- With *L*, *r2*, *r1*, and *b* are constant
- Horizontal motion (*x*) of *A* is twice the vertical motion (*y*) of *B*
- Only **one variable** (*x* or *y*) is needed to specify the **positions of all parts** of the system



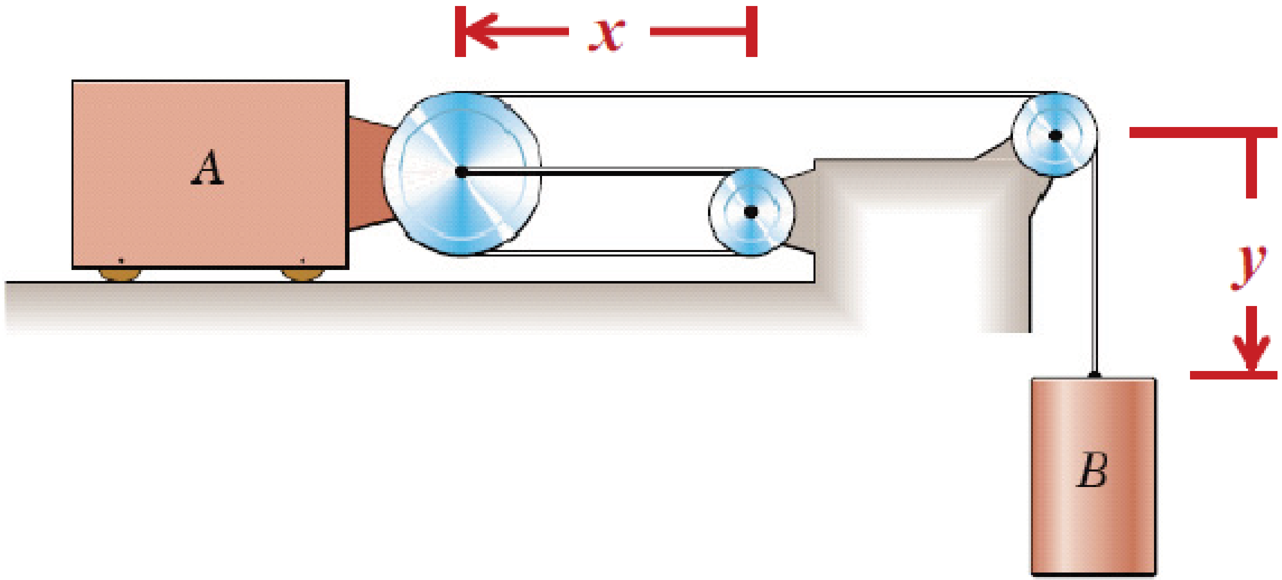
$$L = x + \frac{\pi}{2} r_2 + 2y + \pi r_1 + b$$

$$0 = \dot{x} + 2\dot{y} \qquad 0 = v_A + 2v_B$$

$$0 = \ddot{x} + 2\ddot{y} \qquad 0 = a_A + 2a_B$$

Constrained Motion

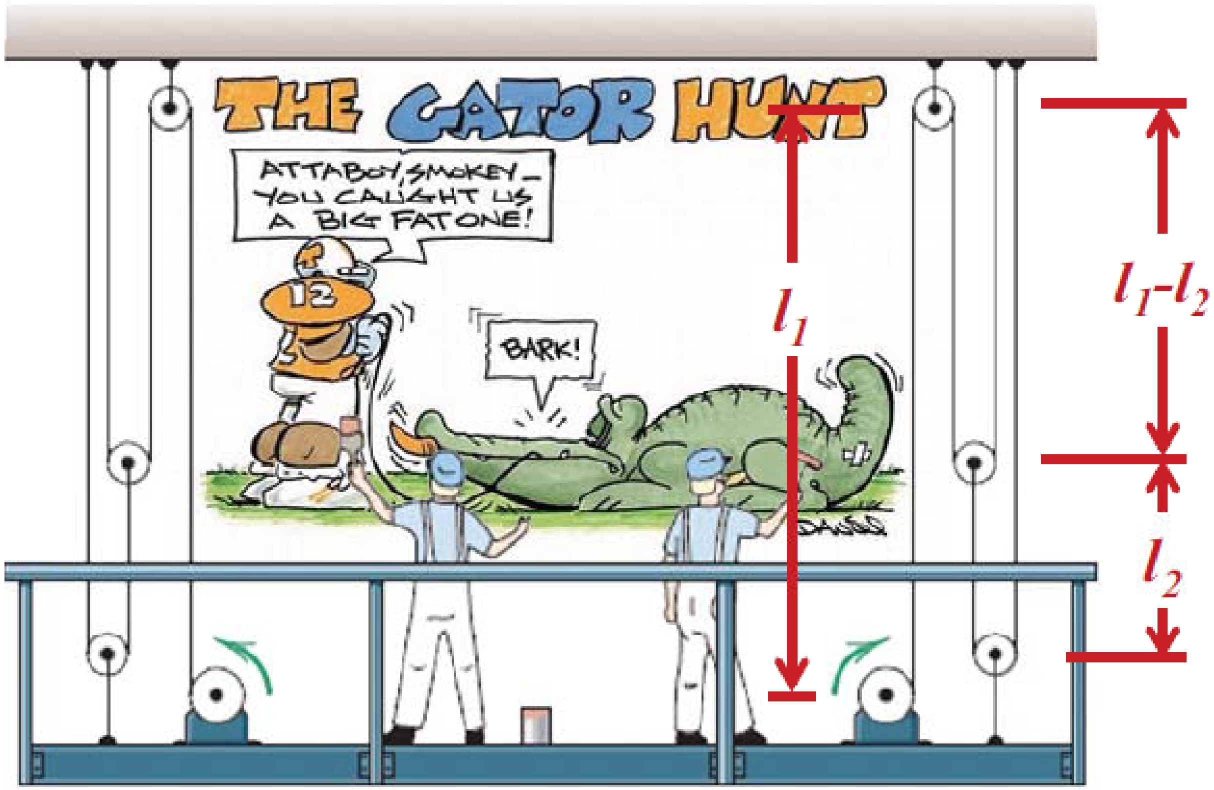
One Degree of Freedom: Exercise



Block *A* has a velocity of 3.6 ft/s to the right.
Determine the velocity of cylinder *B*.

Constrained Motion

One Degree of Freedom: Another Exercise



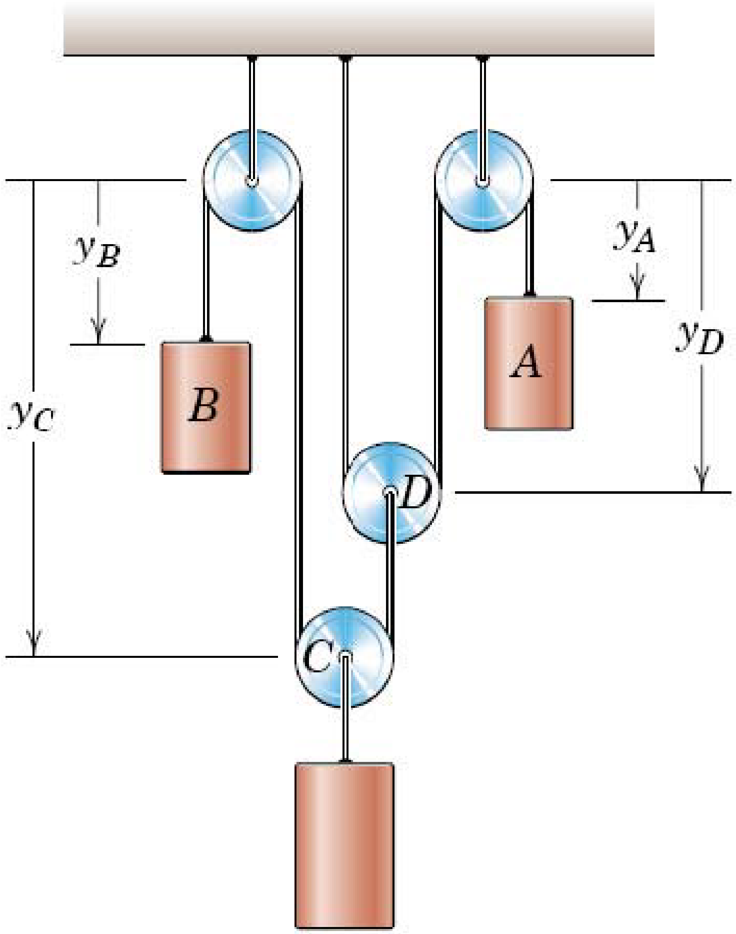
The scaffold is being raised. Each winch drum has a diameter of 200 mm and turns at the rate of 40 rpm.

Determine the upward **velocity** of the scaffold.

Constrained Motion

2. Two Degree of Freedom

Position of lower cylinder depends on **two variables** (y_A and y_B)



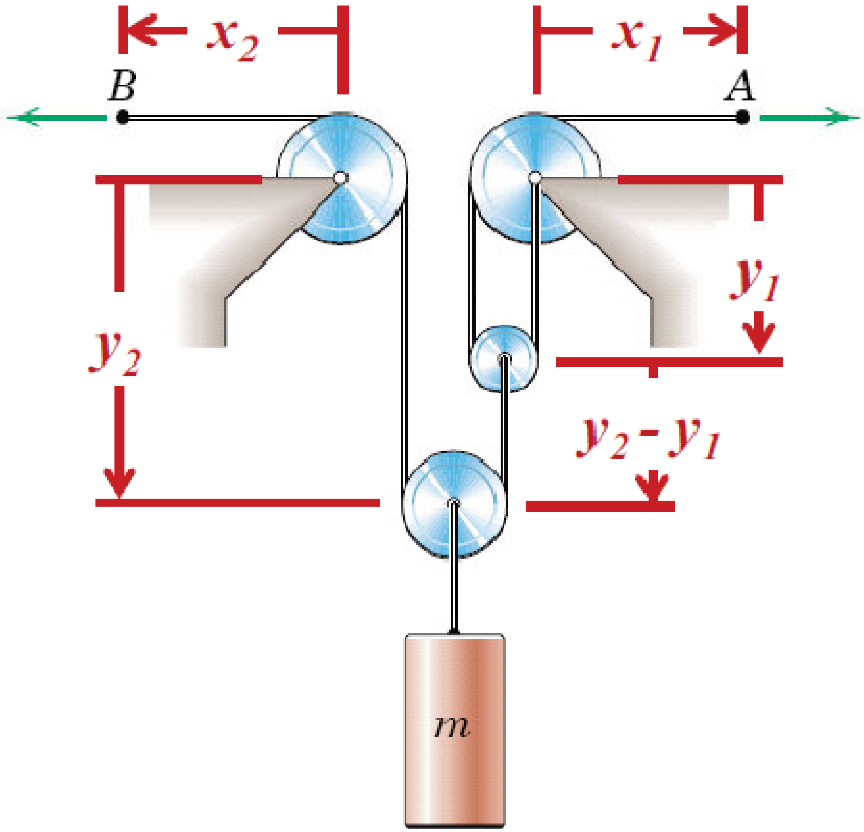
$$L_A = y_A + 2y_D + \text{constant}$$
$$L_B = y_B + y_C + (y_C - y_D) + \text{constant}$$

$$0 = \dot{y}_A + 2\dot{y}_D \qquad 0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$$
$$0 = \ddot{y}_A + 2\ddot{y}_D \qquad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

$$0 = \dot{y}_A + 2\dot{y}_B + 4\dot{y}_C$$
$$0 = \ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C$$

Constrained Motion

Two Degree of Freedom: Exercise

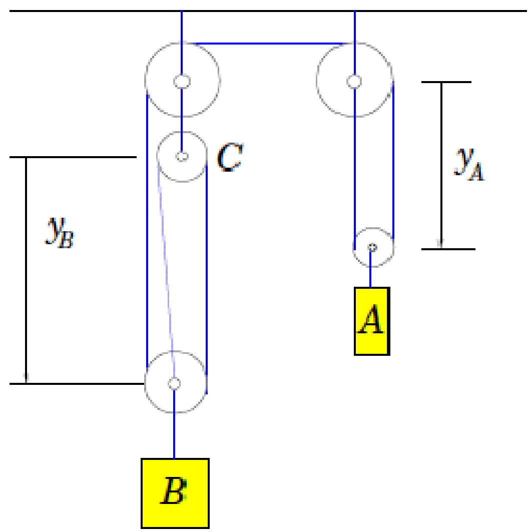


Each of the cables at *A* and *B* is given a **velocity** of 2 m/s in the direction of the arrow. Determine the upward **velocity** of load *m*.

Constrained Motion

Example 1:

In the pulley configuration shown besides, cylinder **A** has a downward **velocity** of 0.3 m/s. Determine the velocity of **B**.



Solution.

The centers of the pulleys at **A** and **B** are located by the coordinates ***yA*** and ***yB*** measured from fixed positions. The total constant length of cable in the pulley system is

$$L = 3y_B + 2y_A + \text{constants}$$

where the constants account for the fixed lengths of cable in contact with the circumferences of the pulleys and the constant vertical separation between the two upper left-hand pulleys. Differentiation with time gives

$$0 = 3\dot{y}_B + 2\dot{y}_A$$

Substitution of $\dot{y}_A = 0.3$ m/s gives

$$\dot{y}_B = -0.2 \text{ m/s}$$

Constrained Motion

Example 2: (Beer/Johnston, 11.59**)**

Collar A starts from rest at t = 0 and moves upward with a constant acceleration of 9 cm/s^2. Knowing that collar B moves downward with a constant velocity of 45.7 cm/s, determine:

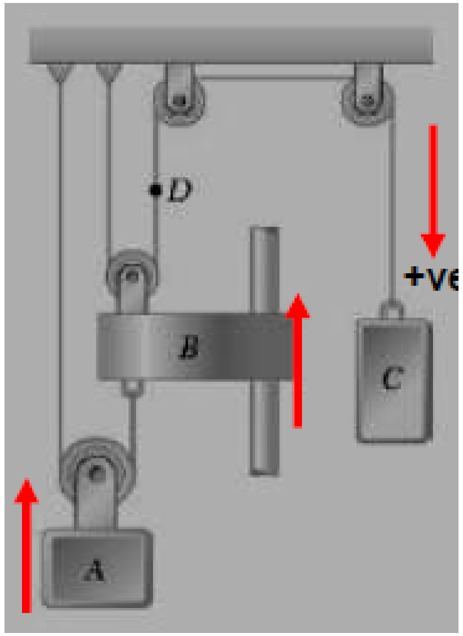
(a) the time at which the velocity of block C is zero,

(b) the corresponding position of block C.

Ans. *t = 5.07 s* $x_C-(x_C)_o = 0.036$ *cm*

Constrained Motion

Example 3: (Beer/Johnston, 11.48**)**



Block **C** starts from rest and moves down with a constant acceleration. Knowing that after block **A** has moved 0.5 m its velocity is 0.2 m/s, determine (a)the accelerations of **A** and **C**, (b)the velocity and the change in position of block **B** after 2 s.

Solution:

Block/cable **A**: *XA+(XA-XB) = Const;*

2VA-VB = 0 => *VA = VB/2; aA=aB/2*

Block/cable **B**: *2XB+XC = const*

2VB+VC = 0 => *VC = -2 VB; aC = -2aB*

a)

$$a_A = \frac{v_A^2 - (v_A)_0^2}{2\left[x_A - (x_A)_0\right]} = \frac{(0.2)^2 - 0}{(2)(-0.5)} = -0.04 \text{ m/s}^2 \qquad a_A = 0.04 \text{ m/s}^2$$

$$a_C = -4a_A \qquad a_C = 0.16 \text{ m/s}^2$$

b)

$$a_B = 2a_A = (2)(-0.04) = -0.08 \text{ m/s}^2$$

$$\Delta v_B = a_B t = (-0.08)(2) = -0.16 \text{ m/s} \qquad \Delta v_B = 0.16 \text{ m/s}$$

$$\Delta x_B = \frac{1}{2}a_B t^2 = \frac{1}{2}(-0.08)(2)^2 = -0.16 \text{ m} \qquad \Delta x_B = 0.16 \text{ m}$$

Problem Solution of Chapter Two Lectures

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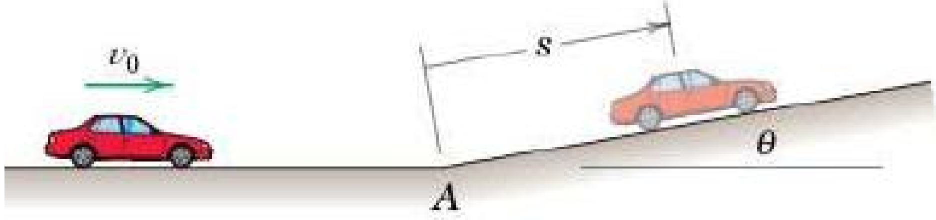
Rectilinear Motion

H.W 1: The car traveling at a constant speed $v_o = 100\,\text{km/h}$ on the level portion of the road. When the 6-percent ($\tan\theta = 6/100$) incline is encountered, the driver does not change the throttle setting and consequently the car decoration at the constant rate $g\sin\theta$. Determine the speed of the car (a) 10 seconds after passing point A and (b) when $s = 100\text{m}$.



Solution:

$$v_o = 100/3.6 = 27.8 \text{ m/s}$$

$$a = -g\sin\theta = -9.81\sin\left[\tan^{-1}\frac{6}{100}\right] = -0.588 \text{ m/s}^2$$

(a)

$$v = v_o + at = 27.8 - 0.588(10) = \underline{21.9 \text{ m/s}}$$

(b)

$$v^2 = v_o^2 + 2a(s - s_o) = 27.8^2 + 2(-0.588)(100)$$

$$v = \underline{25.6 \text{ m/s}}$$

H.W 2:

2/5 The acceleration of a particle is given by $a = 2t - 10$, where a is in meters per second squared and t is in seconds. Determine the velocity and displacement as functions of time. The initial displacement at $t = 0$ is $s_0 = -4$ m, and the initial velocity is $v_0 = 3$ m/s.

Solution:

$$a = \frac{dv}{dt} = 2t - 10$$

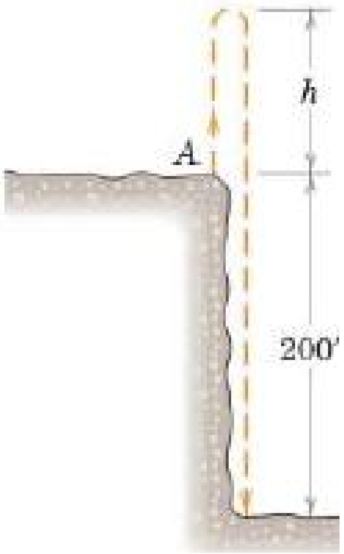
$$\int_{v_0=3}^{v} dv = \int_0^t (2t-10)\,dt, \quad \underline{v = 3 - 10t + t^2} \ (m/s)$$

$$\frac{ds}{dt} = 3 - 10t + t^2$$

$$\int_{s_0=-4}^{s} ds = \int_0^t (3 - 10t + t^2)\,dt$$

$$\underline{s = -4 + 3t - 5t^2 + \tfrac{1}{3}t^3} \ (m)$$

2/10 A ball is thrown vertically up with a velocity of 80 ft/sec at the edge of a 200-ft cliff. Calculate the height h to which the ball rises and the total time t after release for the ball to reach the bottom of the cliff. Neglect air resistance and take the downward acceleration to be 32.2 ft/sec^2.



Solution:

$\uparrow +y$ $\quad y = v_0 t + \frac{1}{2} a t^2, \quad y = 80t - \frac{1}{2} 32.2 t^2$

for $y = -200$ ft,

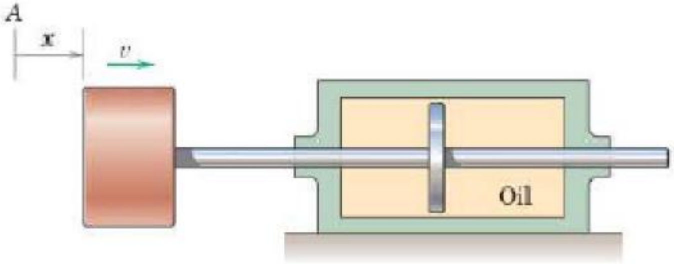
$$-200 = 80t - 16.1 t^2$$

or $\quad 16.1 t^2 - 80t - 200 = 0$

$$t = \frac{80 \pm \sqrt{(80)^2 + 4(16.1)(200)}}{2(16.1)} = \underline{6.80 \text{ sec}} \quad (\text{or } -1.83 \text{ s})$$

For $\dot{y} = 0$, $\quad v^2 = v_0^2 + 2ay$, $\quad y = h = \dfrac{0 - 80^2}{-2(32.2)} = \underline{99.4 \text{ ft}}$

2/44 The horizontal motion of the plunger and shaft is arrested by the resistance of the attached disk which moves through the oil bath. If the velocity of the plunger is v_0 in the position A where $x = 0$ and $t = 0$, and if the deceleration is proportional to v so that $a = -kv$, derive expressions for the velocity v and position coordinate x in terms of the time t. Also express v in terms of x.



Solution:

$$a = \frac{dv}{dt} = -kv, \quad \int_{v_0}^{v} \frac{dv}{v} = -k\int_0^t dt$$

$$\ln \frac{v}{v_0} = -kt, \quad \underline{v = v_0 e^{-kt}}$$

$$v = \frac{dx}{dt} = v_0 e^{-kt}, \quad \int_0^x dx = \int_0^t v_0 e^{-kt} dt$$

$$\underline{x = \frac{v_0}{k}\left[1 - e^{-kt}\right]}$$

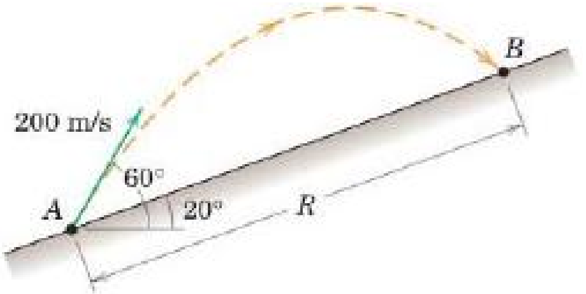
$$v\,dv = a\,dx, \quad \frac{v\,dv}{v} = -k\,dx$$

$$\int_{v_0}^{v} dv = -k\int_0^x dx, \quad \underline{v = v_0 - kx}$$

1. Rectangular Coordinates (x-y)

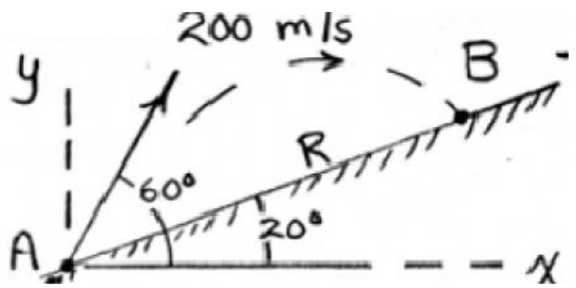
2/85 A projectile is launched with an initial speed of 200 m/s at an angle of 60° with respect to the horizontal. Compute the range R as measured up the incline.

Ans. $R = 2970$ m



Solution:

$$\begin{cases} v_{x_0} = 200\cos 60^\circ = 100 \text{ m/s} \\ v_{y_0} = 200\sin 60^\circ = 173.2 \text{ m/s} \end{cases}$$



t_f = flight time

$x = x_0 + v_{x_0} t$ @ B: $R\cos 20^\circ = 100\, t_f$ (1)

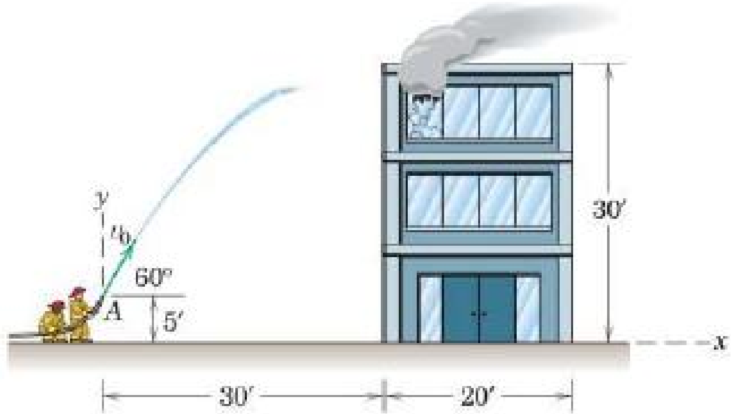
$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $R\sin 20^\circ = 173.2\, t_f - \frac{9.81}{2} t_f^2$ (2)

(1): $t_f = 0.00940\, R$

(2): $R\sin 20^\circ = 173.2(0.00940\,R) - \frac{9.81}{2}(0.00940\,R)^2$

$\underline{R = 2970 \text{ m}}$

2/74 Water issues from the nozzle at A, which is 5 ft above the ground. Determine the coordinates of the point of impact of the stream if the initial water speed is (*a*) $v_0 = 45$ ft/sec and (*b*) $v_0 = 60$ ft/sec.



Solution: Use x-y coordinates of the figure.

(a) $v_0 = 45$ ft/sec

$x = x_0 + v_{x_0} t$ @ left wall : $30 = 0 + 45\cos 60°\, t$

$t = 1.333$ sec

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$: $y = 5 + 45\sin 60°(1.333) - 16.1(1.333)^2 = 28.3$ ft (hits wall)

Ans. : $(x, y) = (30', 28.3')$

(b) $v_0 = 60$ ft/sec

Repeat above procedure to find $y = 40.9'$ when $x = 30'$, so water clears left wall.

$x = x_0 + v_{x_0} t$ @ right wall : $50 = 0 + 60\cos 60° t$

$t = 1.667$ sec

y eq. yields $y = 46.9$ ft @ $t = 1.667$ sec, so water clears building! For horizontal range:

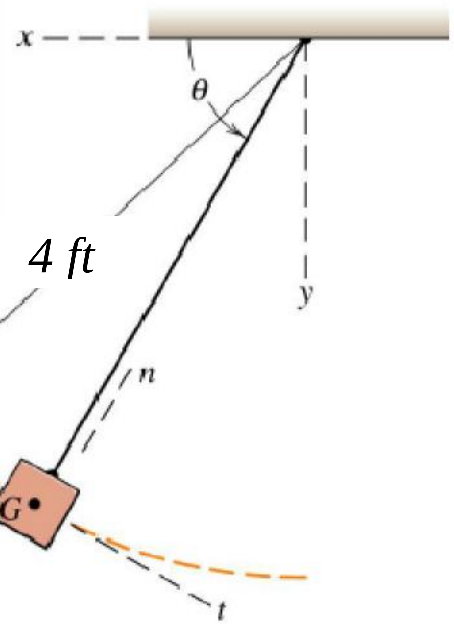
From $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ $y = 0$, $y_0 = 5$ ft, we find $t = -0.0935$ s & $t = 3.32$ s. From

$x = x_0 + v_{x_0} t$: $x = 0 + 60\cos 60°(3.32) = 99.6$ ft

2. Normal And Tangential Coordinate (*n*-*t*)

Example 2: Pendulum

Write the vector expression of the acceleration **a** of the mass center G of the simple pendulum in both *n*-*t* and *x*-*y* when $\theta = 60^\circ, \dot{\theta} = 2\ \text{rad/s}$ and $\ddot{\theta} = 2.45\ \text{rad/s}^2$



Solution:

$$a_n = r\dot{\theta}^2 = 4(2.00)^2 = 16.00\ ft/sec^2$$

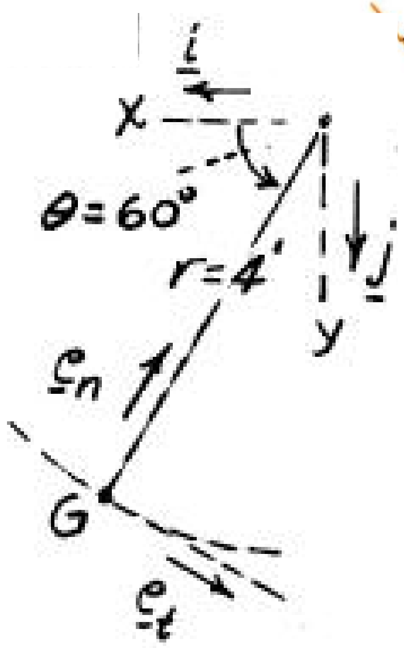
$$a_t = r\ddot{\theta} = 4(4.025) = 16.10\ ft/sec^2$$

$$\underline{a} = 16.00\,\underline{e}_n + 16.10\,\underline{e}_t\ \ ft/sec^2$$

$$a_x = -16.00\cos 60^\circ - 16.10\sin 60^\circ = -21.9\ ft/sec^2$$

$$a_y = 16.10\cos 60^\circ - 16.00\sin 60^\circ = -5.81\ ft/sec^2$$

$$\underline{a} = -21.9\,\underline{i} - 5.81\,\underline{j}\ \ ft/sec^2$$

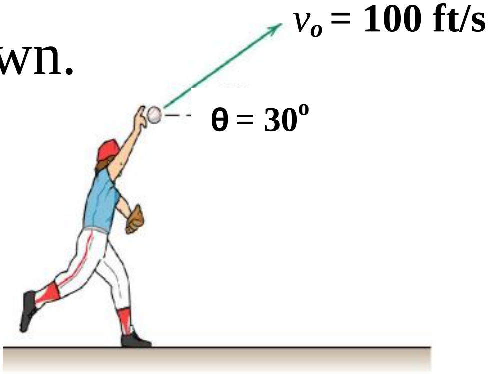


Example 4: Baseball Player

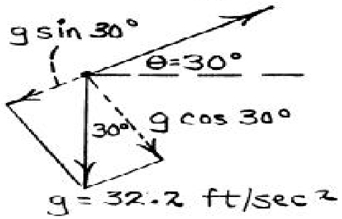
A baseball player releases a ball with the initial conditions shown. Determine the radius of curvature of the trajectory:

a) Just after release **b)** At the apex. **c)** At **t = 1 sec**.

For each case, compute the time rate of change of the speed $\dot{v}$.



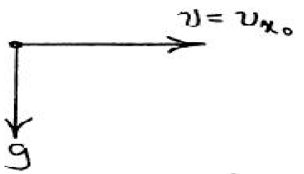
Solution: **a) & b)**



(a) $a_n = g\cos 30° = \dfrac{v^2}{\rho}$

$\rho = \dfrac{100^2}{g\cos 30°} = \underline{359\ \text{ft}}$

$\dot{v} = -g\sin 30° = \underline{-16.1\ \text{ft/sec}^2}$



(b) $a_n = g = \dfrac{v^2}{\rho}$

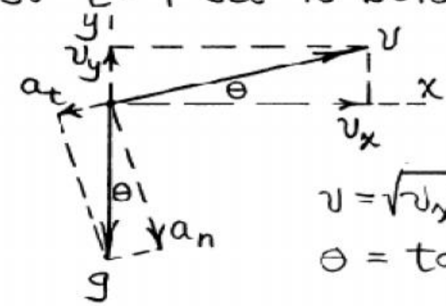
$\rho = \dfrac{(100\cos 30°)^2}{32.2} = \underline{233\ \text{ft}}$

$\underline{\dot{v} = 0}$

c)

$v_y = v_{y_0} - gt$: $0 = 100\sin 30° - 32.2\, t_{up}$, $t_{up} = 1.553$ sec

So $t = 1$ sec is before apex and $t = 2.5$ sec is after.



(a) $t = 1$ sec

$v_x = 100\cos 30° = 86.6$ ft/sec

$v_y = 100\sin 30° - 32.2(1) = 17.80$ ft/sec

$v = \sqrt{v_x^2 + v_y^2} = 88.4$ ft/sec

$\theta = \tan^{-1}\dfrac{v_y}{v_x} = 11.61°$

$a_n = g\cos\theta = 32.2\cos 11.61° = 31.5\ \text{ft/sec}^2$

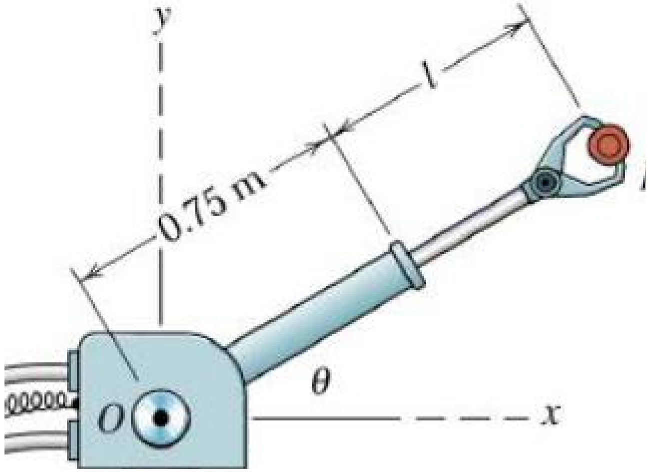
$\rho = \dfrac{v^2}{a_n} = \dfrac{88.4^2}{31.5} = \underline{248\ \text{ft}}$

$a_t = -g\sin\theta = -32.2\sin 11.61° = \underline{-6.48\ \text{ft/sec}^2}$

3. Polar Coordinates (r-θ)

Example 1: Robot Arm

The robot arm is elevating and extending simultaneously. At a given instant, $\theta = 30°$, $\dot{\theta} = 10$ deg/s constant, $l = 0.5$ m, $\dot{l} = 0.2$ m/s, and $\ddot{l} = -0.3$ m/s^2. Compute the magnitude of the velocity, $\vec{v}$, and acceleration, $\vec{a}$, of the gripped part P. In addition, express $\vec{v}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$.



Solution:

$$\begin{cases} r = 0.75 + 0.5 = 1.25\text{ m} \\ \dot{r} = 0.2\text{ m/s} \\ \ddot{r} = -0.3\text{ m/s}^2 \end{cases} \qquad \begin{aligned} \theta &= 30° \\ \dot{\theta} &= 0.1745\ \tfrac{\text{rad}}{\text{s}} \\ \ddot{\theta} &= 0 \end{aligned}$$

$$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$$

$$= 0.2\underline{e}_r + 1.25(0.1745)\underline{e}_\theta = 0.2\,\underline{e}_r + 0.218\,\underline{e}_\theta\ \frac{\text{m}}{\text{s}}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296\text{ m/s}}$$

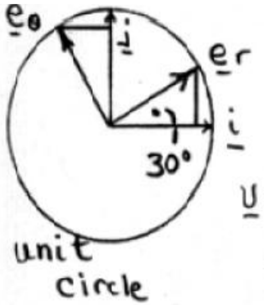
$$\underline{a} = a_r \underline{e}_r + a_\theta \underline{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

$$= [-0.3 - 1.25(0.1745)^2]\,\underline{e}_r + [1.25(0) + 2(0.2)(0.1745)]\,\underline{e}_\theta$$

$$= -0.338\,\underline{e}_r + 0.0698\,\underline{e}_\theta\ \text{m/s}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345\text{ m/s}^2}$$

in terms of i,j :



$$\underline{e}_r = \underline{i}\cos 30° + \underline{j}\sin 30°$$

$$\underline{e}_\theta = -\underline{i}\sin 30° + \underline{j}\cos 30°$$

$$\underline{v} = 0.2[\underline{i}\,c\,30° + \underline{j}\,s\,30°] + 0.218[-\underline{i}\,s\,30° + \underline{j}\,c\,30°]$$

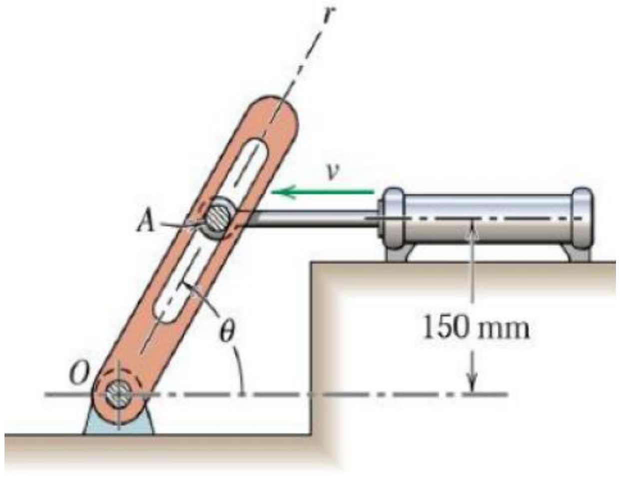
$$= 0.064\,\underline{i} + 0.289\,\underline{j}\ \text{m/s}$$

$$\underline{a} = -0.338[\underline{i}\,c\,30° + \underline{j}\,s\,30°] + 0.0698[-\underline{i}\,s\,30° + \underline{j}\,c\,30°]$$

$$= \underline{-0.328\,\underline{i} - 0.1086\,\underline{j}\ \text{m/s}^2}$$

Example 2: Hydraulic Cylinder

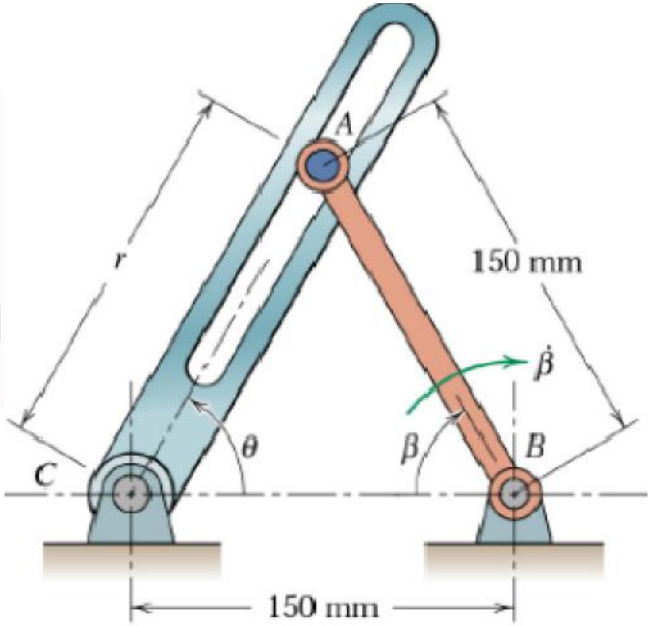
The piston of the hydraulic cylinder gives pin A a constant velocity v = 1.5 m/s in the direction shown for an interval of its motion. For the instant when θ = 60°, determine $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$, where $r = \overline{OA}$



Solution:

Example 3: Two links

Link AB rotates through a limited range of the angle β, and its end A causes the slotted link AC to rotate also. For the instant represented where $\beta = 60^\circ$ and $\dot{\beta} = 0.6$ rad/s constant, determine the corresponding values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$.



Solution:

For $\beta = 60^\circ$, $\theta = 60^\circ$, $r = 150$ mm

$v_A = 150(0.6) = 90$ mm/s

$v_\theta = r\dot{\theta} = -v_A \cos 60^\circ$, $\dot{\theta} = \dfrac{-90 \cos 60^\circ}{150} = -0.3 \dfrac{\text{rad}}{\text{s}}$

$v_r = \dot{r} = v_A \sin 60^\circ = 90 \sin 60^\circ = 77.9$ mm/s

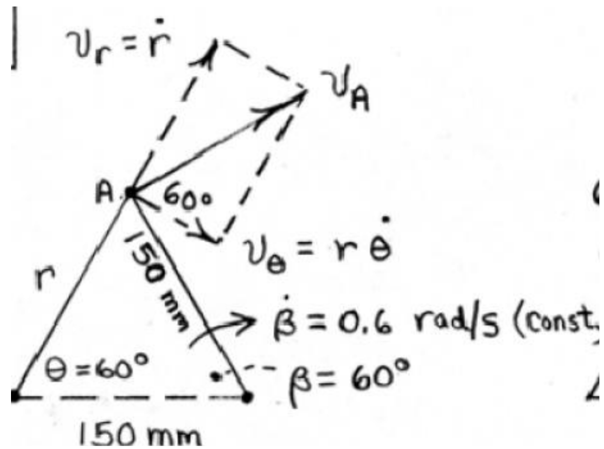
$a_A = a_n = 150(0.6)^2 = 54$ mm/s^2

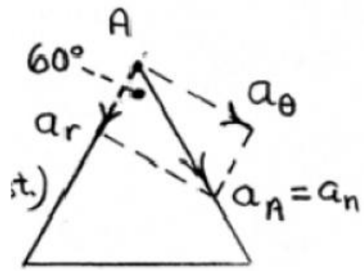
$a_r = \ddot{r} - r\dot{\theta}^2$: $-54 \cos 60^\circ = \ddot{r} - 150(-0.3)^2$

$\ddot{r} = -13.5$ mm/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $-54 \sin 60^\circ = 150\ddot{\theta} + 2(77.9)(-0.3)$

$\ddot{\theta} = 0$

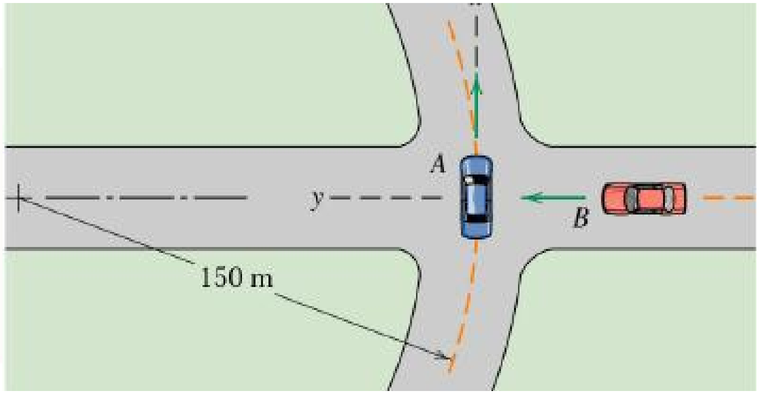




Relative Motion

Example 2: Two cars

Car A rounds a curve of 150-m radius at a constant speed of 54 km/h. At the instant represented, car B is moving at 81 km/h but is slowing down at the rate of 3 m/s^2. Determine the velocity and acceleration of car A as observed from car B. [Extra] Find the curvature of A as observed from B.

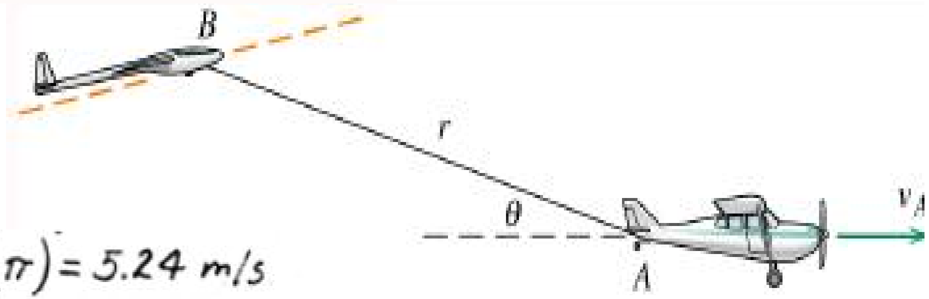


Solution:

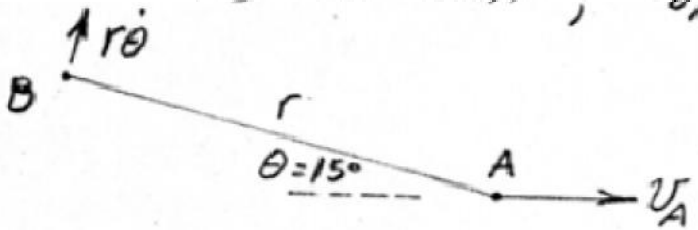
Example 3: Two planes

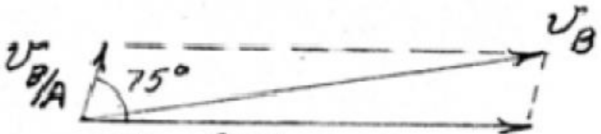
Airplane A is flying horizontally with a constant speed of 200 km/h and is towing the glider B, which is gaining altitiude. If the tow cable has a length r = 60 m and θ is increasing at the constant rate of 5 degree per second, determine the magnitudes of the velocity $\vec{v}$ and acceleration $\vec{a}$ of the glider for the instant when $\theta = 15°$.

Solution:



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}, \quad v_{B/A} = r\dot{\theta} = 60\left(\frac{5}{180}\pi\right) = 5.24 \text{ m/s}$$





$$v_A = \frac{200}{3.6} = 55.6 \text{ m/s}$$

$$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75° = 3264 \text{ (m/s)}^2$$

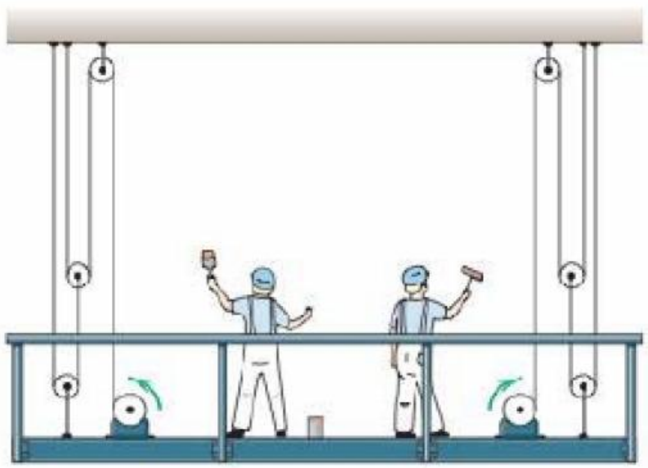
$$v_B = 57.1 \text{ m/s} \quad \text{or} \quad v_B = 57.1(3.6) = \underline{206 \text{ km/h}}$$

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}, \quad a_A = 0, \quad a_{B/A} = r\dot{\theta}^2 = 60\left(\frac{5\pi}{180}\right)^2 = 0.457 \text{ m/s}^2$$

Thus $a_B = a_{B/A} = \underline{0.457 \text{ m/s}^2}$ from B to A

Constrained Motion

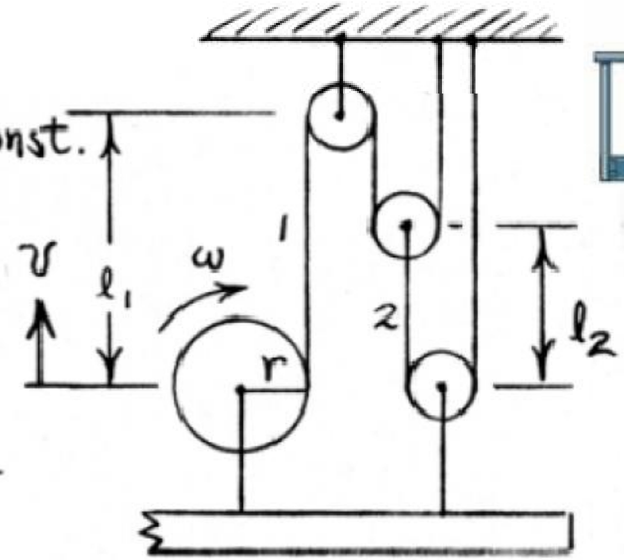
Exercise2:



The scaffold is being raised. Each winch drum has a diameter of 200 mm and turns at the rate of 40 rpm.

Determine the upward **velocity** of the scaffold.

Solution:



Length $L_1 = l_1 + 2(l_1 - l_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 3\dot{l}_1 - 2\dot{l}_2$

Length $L_2 = l_2 + l_1 + \text{const.}$

$\dot{L}_2 = 0 = \dot{l}_2 + \dot{l}_1, \quad -\dot{l}_1 = \dot{l}_2$

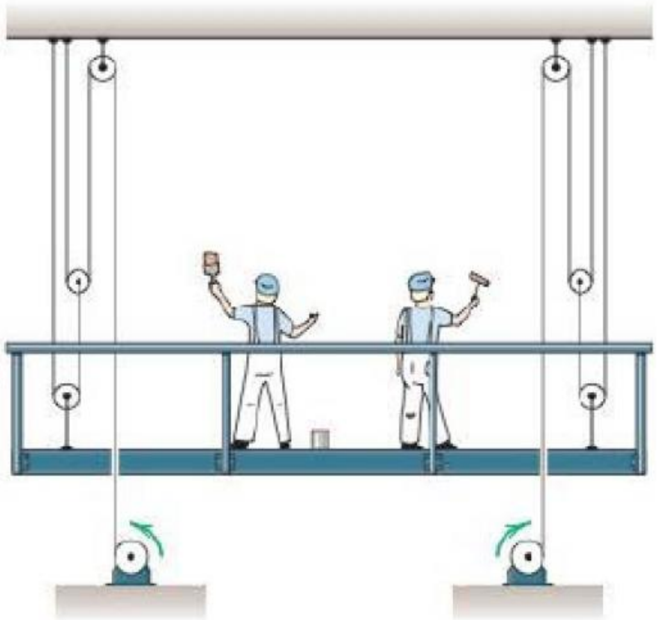
But $v = -\dot{l}_1$, so

$-r\omega = 3(-v) - 2v, \quad r\omega = 5v$

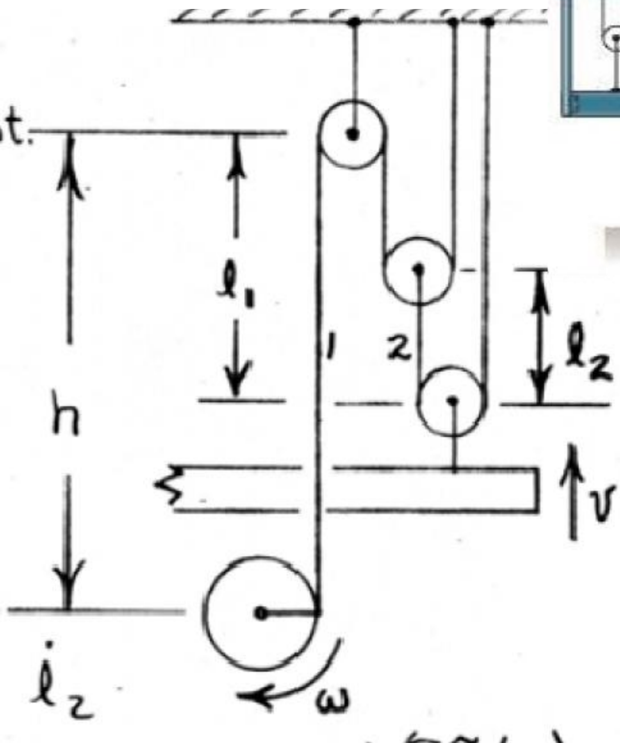
$$v = \frac{r\omega}{5} = \frac{0.1(40)(2\pi/60)}{5} = 0.0838 \ \frac{m}{s}$$

or $\underline{v = 83.8 \ mm/s}$

2/226 The scaffold of Prob. 2/225 is modified here by placing the power winches on the ground instead of on the scaffold. Other conditions remain the same. Calculate the upward velocity v of the scaffold.



Solution:



Length $L_1 = h + 2(\ell_1 - \ell_2) + \text{const.}$

$\dot{L}_1 = -r\omega = 0 + 2\dot{\ell}_1 - 2\dot{\ell}_2$

But $v = -\dot{\ell}_1$

So $-r\omega = -2v - 2\dot{\ell}_2$

Length $L_2 = \ell_1 + \ell_2 + \text{const.}$

$\dot{L}_2 = 0 = \dot{\ell}_1 + \dot{\ell}_2$, $v = \dot{\ell}_2$

$\therefore r\omega = 2v + 2v$, $v = \dfrac{r\omega}{4} = \dfrac{0.1(40)(2\pi/60)}{4}$

$v = 0.1047$ m/s or $v = 104.7 \dfrac{\text{mm}}{\text{s}}$

Chapter 3
Kinetics of Particles

3. Kinetics of Particles

- Introduction
  - *Kinetics* is the study of the relations between unbalanced force and the resulting changes in motion, i.e. **F** vs **r**, **v**, **a**.
- The three approaches
  - A. Direct Application or Force-Mass-Acceleration
  - B. Work and Energy
  - C. Impulse and Momentum
- Special Applications
  - Impact

3-1 Force, Mass, and Acceleration

3-1. Force, Mass, and Acceleration

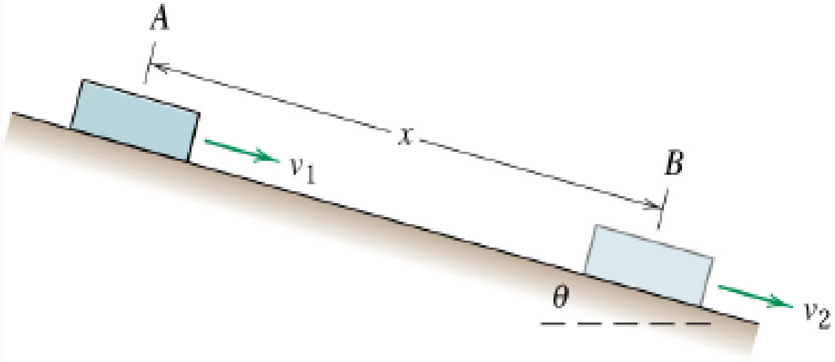
- The main equation is the Newton's second law.

Newton's Second Law

$$\Sigma\vec{F} = m\vec{a}$$

- Combine it with coordinate systems studied in Chapter 2 to solve engineering problems

Suppose the block shown starts from rest at point A and slides down the incline due to the force of gravity. Find the speed of this block as a function of time, if θ = 15°.



3-1. Force, Mass, and Acceleration

Free Body Diagram

- A free-body diagram must be drawn to correctly evaluating all forces involved in Newton's second law.
- Procedures
  - Clearly draw an isolated body
  - Define coordinate system and their positive directions
  - Add all the forces (contact and non-contact) acting <u>on</u> that body

3-1. Force, Mass, and Acceleration

Rectilinear vs Curvilinear

$$\begin{aligned}\Sigma F_x &= ma_x\\ \Sigma F_y &= 0\\ \Sigma F_z &= 0\end{aligned}$$

Normal and Tangential Coordinates

$$\Sigma \vec{F}_n = m\vec{a}_n$$
$$\Sigma \vec{F}_t = m\vec{a}_t$$

where $a_n = \rho\dot{\beta}^2 = v^2/\rho = v\dot{\beta}$, $a_t = \dot{v}$, and $v = \rho\dot{\beta}$

Rectangular Coordinates

$$\Sigma \vec{F}_x = m\vec{a}_x$$
$$\Sigma \vec{F}_y = m\vec{a}_y$$

Polar Coordinates

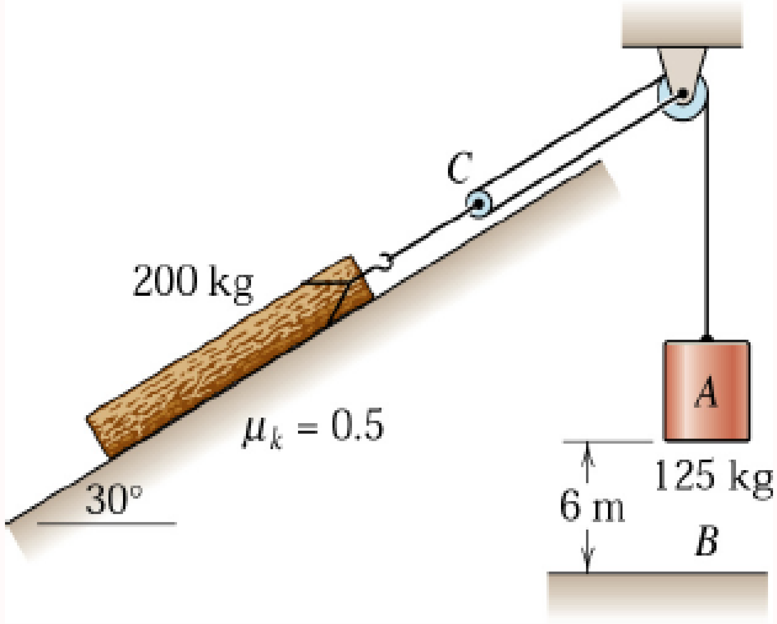
$$\Sigma \vec{F}_r = m\vec{a}_r$$
$$\Sigma \vec{F}_\theta = m\vec{a}_\theta$$

where $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

3-1. Force, Mass, and Acceleration

Example 1: A log and a pulley

The 125-kg concrete block *A* is released from rest in the position shown and pulls the 200-kg log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at *B*.

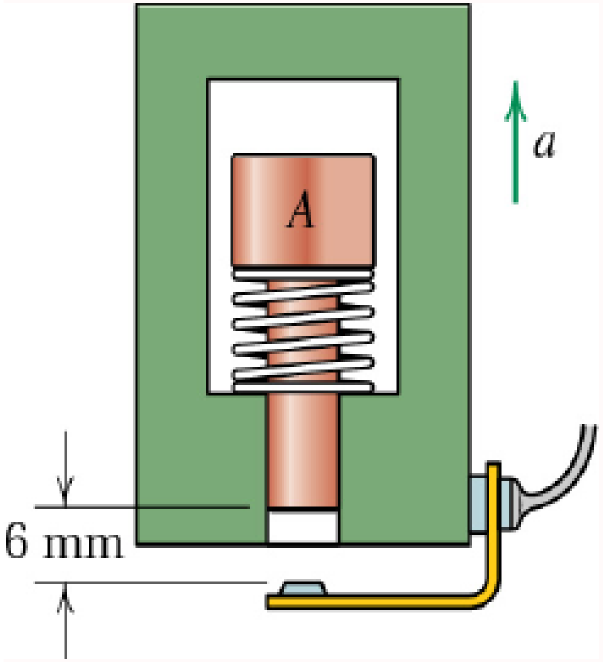


Ans: 4.62 m/s

3-1. Force, Mass, and Acceleration

Example 2: An accelerometer

The device shown is used as an accelerometer and consists of a 100-g plunger *A* which deflects the spring as the housing of the unit is given an upward acceleration *a*.



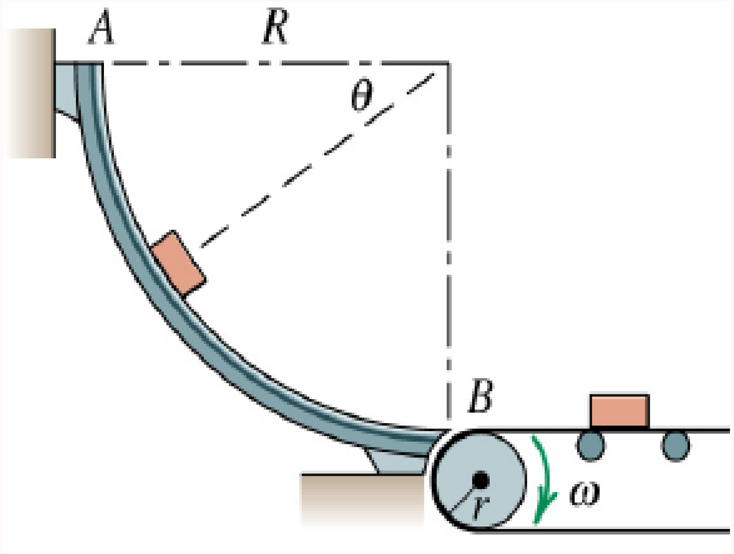
Specify the neccessary spring stiffness *k* which will permit the plunger to deflect 6 mm beyond the equilibrium position and touch the electical contact when the steadily but slowly increasing upward acceleration reaches 5g. Friction may be neglected.

Ans: 818 N/m

3-1. Force, Mass, and Acceleration

Example 3: A Conveyor

Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B. Determine the expression for the normal contact force N between the guide and each object in terms of θ and specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.

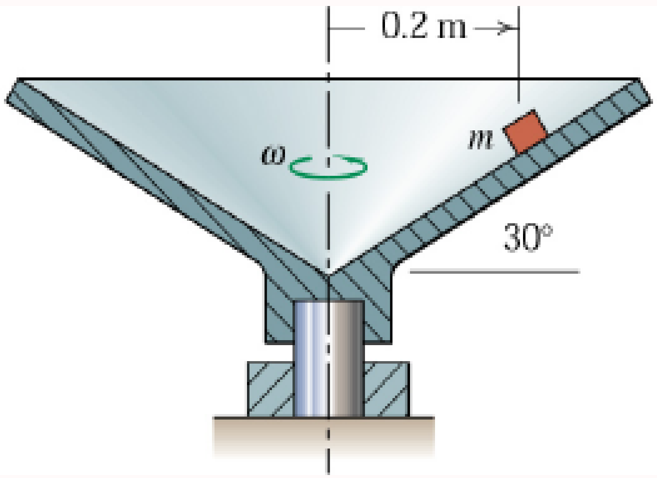


Ans: $N = 3mg\sin(\theta)$, $\omega = \sqrt{2gR}/r$

3-1. Force, Mass, and Acceleration

Example 4: A Conical dish

The small object is placed on the inner surface of the conical dish at the radius shown. If the coefficient of static friction between the object and the conical surface is 0.30, for what range of angular velocities ω about the vertical axis will the block remain on the dish without slipping? Assume that speed changes are made slowly so that any angular acceleration may be neglected.

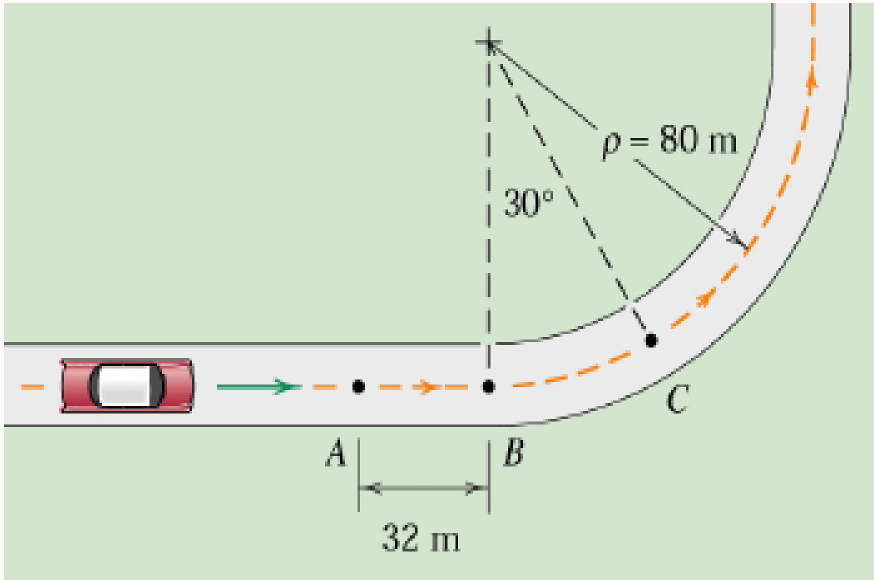


Ans: $3.41 \le \omega \le 7.21$ rad/s

3-1. Force, Mass, and Acceleration

Example 5: A car on a curve

The 1500-kg car is traveling at 100 km/h on the straight portion of the road, and then its speed is reduced uniformly from *A* to *C*, at which point it comes to rest. Compute the magnitude *F* of the total friction force exerted by the road on the car (a) just before it passes point *B*, (b) just after it passes point *B*, and (c) just before it stops at point *C*.



Ans: 7.83 kN, 11.34 kN, 7.83 kN

3/6 Work and Energy

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

3-6. Work and Energy

- 1. Work and Kinetic Energy
  - Definition of Work
  - Calculation of Work
  - Work of External Force
  - Work of Weight
  - Work of Linear Spring
  - Work and Curvilinear Motion
  - Principle of Work and Kinetic Energy
  - Advantage of Work-Energy Method
  - Power
  - Examples

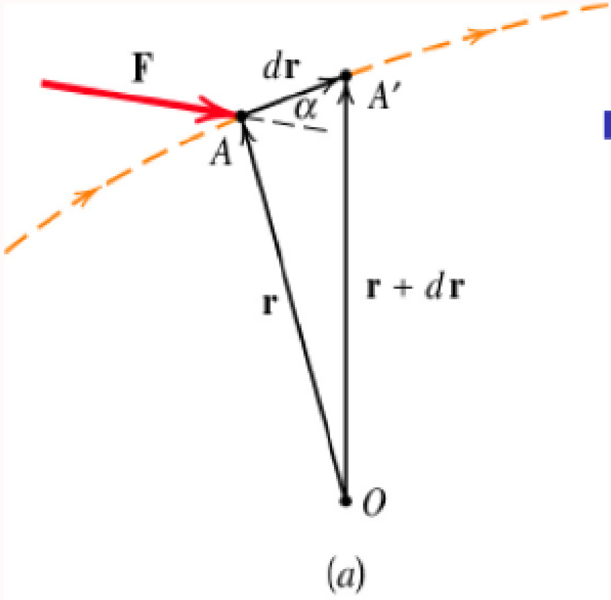
3-6. Work and Energy

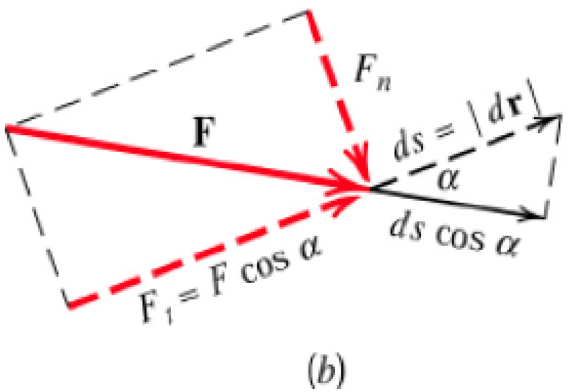
1. Introduction

- Recall Newton's second law and notice that this is an **instantaneous relationship**.
- When we want to see **changes** in velocity or position due to motion, we have to **integrate** Newton's second law by using appropriate kinematic equations.
- However, we may integrate Newton's second law directly and avoid solving for acceleration first.
- In general, there is two classes of problems
  - Integration with respect to displacement → Work-Energy equation → velocity between two positions of a particle or system's configurations.
  - Integration with respect to time → Impulse-Momentum equation → changes in velocity between two points in time.

3-6. Work and Energy

2. Definition of Work





- Work done by force $\vec{F}$ during a small displacement $d\vec{r}$ is defined as

$$dU = \vec{F} \cdot d\vec{r}$$

or

$$dU = F\, ds \cos(\alpha)$$

- Displacement in the direction of force, OR
- Force in the direction of ds; i.e.,

$$dU = F_t\, ds$$

3-6. Work and Energy

3. Calculation of Work

- In general,

$$U = \int \vec{F} \cdot d\vec{r}$$

- In x-y coordinate, we could

$$U = \int (F_x\, dx + F_y\, dy + F_z\, dz)$$

- Make sure F_x is positive in x direction
- Or, in n-t coordinate

$$U = \int F_t\, ds$$

- F_t is positive in +s direction.

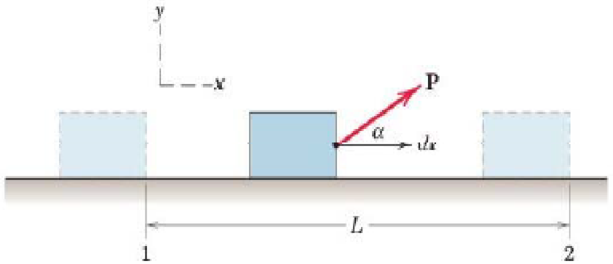
Notes:

- U is positive when F_t and ds are in the same direction
- Active force = force that does work.
- Reactive force = constraint force that does not do work.
- In SI units, unit of work is N-m or Joule (J)

3-6. Work and Energy

4. Work Constant External Force

- Consider the constant force ***P*** applied to the body as it moves from position 1 to 2.



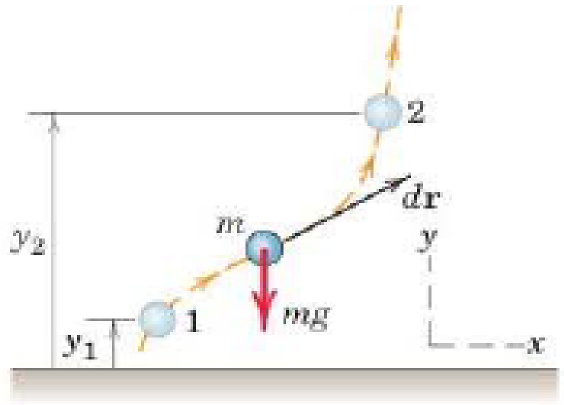
- The work done on the body by the force is:

$$U_{1\text{-}2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 [(P\cos\alpha)\mathbf{i} + (P\sin\alpha)\mathbf{j}]\cdot dx\mathbf{i}$$
$$= \int_{x_1}^{x_2} P\cos\alpha\, dx = P\cos\alpha(x_2 - x_1) = PL\cos\alpha$$

5. Work of Weight

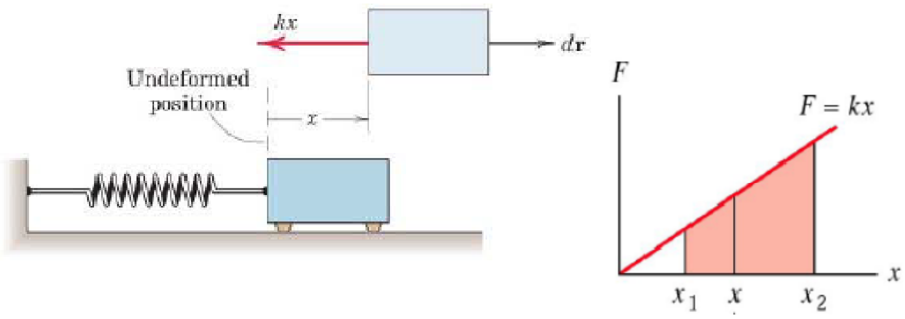
- The work done due to Weight is:

$$U_{1\text{-}2} = \int_1^2 \mathbf{F}\cdot d\mathbf{r} = \int_1^2 (-mg\mathbf{j})\cdot(dx\mathbf{i} + dy\mathbf{j})$$
$$= -mg\int_{y_1}^{y_2} dy = -mg(y_2 - y_1)$$



3-6. Work and Energy

6. Work of Linear Spring



- Work done on the spring by the body → use *F*
- Work done on the body by the spring → use *P* = -*F*
- Thus **work done on the body by the spring** is → → → →

- Linear spring $F = kx$

where *F* is the force acting on the spring to compress/extend

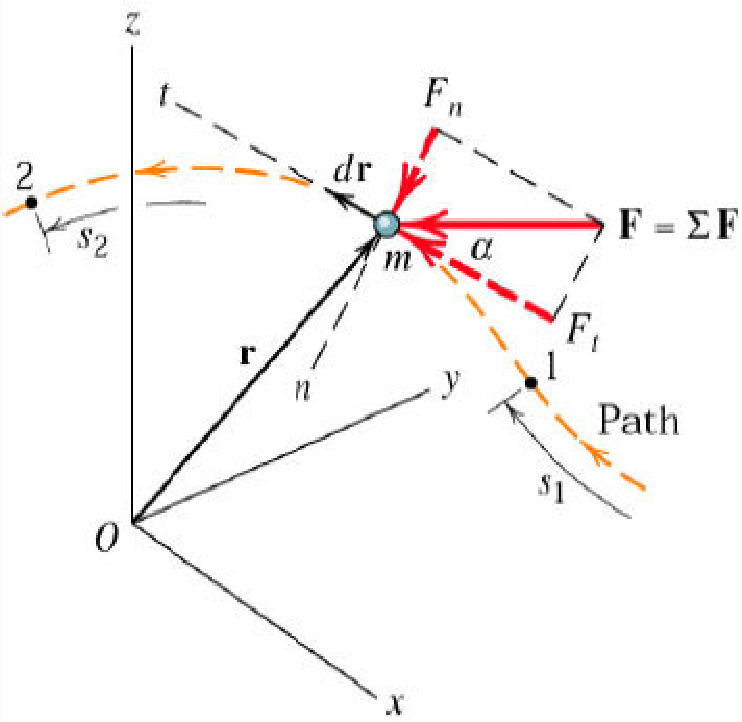
Extension from x_1 to x_2

$$
\begin{aligned}
U_{1-2} &= -\int_{x_1}^{x_2} F\,dx \\
&= -\int_{x_1}^{x_2} kx\,dx \\
&= \boxed{-\frac{1}{2}k(x_2^2 - x_1^2)}
\end{aligned}
$$

3-6. Work and Energy

7. Work and Curvilinear Motion

- Movement from 1 to 2



$$
\begin{aligned}
U_{1-2} &= \int_1^2 \vec{F} \cdot d\vec{r} \\
&= \int_{s_1}^{s_2} F_t \, ds \\
&= \int_1^2 m\vec{a} \cdot d\vec{r} \\
&= \int_1^2 m a_t \, ds \\
&= \int_1^2 mv \, dv
\end{aligned}
$$

$$U_{1-2} = \frac{1}{2} m (v_2^2 - v_1^2)$$

3-6. Work and Energy

8. Work and Kinetic Energy

Kinetic Energy

$$T = \frac{1}{2}mv^2$$

- Recall: $U_{1-2} = \frac{1}{2}m(v_2^2 - v_1^2)$
- T is the work done on a particle to accelerate it from rest to the velocity v
- The unit of work is N·m

Work-Energy Equation

$$U_{1-2} = T_2 - T_1 = \Delta T$$

3-6. Work and Energy

8. Work and Kinetic Energy

Work-Energy Equation

$$T_1 + U_{1-2} = T_2$$

- Positive work done on the body, increase kinetic energy.
- Negative work done on the body, reduce kinetic energy.

Advantage of Work-Energy Method

- No need to find acceleration first
- Get change in velocity directly from active force
- Can be applied to system of particle joined using frictionless and non-deformable link

3-6. Work and Energy

9. Power

- Power is defined as **time rate of work**.

Power

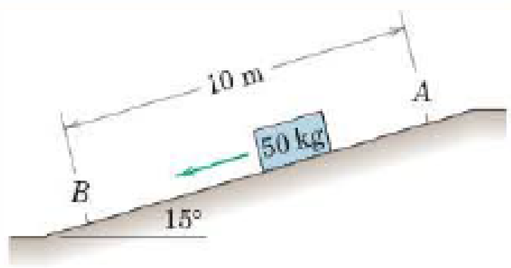
$$P = \vec{F} \cdot \vec{v}$$

- Unit = N · m/s or J/s
- For a machine, power tells how much **work** it can do in a **period of time**.

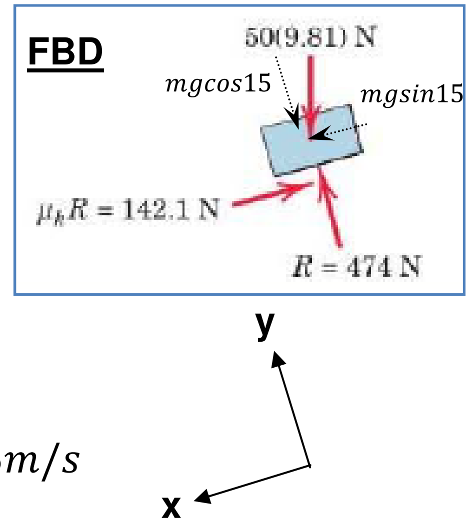
3-6. Work and Energy

Example 1: Crate and Chute

Calculate the velocity $\boldsymbol{v}$ of the $\mathbf{50}\,\boldsymbol{kg}$ crate when it reaches the bottom of the chute at $\boldsymbol{B}$ if it is gives an initial velocity of $\mathbf{4}\,\boldsymbol{m/s}$ down the at $\boldsymbol{A}$. The coefficient of kinetic friction is $\mathbf{0.30}$.



Solution



$$\nwarrow \sum Fy = 0 \Rightarrow R - mg\cos 15 = 0; \quad R = (50)(9.81)\cos 15 \Rightarrow R = 474\ N$$

$$\swarrow \sum Fx = mg\sin 15 - \mu R = (50)(9.81)\sin 15 - (0.3)(474) \Rightarrow Fx = -15.19\ N$$

$$U_{A-B} = \int_A^B F.dr = \int_A^B (Fx\,dx + Fy\,dy) \Rightarrow U_{A-B} = \int_{x1}^{x2} Fx\,dx = Fx\,(x_2 - x_1)$$

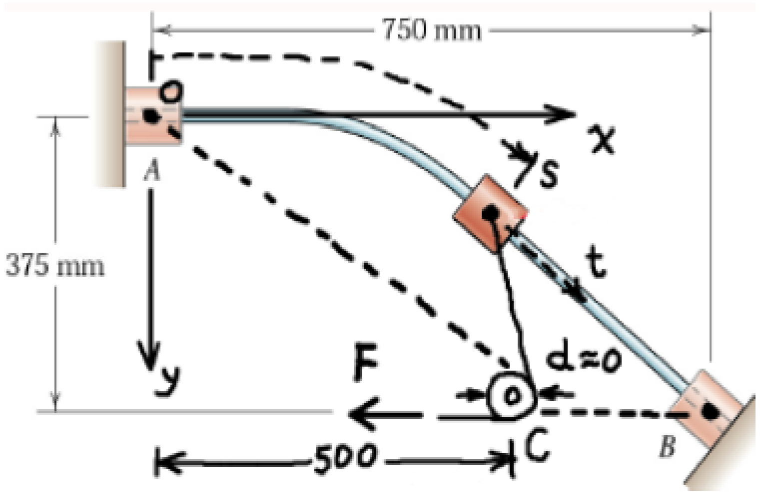
$$U_{A-B} = (-15.19)(10) = -151.9\ N.m$$

$$U_{A-B} = T_A - T_B = \frac{1}{2}m({v_2}^2 - {v_1}^2) \Rightarrow v_2 = \sqrt{U_{A-B}/(1/2)\,m + {v_1}^2}\,; \quad v_2 = 3.15m/s$$

3-6. Work and Energy

Example 2: Collar and Guide

Find the work done by the force F on the collar when it moves from point A to any point.



Solution:

$$U = \int_A^B \vec{F} \cdot d\vec{r}$$

$$\begin{aligned} U &= \int_A^B F_r\,\hat{e}_r \cdot d(r\,\hat{e}_r) \\ &= \int_A^B -F\,\hat{e}_r \cdot (dr\,\hat{e}_r + rd\theta\,\hat{e}_\theta) \\ &= \int_A^B -F\,dr \end{aligned}$$

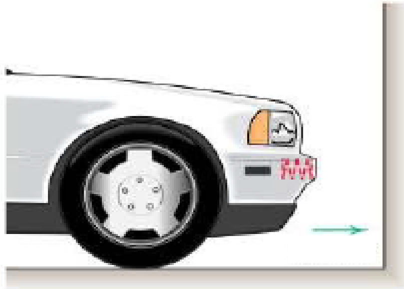
if F is a constant.

$$\begin{aligned} U &= \int_A^B -F\,dr = -F(\overline{BC} - \overline{AC}) \\ &= F(\overline{AC} - \overline{BC}) \end{aligned}$$

3-6. Work and Energy

Example 3: Spring Bumper

In the design of the spring bumper for a $\mathbf{1500}\ \boldsymbol{Kg}$ car, it is desired to bring the car to stop from a speed of $\mathbf{8}\ \boldsymbol{Km/h}$ in a distance equal to a $\mathbf{150}\ \boldsymbol{mm}$ of spring deformation. Specify the required stiffness $\boldsymbol{K}$ for each of the two springs behind the bumper.



Solution

$$U_{1-2} = T_2 - T_1;\ U_{1-2} = \int_1^2 F.dr = 2\left(\frac{1}{2}Kx^2\right)$$

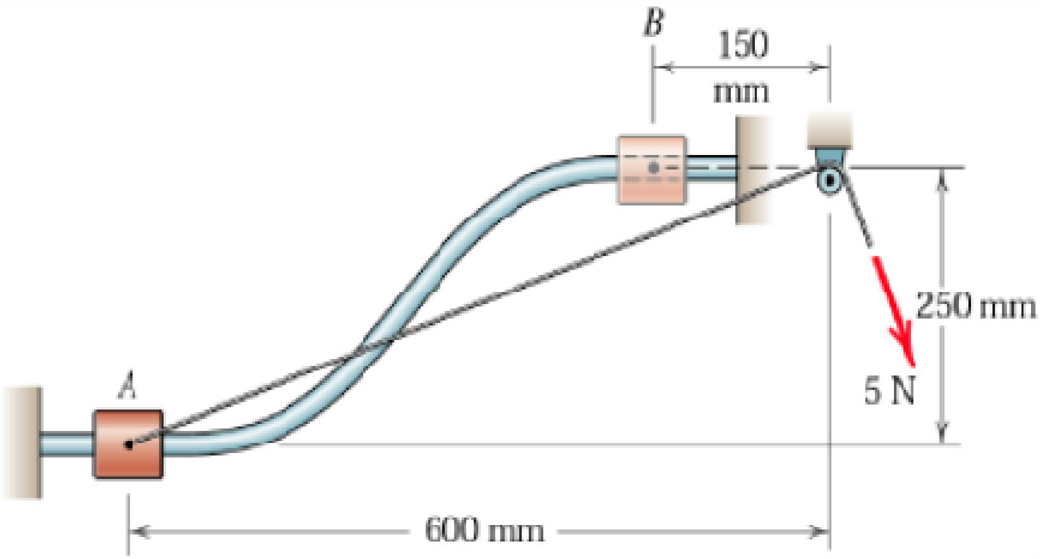
$$T_2 = \frac{1}{2}m{v_2}^2 = \frac{1500}{2}\left(8\times\frac{5}{18}\right)^2 = 3703.7\ N.m\ ;\ \ T_1 = \frac{1}{2}m{v_1}^2 = 0$$

$$Kx^2 = \frac{1}{2}m{v_2}^2 \ \Rightarrow\ k = \frac{1}{2}\frac{m{v_2}^2}{x^2} = 0.5\times\frac{1500(8\times\frac{5}{18})^2}{(0.150)^2} \ \Rightarrow\ k = 164.6\ kN/m$$

3-6. Work and Energy

Example 4: Slider

The 0.2-kg slider moves freely along the fixed curved rod from *A* to *B* in the vertical plane under the action of the constant 5-N tension in the cord. If the slider is released from rest at *A*, calculate its velocity *v* as it reaches *B*.

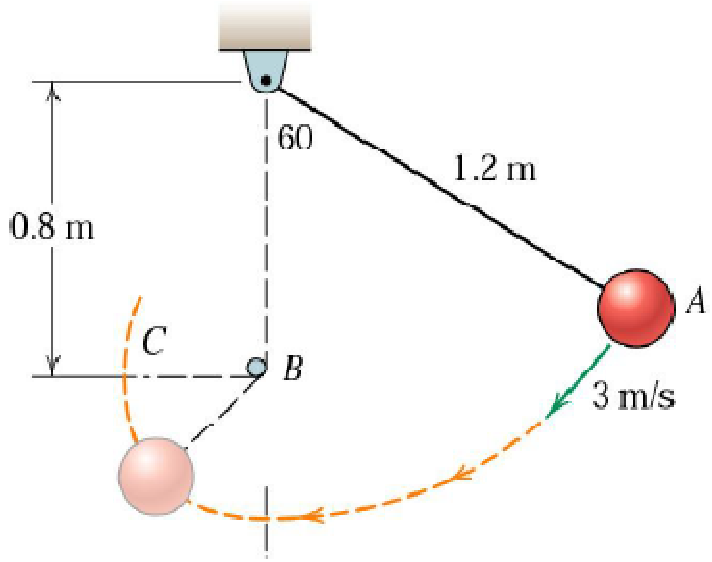


E

Ans: 4.48 m/s

3-6. Work and Energy

The ball is released from position *A* with a velocity of 3 m/s and swings in a vertical plane. At the bottom position, the cord strikes the fixed bar at *B*, and the ball continues to swing in the dashed arc. Calculate the velocity v_C of the ball as it passes position *C*.



Ans: 3.59 m/s

3/7 Potential Energy (PE)

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

3-7. Potential Energy

- 2. Potential Energy
  - Gravitational Potential Energy (V_g)
  - Elastic Potential Energy (V_e)
  - Alternate form of Work-KE equation
  - Examples

3-7. Potential Energy

1. Potential Energy

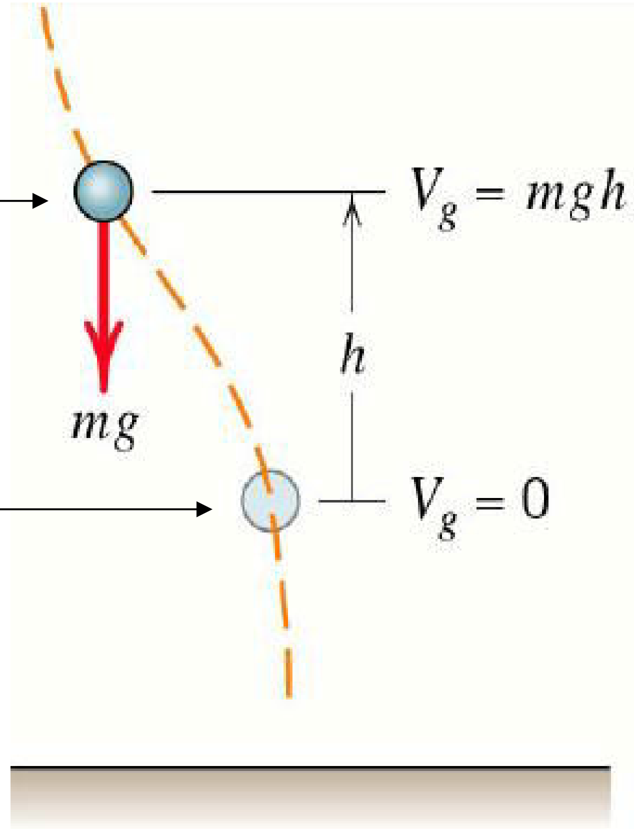
- Gravitational PE
- Elastic PE

1.1 Gravitational PE (V_g)

Potential Energy

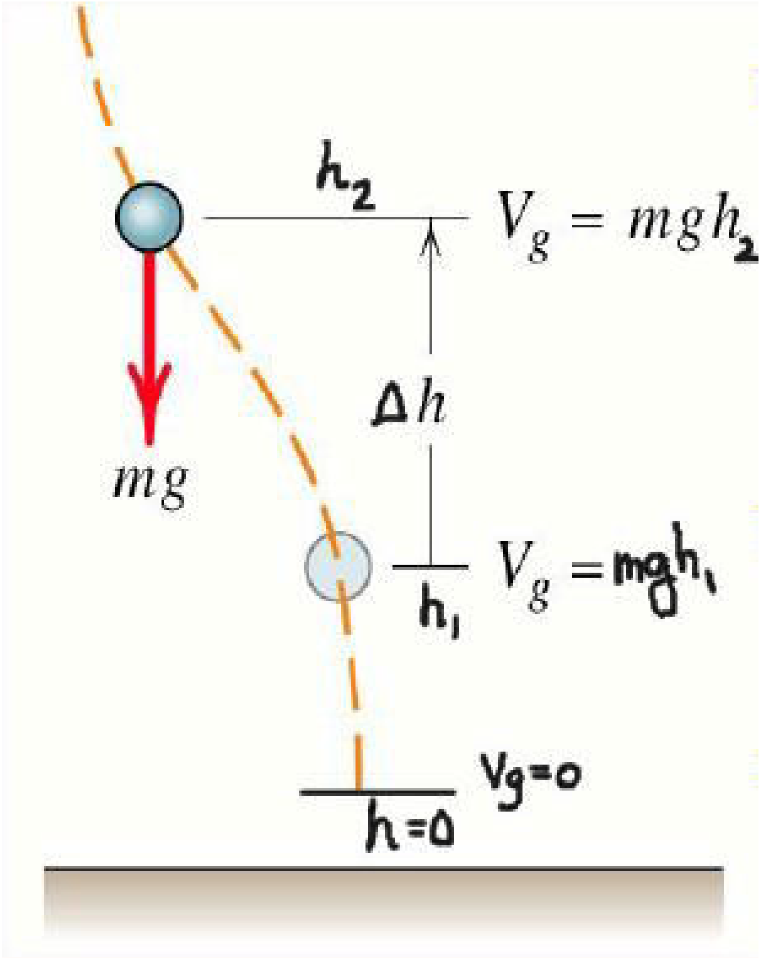
$$V_g = mgh$$

Reference



3-7. Potential Energy

1.1 Gravitational PE (V_g)



- Change in potential when going from $h = h_1$ to $h = h_2$

$$\Delta V_g = mg(h_2 - h_1) = mg\Delta h$$

- Start low finish high = go up

$$\Delta V_g = +$$

- Start high finish low = go down

$$\Delta V_g = -$$

3-7. Potential Energy

1.2 Elastic PE (V_e)

Elastic Potential Energy (of a linear spring)

$$V_e = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

x is how much the spring is compressed or extended from its relaxed (original length)

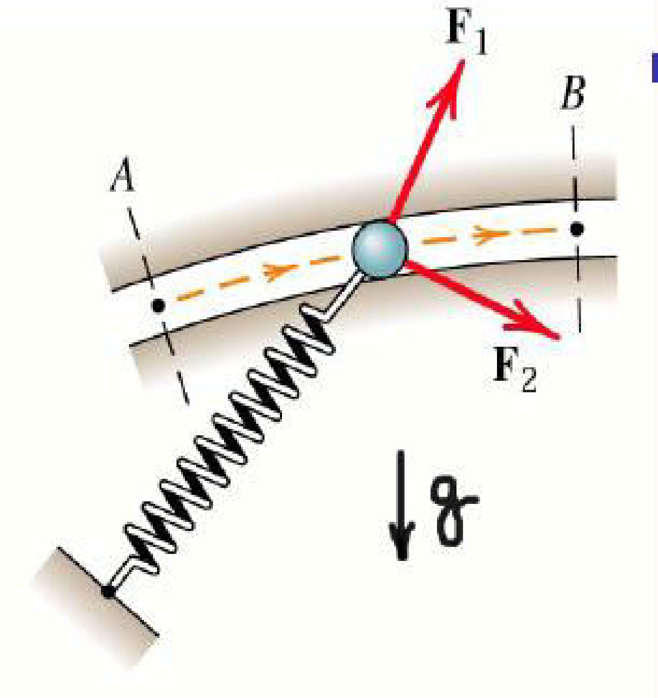
- Change in potential from x_1 to x_2

$$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2)$$

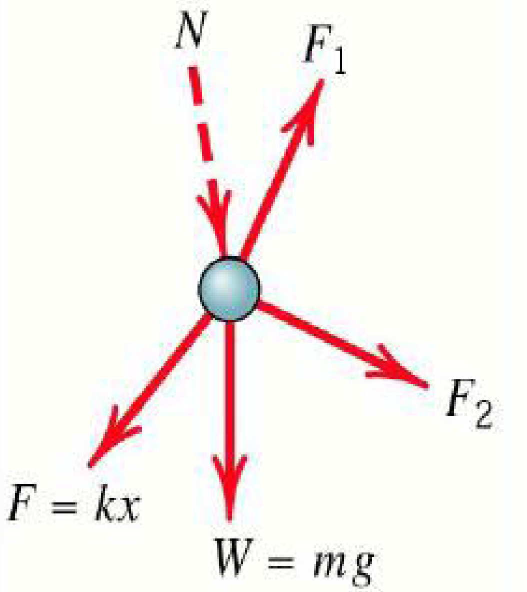
3-7. Potential Energy

2. Alternate form of Work-KE equation

- Consider a system



- Apply the work-energy equation to the particle



$$U_{1\text{-}2} = \Delta T$$

- We have $U_{1-2} = \Delta T$
- Recall that V_g is neg. of work by mg
- and, V_e is neg. of work on the particle.

$$U_{1-2} = U'_{1-2} - \Delta V_g - \Delta V_e = \Delta T$$

Work-Energy Equation

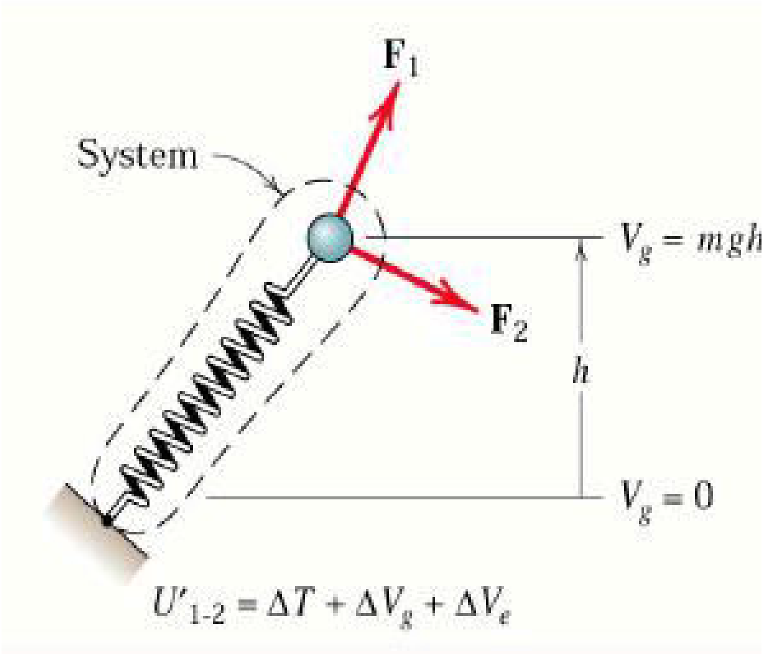
$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

- U'_{1-2} work of all external forces **other than** gravitational and spring forces.

3-7. Potential Energy

2. Alternate form of Work-KE equation

- Convenient to setup system with particle and spring



- Think of ΔV_e as energy in the spring
- Make sure you clearly define your system

- System with springs use: $U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

3-7. Potential Energy

2. Alternate form of Work-KE equation

Special Case (when there is no work from the external force other than mg and spring).

Law of Conservation of Mechanical Energy

- $U'_{1-2} = 0$
- Define E as the total energy of the system

$$E = T + V_g + V_e$$

- When no external force other than mg and spring

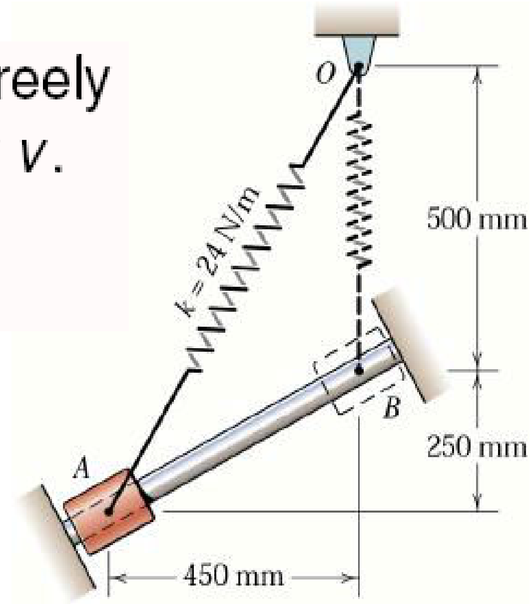
$$\Delta E = 0$$

- The energy of the system is conserved!

3-7. Potential Energy

Example 1: Spring and Slider

The 0.9-kg collar is released from rest at A and slides freely up the inclined rod, striking the stop at B with a velocity v. The spring of stiffness $k = 24 N/m$ has an unstreched length of 375 mm. Calculate v.



Solution:

$\overline{AO} = \sqrt{0.45^2 + 0.75^2} = 0.875\, m$

$x_2 = 0.5 - 0.375 = 0.125\, m; \qquad x_1 = 0.875 - 0.375 = 0.5\, m$

$$U'_{1-2} = 0 = \Delta T + \Delta V_g + \Delta V_e \Rightarrow \frac{1}{2} m({v_2}^2 - {v_1}^2) + mgh + \frac{1}{2} k({x_2}^2 - {x_1}^2) = 0$$

$U'_{1-2} = 0$ *(Law of Conservation of Mechanical Energy)*

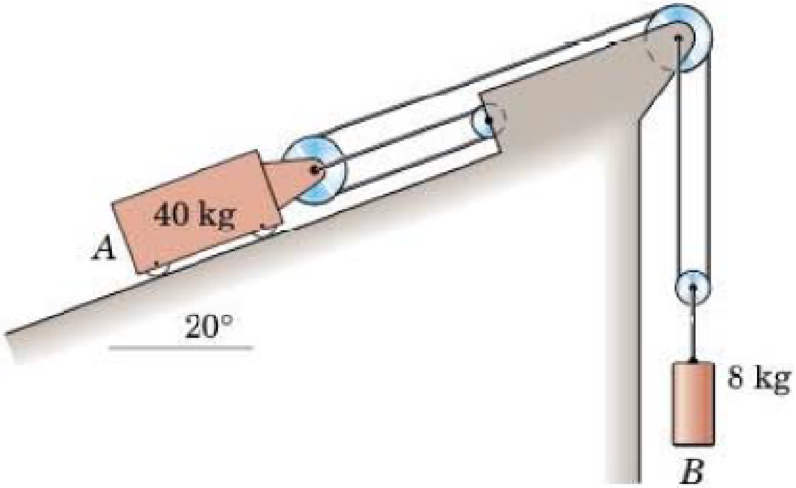
$$0.9({v_2}^2 - 0^2) + 2 \times 0.9 \times 9.81 \times 0.25 + 24(0.125^2 - 0.5^2) = 0$$

$${v_2}^2 = -(2 \times 0.9 \times 9.81 \times 0.25 + 24(0.125^2 - 0.5^2))/0.9 \quad \Rightarrow \quad v_2 = 1.16 m/s$$

3-7. Potential Energy

Example 2: Pulleys

If the system is released from **rest**, determine the **speeds** of both masses after **B** has moved **1** ***m***. Neglect friction and the masses of pulleys.



Solution: A force analysis reveals that A will move down & B will move up.

Kinematics : $3v_A = 2v_B$ (speeds)

$T_1 + V_1 = T_2 + V_2$, datum @ initial position

$$0 + 0 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B \left(\frac{3}{2} v_A\right)^2 + m_B g h_B - m_A g h_A$$

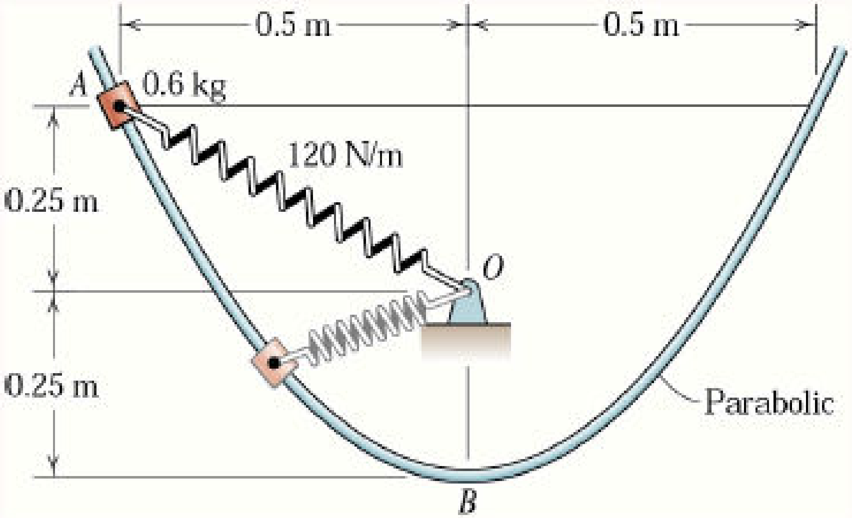
$$0 = \frac{1}{2}(40) v_A^2 + \frac{1}{2} 8 \frac{9}{4} v_A^2 + 8(9.81)(1) - 40(9.81)\left(\frac{2}{3}(1)\sin 20°\right)$$

$v_A = 0.616$ m/s, $v_B = \frac{3}{2} v_A = 0.924$ m/s

3-7. Potential Energy

Example 3: Spring and Slider

The 0.6-kg slider is released from rest at *A* and slides down the smooth parabolic guide (which lies in a vertical plane) under the influence of its own weight and of the spring of constant 120 N/m.



Determine the speed of the slider as it passes point *B* and the corresponding normal force exerted on it by the guide. The unstretched length of the spring is 200 mm.

Ans: $v_B = 5.92$ m/s; $N = 84.1$ N

3-7. Potential Energy

Example 4: Pole Vault

A 175 *lb* pole vaulter carrying a uniform 16 ft, 10 *lb* pole approaches the jump with a velocity v and manages to barely clear the bar set at a height of 18 ft. As he clears the bar, his velocity and that of the pole are essentially *zero*. Calculate the minimum possible value of v required for him to make the jump. Both the horizontal pole and the center of gravity of the vaulter are 42 *in*. above the ground during the approach.

Solution:

$U'_{1-2} = 0$ so $T_1 + V_{g_1} = T_2 + V_{g_2}$

Take datum $V_g = 0$ at ground level.

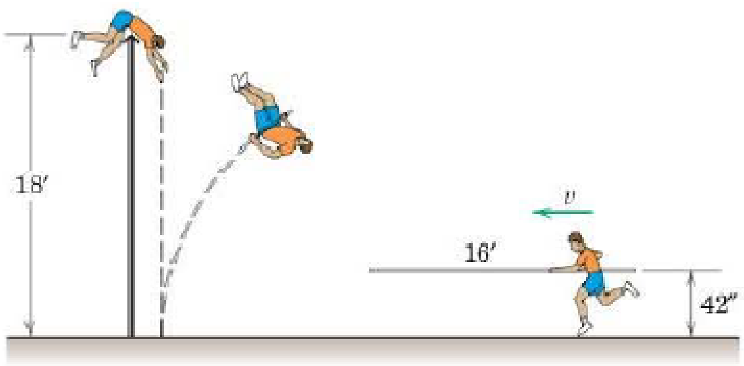
$T_1 = \frac{1}{2}\frac{175+10}{32.2}v^2 = 2.87v^2$, $T_2 = 0$

$V_{g_1} = (175+10)\frac{42}{12} = 648$ ft-lb

$V_{g_2} = 175(18) + 10(8) = 3230$ ft-lb

So $2.87v^2 + 648 = 0 + 3230$

$v = 30.0$ ft/sec or 20.4 mi/hr



Chapter 3
Kinetics of Particles

3-3 Impulse and Momentum

3-3. Impulse and Momentum

- 1. Linear Impulse/Momentum
  - Introduction
  - Definitions
  - Impulse-Momentum Equation
  - Conservation of Linear Momentum
- 2. Angular Impulse/Momentum
  - Definitions
  - Rate of Change of Angular Momentum
  - Angular Impulse-Momentum Principle
  - Plane Motion Applications
  - Conservation of Angular Momentum

3-3. Impulse and Momentum

- 3. Impact*
  - Direct Central Impact
  - Coefficient of Restitution
  - Energy Loss
  - Oblique Central Impact

3-3. Impulse and Momentum

1.1 Introduction

Work-Energy

- Recall: We integrated $\Sigma \vec{F} = m\vec{a}$ w.r.t displacement to get the work-engergy equation.
- Changes in velocity (or kinetic energy) can be found directly from work done on the body.
- Suitable when forces involved are functions of position.
- Not suitable if forces are functions of time!

Impulse-Momentum

- Here: integrate w.r.t. time to get impulse-momentum equation
- Good when forces are functions of time.
- Also good when forces are applied during very short period of time (impact problems)

3-3. Impulse and Momentum

1.2 Definitions

Linear Momentum

$$\vec{G} = m\vec{v}$$

← This is a **VECTOR**

Newton's second law $\Sigma\vec{F} = m\frac{d}{dt}\vec{v}$

Time Rate of Change of Linear Momentum

$$\Sigma\vec{F} = \frac{d\vec{G}}{dt}$$

- **The resultant force on a particle equals to its time rate of change of linear momentum.**
- Unit of linear momentum (SI), kg·m/s or N·s
- In components form, e.g. $\Sigma F_x = \dot{G}_x \qquad \Sigma F_y = \dot{G}_y$

3-3. Impulse and Momentum

1.3 Linear Impulse-Momentum Equation

- Integrate w.r.t. time $\Sigma\vec{F} = d\vec{G}/dt$

Impulse-Momentum Equation

$$\int_{t_1}^{t_2} \Sigma\vec{F}\, dt = G_2 - G_1 = \Delta G$$

- G_1 = linear momentum at time t_1
- G_2 = linear momentum at time t_2
- $\int_{t_1}^{t_2} \Sigma\vec{F}\, dt$ = (Total) linear impulse (from time t_1 to t_2)
- **The total linear impulse on m equals the corresponding change in linear momentum of m**

3-3. Impulse and Momentum

1.4 Conservation of Linear Momentum Equation

- When no resultant force

Conservation of Linear Momentum

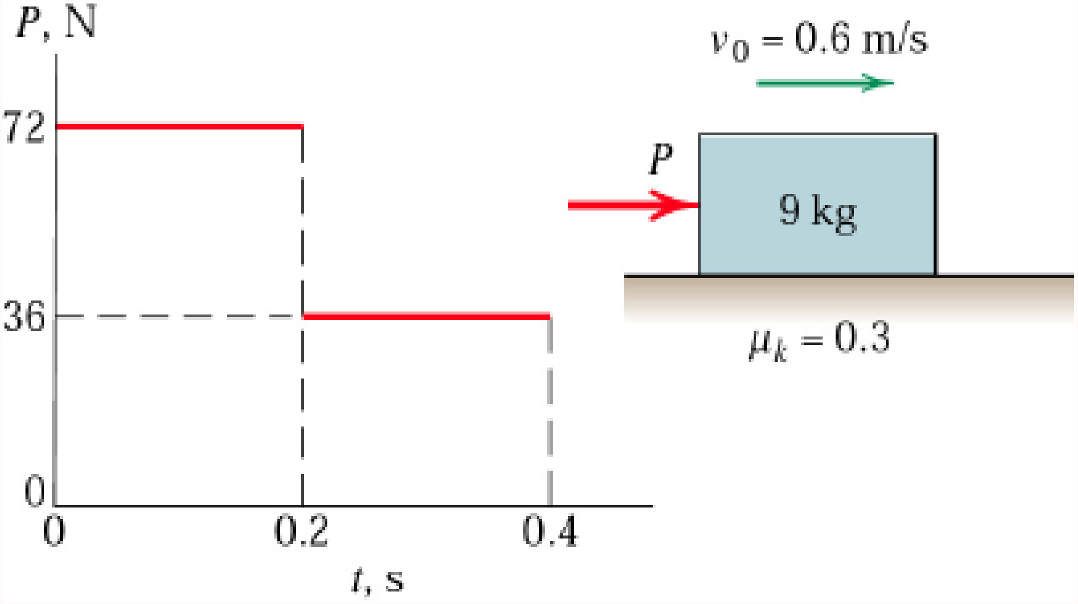
$$\Delta \vec{G} = 0$$

- For a system of particles,
- If only interactive forces $\vec{F}$ and $-\vec{F}$ are involved
- Linear Momentum of the system will be conserved.

3-3. Impulse and Momentum

Example 1: Sliding block

The 9-kg block is moving to the right with a velocity of 0.6 m/s on a horizontal surface when a force P is applied to it at time $t = 0$. Calculate the velocity v of the block when $t = 0.4s$. The kinetic coefficient of friction is $\mu_k = 0.3$

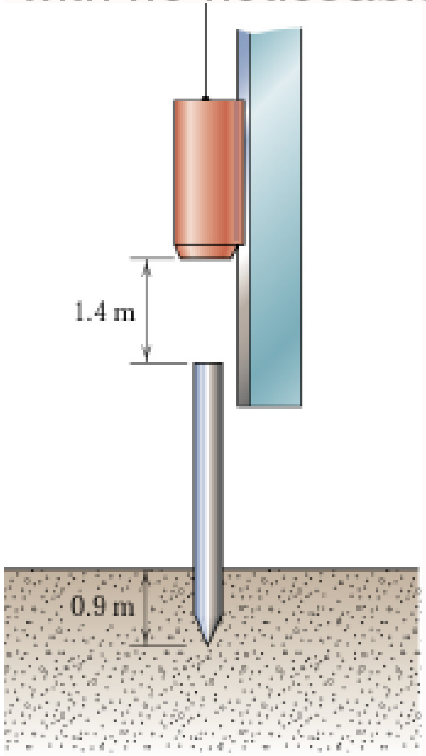


Ans: 1.823 m/s

3-3. Impulse and Momentum

Example 2: Sliding block

The 450-kg ram of a pile driver falls 1.4 m from rest and strikes the top of a 240-kg pile embeded 0.9 m in the ground. Upon impact the ram is seen to move with the pile with no noticeable rebound.



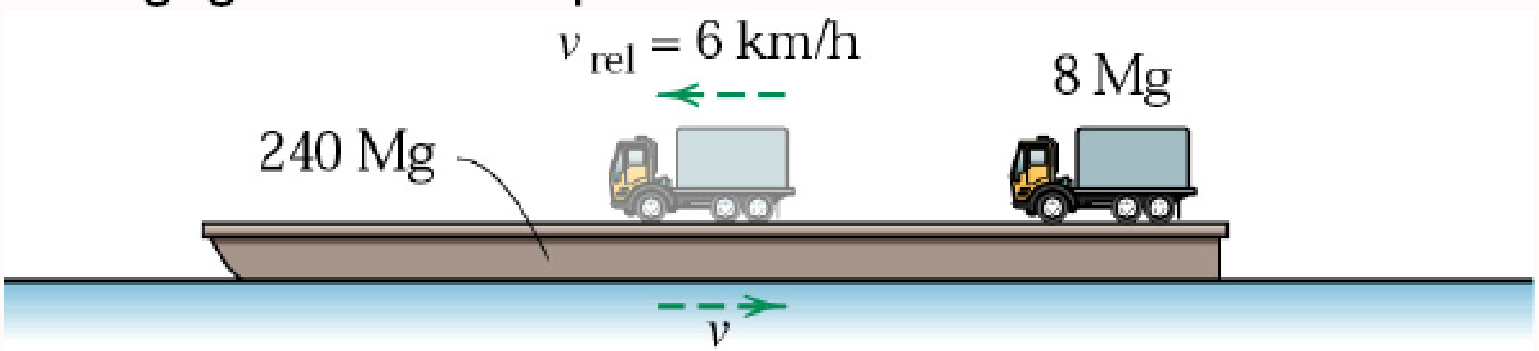
Determine the velocity *v* of the pile and ram immediately after impact. Can you justify using the principle of conservation of momentum even though the weights act during the impact?

Ans: 3.42 m/s

3-3. Impulse and Momentum

Example 3: Truck on a barge

An 8-Mg truck is resting on the deck of a barge which displaces 240 Mg and is at rest in still water. If the truck starts and drives toward the the bow at a speed relative to the barge $v_{rel} = 6$ km/h, calculate the speed v of the barge. The resistance to the motion of the barge through the water is negligible at low speeds.



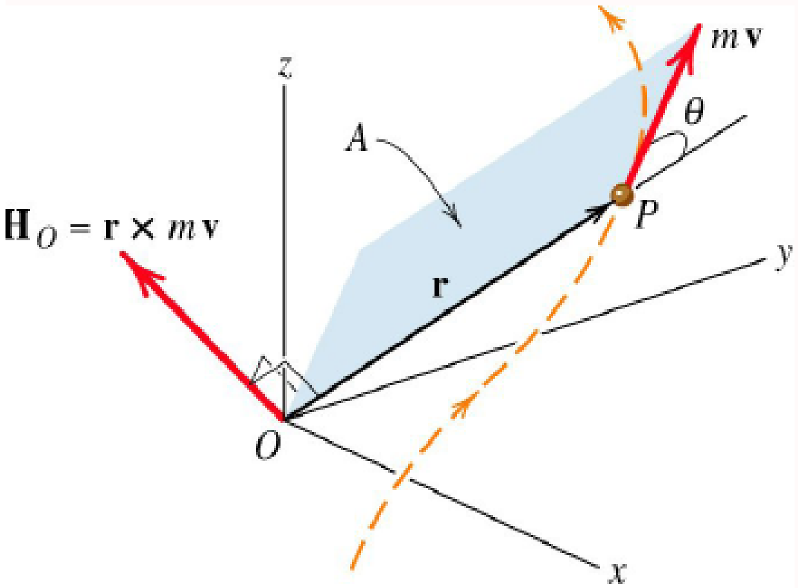
Ans: 0.1935 m/s

3-3. Impulse and Momentum

2. Angular Impulse and Momentum

2.1 Definitions

- Define: Moment of linear momentum = Angular momentum



- Recall: Linear Momentum

$$\vec{G} = m\vec{v}$$

- Moment about a point O

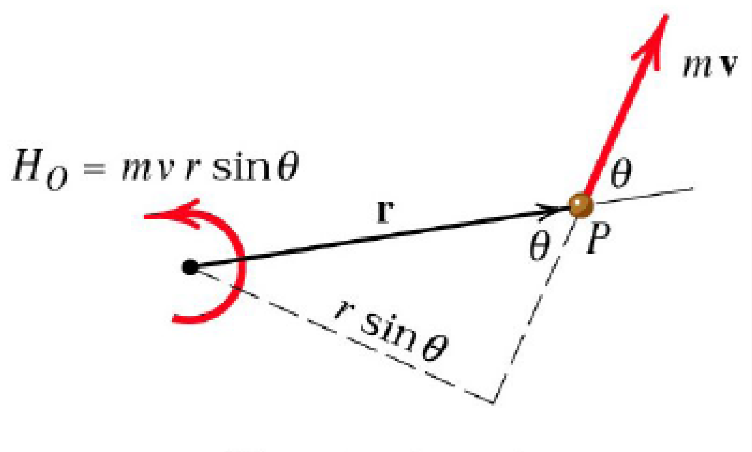
Angular Momentum

$$\vec{H}_O = \vec{r} \times m\vec{v}$$

3-3. Impulse and Momentum

2.1 Definitions

In the plane A,

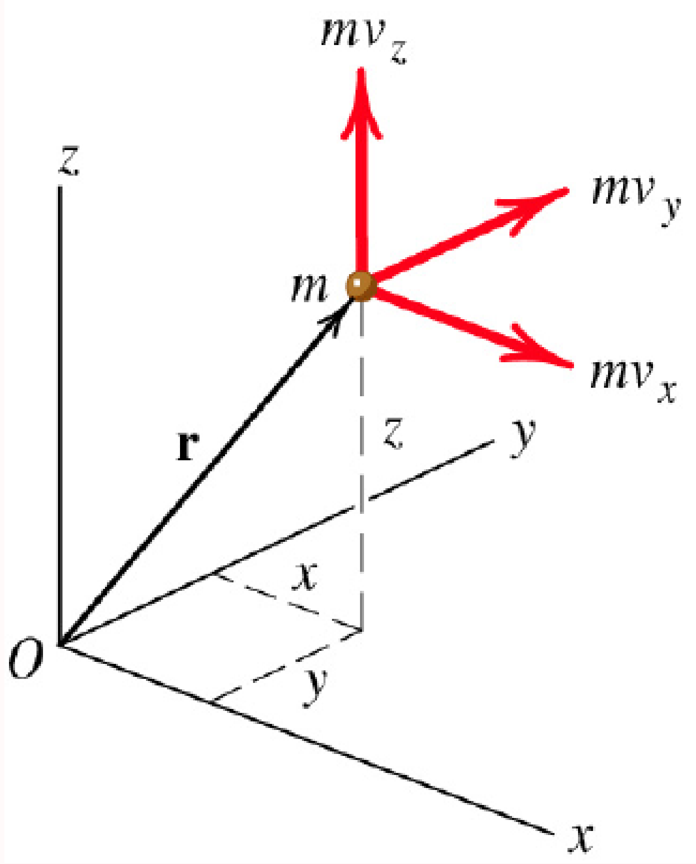


View in plane A

$$\vec{H}_O = \vec{r} \times m\vec{v}$$
$$H_O = mvr\sin\theta$$

3-3. Impulse and Momentum

2.1 Components of Angular Momentum*



- $\vec{H}_O = \vec{r} \times m\vec{v}$

$$\vec{H}_O = m\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix}$$

$$\begin{aligned} \vec{H}_O &= m(v_z y - v_y z)\hat{\imath} \\ &+ m(v_x z - v_z x)\hat{\jmath} \\ &+ m(v_y x - v_x y)\hat{k} \end{aligned}$$

$$H_x = m(v_z y - v_y z), \quad H_y = m(v_x z - v_z x), \quad H_z = m(v_y x - v_x y)$$

3-3. Impulse and Momentum

2.2 Rate of Change of Angular Momentum

For a particle with a resultant force $\Sigma\vec{F}$

- Moment about point O

$$\Sigma\vec{M}_O = \vec{r} \times \Sigma\vec{F}$$

- From $\Sigma\vec{F} = m\dot{\vec{v}}$

$$\Sigma\vec{M}_O = \vec{r} \times m\dot{\vec{v}}$$

- See that $\dot{\vec{H}}_O = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}}$
- Since $\dot{\vec{r}} = \vec{v}$, we have $\dot{\vec{H}}_O = \vec{r} \times m\dot{\vec{v}}$

Rate of Change of Angular Momentum

$$\Sigma\vec{M}_O = \dot{\vec{H}}_O$$

3-3. Impulse and Momentum

2.2 Rate of Change of Angular Momentum

Rate of Change of Angular Momentum

$$\Sigma \vec{M}_O = \dot{\vec{H}}_O$$

- **The moment about the fixed point O of all forces acting on m equals the time rate of change of angular momentum of m about O**

- In component form,

$$\Sigma M_{O_x} = \dot{H}_{O_x}, \quad \Sigma M_{O_y} = \dot{H}_{O_y}, \quad \Sigma M_{O_z} = \dot{H}_{O_z}$$

3-3. Impulse and Momentum

2.3 Angular Impulse-Momentum Principle

- Recall: $\Sigma \vec{M}_O = \dot{\vec{H}}_O$
- Integrate w.r.t. to time

Angular Impulse-Momentum Principle

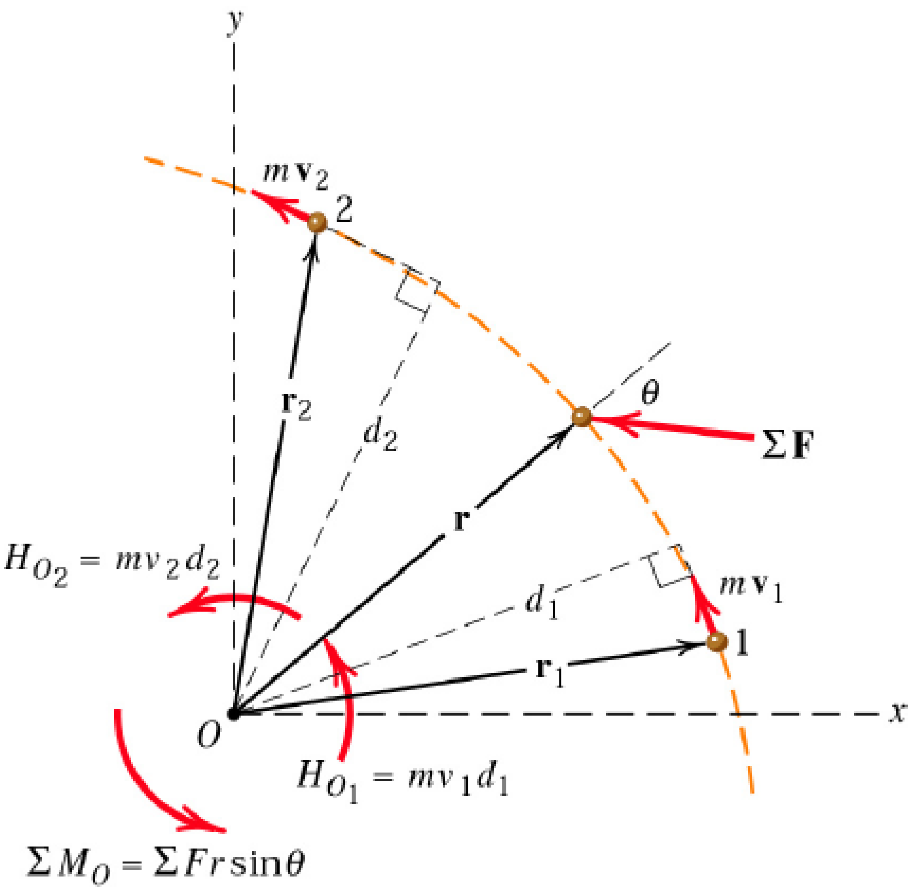
$$\int_{t_1}^{t_2} \Sigma \vec{M}_O \, dt = \vec{H}_{O_2} - \vec{H}_{O_1} = \Delta \vec{H}_O$$

- **The total angular impulse on m about the fixed point O equals the correponding change in angular momentum of m about O**
- In component form, we have

$$\begin{aligned}\int_{t_1}^{t_2} \Sigma M_{O_x} \, dt &= (H_{O_x})_2 - (H_{O_x})_1 \\ &= m[(v_z y - v_y z)_2 - (v_z y - v_y z)_1]\end{aligned}$$

2.4 Plane Motion Application

For plane motion



- Moment about point *O* in *z* direction (+CCW)

$$\int_{t_1}^{t_2} \Sigma M_O \, dt = H_{O_2} - H_{O_1}$$

- Or,

$$\int_{t_1}^{t_2} \Sigma Fr\sin\theta \, dt = mv_2 d_2 - mv_1 d_1$$

3-3. Impulse and Momentum

2.5 Conservation of Angular Momentum

- When $\Sigma \vec{M}_O = 0$

Conservation of Angular Momentum

$$\Delta \vec{H}_O = 0 \qquad \text{or} \qquad \vec{H}_{O_1} = \vec{H}_{O_2}$$

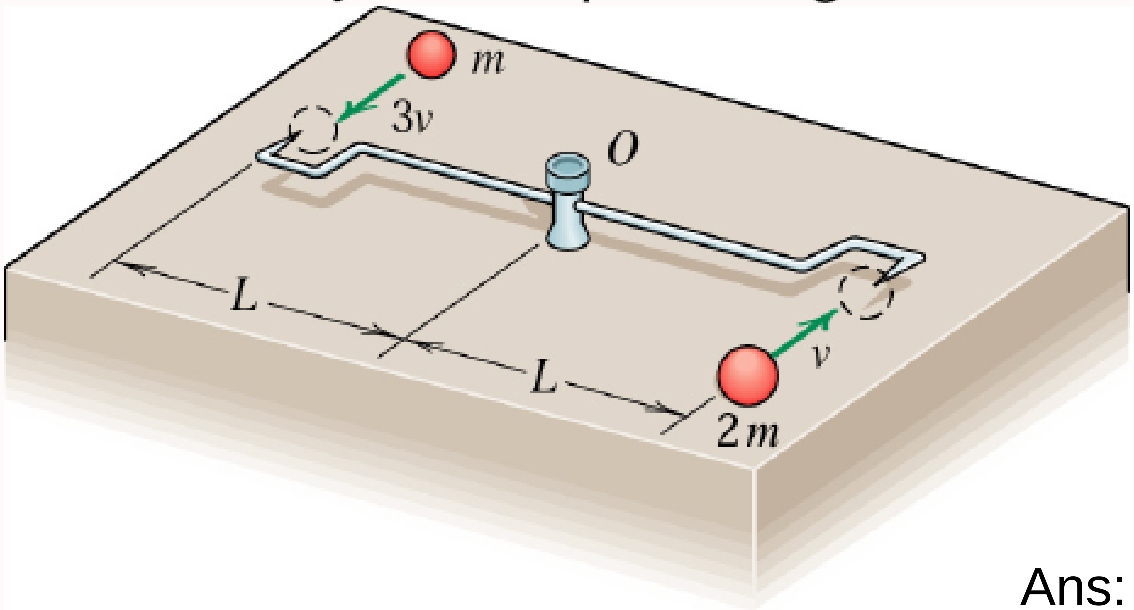
- Momentum may be conserved only about some axis; e.g.,

$$\Sigma M_x = 0 \quad \Longrightarrow \quad \Delta H_x = 0$$

3-3. Impulse and Momentum

Example 4: Rotating spheres and rod

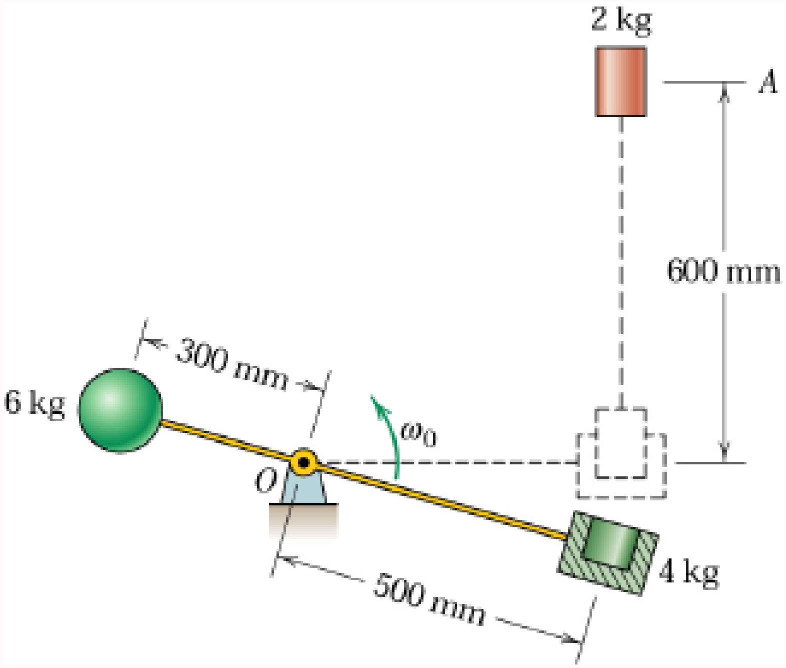
The small spheres, which have the masses and initial velocities shown in the figure, strike and become attached to the spiked ends of the rod, which is freely pivoted at *O* and is initially at rest. Determine the angular velocity ω of the assembly after impact. Neglect the mass of the rod.



Ans: 5v/3L

Example 5: Rotating spheres and rod

The 6-kg sphere and 4-kg block (shown in section) are secured to the arm of negligible mass which rotates in the vertical plane about a horizontal axis at *O*. The 2-kg plug is release from rest at *A* and falls into the recess in the block when the arm has reached the horizontal posiion.



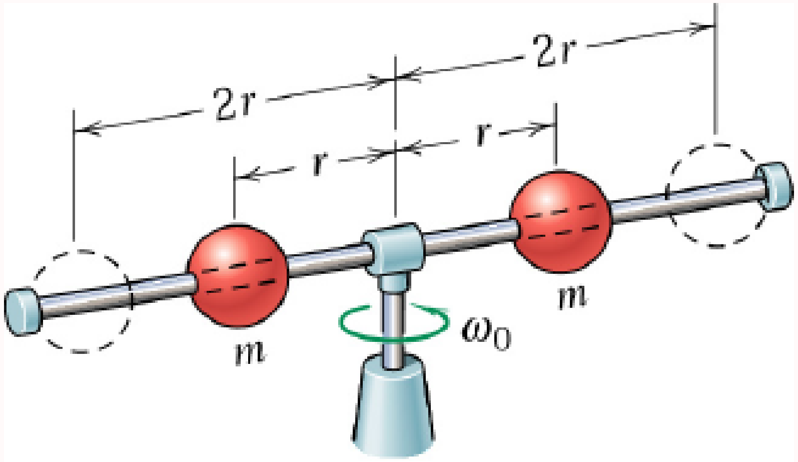
An instant before engagement, the arm has an angular velocity ω_o = 2 rad/s. Determine the angular velocity ω of the arm imediately after the plug as wedged itself in the block.

Ans: 0.172 rad/s CW

3-3. Impulse and Momentum

Example 6: Rotating spheres and rod

The two sheres of equal mass m are able to slide along the horizontal rotating rod. If they are initially latched in position a distance r from the rotating axis with the assembly rotating freely with an angular velocity ω_0, determine the new angular velocity ω after the spheres are released and finally assume positions at the ends of the rod at a radial distance of $2r$.



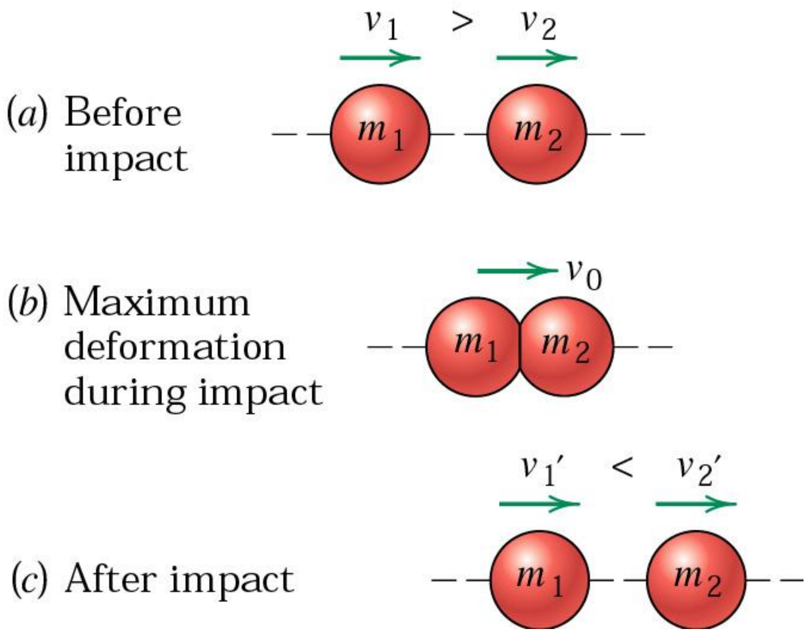
Also find the fraction n of the inital kinetic energy of the system which is lost. Neglect the small mass of the rod and shaft.

3-3. Impulse and Momentum

3. Impact*

- Impact = collision between two bodies
- The impact force is large and acts over short time

3.1 Direct Central Impact



- Conservation of Linear Momentum of the system (two masses) in the impact direction→ $\Delta G_x=0$

$$m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$$

3-3. Impulse and Momentum

3.2 Coefficient of Restitution

- Coefficient of Restitution (*e*) tells how much the bodies can recover from the impact

$$e = \frac{(v_2' - v_1')}{(v_1 - v_2)}$$

$$= \frac{|\text{relative velocity of separation}|}{|\text{relative velocity of approach}|}$$

3-3. Impulse and Momentum

3.3 Energy Loss

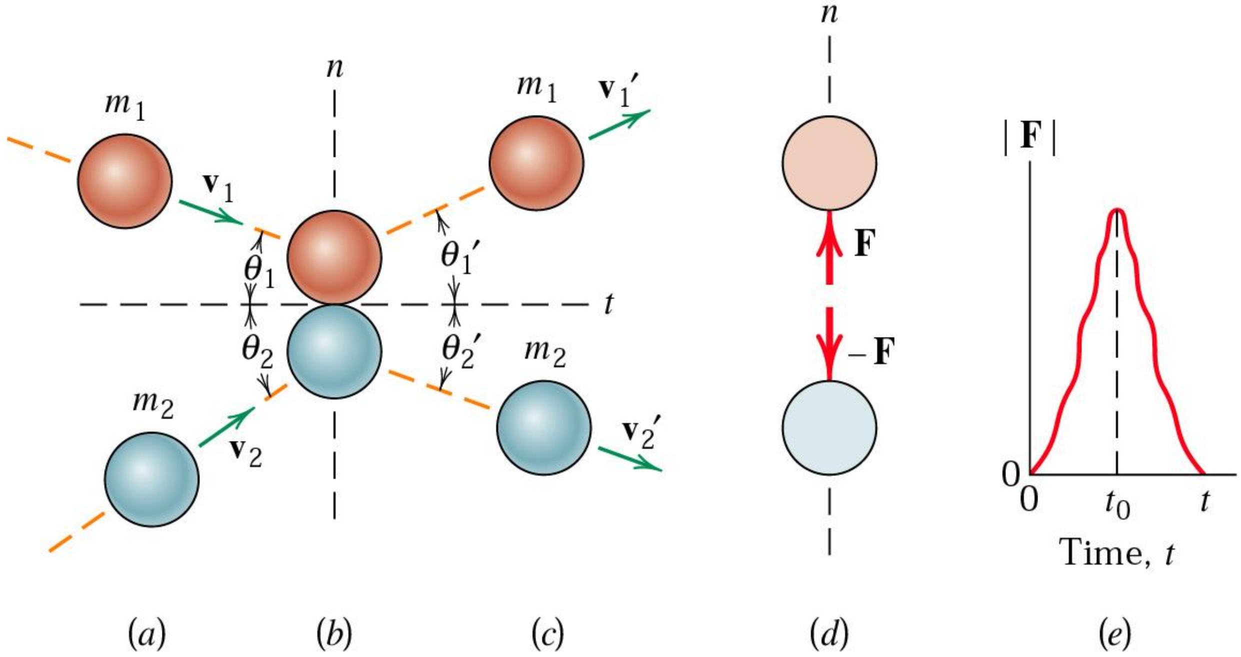
- Usually, KE is lost into heat due to the impact

- If $e = 1 \rightarrow$ No KE is lost $\rightarrow$ elastic impact
- If $0 < e < 1 \rightarrow$ Some KE is lost $\rightarrow$ partially inelastic impact
- If $e = 0 \rightarrow$ KE loss is max $\rightarrow$ plastic or completely/perfectly inelastic impact [bodies sticks together after impact]

- Note: Linear momentum of the system is still conserved!

3-3. Impulse and Momentum

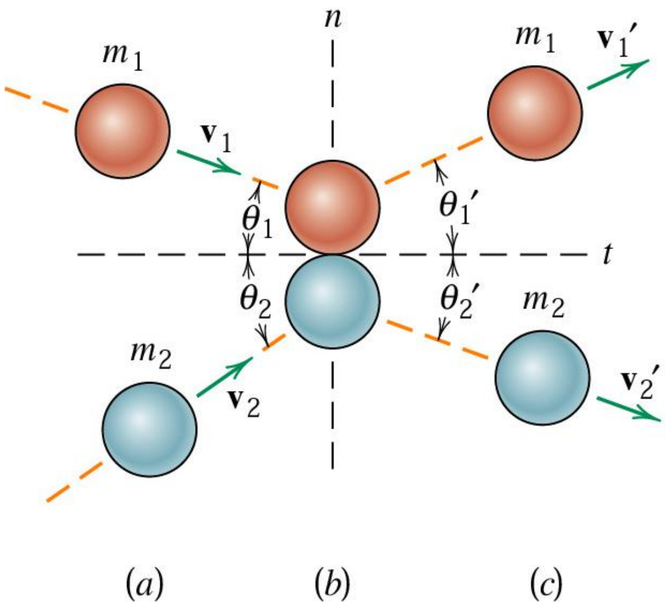
3.4 Oblique Central Impact



- Initial and final velocities are not parallel.
- The impact forces are in the *n*-direction

3-3. Impulse and Momentum

3.4 Oblique Central Impact



- Momentum of the system in the *n*-direction is conserved.

$$m_1 v_{1n} + m_2 v_{2n} = m_1 v_{1n}' + m_2 v_{2n}'$$

- Momentum of each body in the *t*-direction is conserved

$$v_{1t} = v_{1t}'$$

$$v_{2t} = v_{2t}'$$

- Coefficient of Restitution applies to *n*-direction

$$e = \frac{(v_2' - v_1')}{(v_1 - v_2)}$$

Chapter 5

Plane Kinematics of Rigid Bodies

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

5. Plane Kinematics

- Introduction
- 5.1 Rotation
- 5.2 Absolute Motion
- 5.3 Relative Velocity
- 5.4 Relative Acceleration
- 5.5 Instantaneous Center of Zero Velocity
- 5.6 Motion Relative to Rotating Axes

5. Plane Kinematics

1.1 Introduction

	Particle	Rigid Body
Size	Small & Not important in analysis	Big & Important in analysis
Motion	Translation only	Can be Translation or Rotation or both

- Rigid body

  = a body with negligible deformation

  = distance between any two points in a rigid body is constant

5. Plane Kinematics

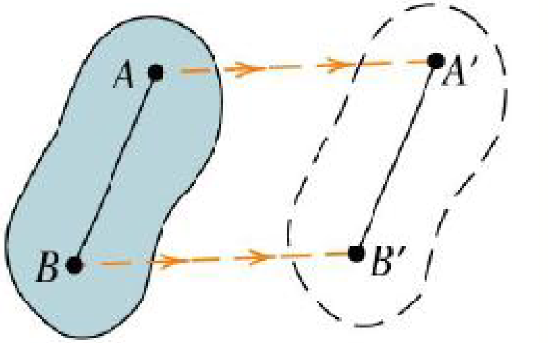
1.2 Motions of a Rigid Body

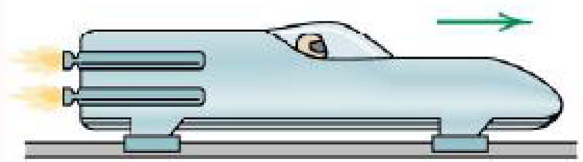
- 1. In space = three dimensions
- 2. In plane = two dimensions
  - Translation
    - Rectilinear
    - Curvilinear
  - Rotation

5. Plane Kinematics

1.3 Plane Motions of a Rigid Body (Type of Motion)

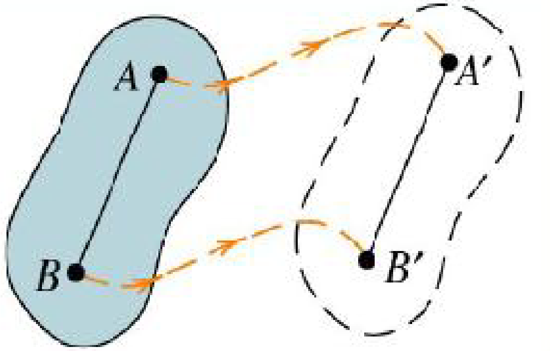
- Translation
  - Rectilinear Translation

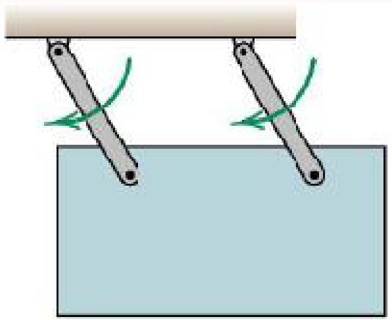




Rocket test sled

  - Curviliear translation



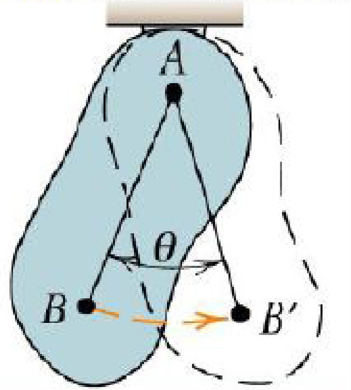


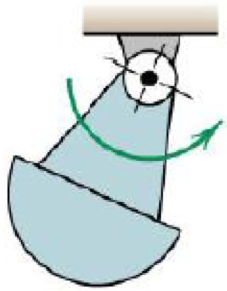
Parallel-link swinging plate

5. Plane Kinematics

1.3 Plane Motions of a Rigid Body

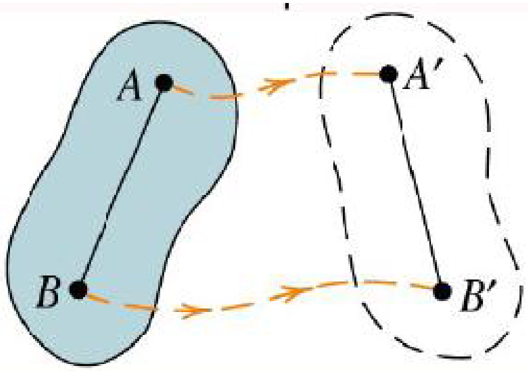
- Rotation
  - Fixed-axis rotation

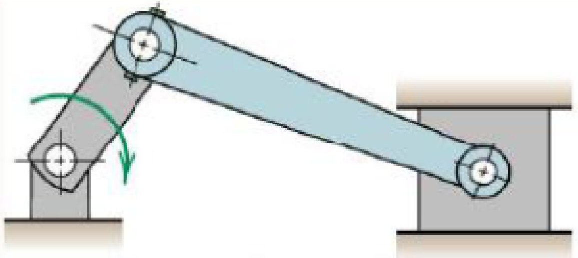




Compound pendulum

- General Plane Motion = Translation + Rotation



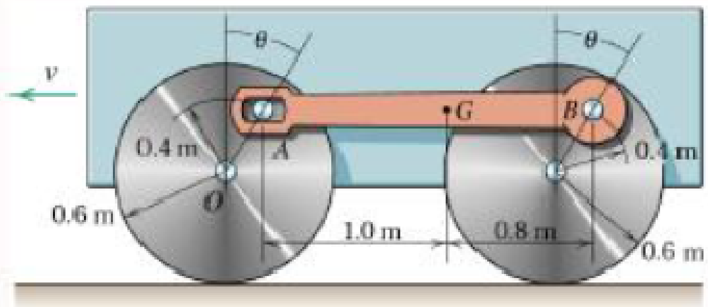


Connecting rod in a reciprocating engine

5. Plane Kinematics

1.3 Plane Motions of a Rigid Body

What is the type of motion of these bodies?

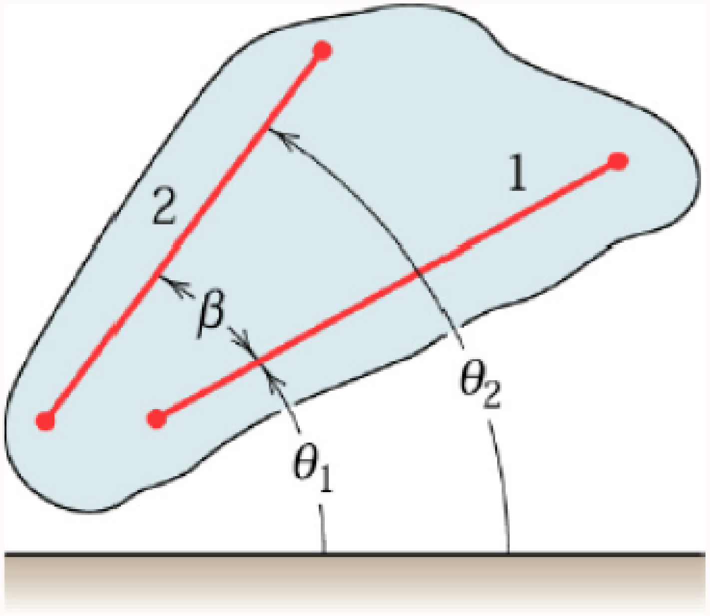


- Wheel?, Car?, Link AB?
- Ferris wheel: the wheel?, the car?

5-1 Rotation

5-1 Rotation

- How to describe rotation of a rigid body?



- **Angle between any line on a body and a reference line can be used to measure rotation of the body.**
- $\theta_2 = \theta_1 + \beta$
- For a rigid body, β = constant.
- Angular velocity $\dot{\theta}_2 = \dot{\theta}_1$
- Angular acceleration $\ddot{\theta}_2 = \ddot{\theta}_1$

- ω as well as α is the same for every point

5-1 Rotation

Rotation

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega\, d\omega = \alpha\, d\theta$$

- For constant angular acceleration (α=constant), we have

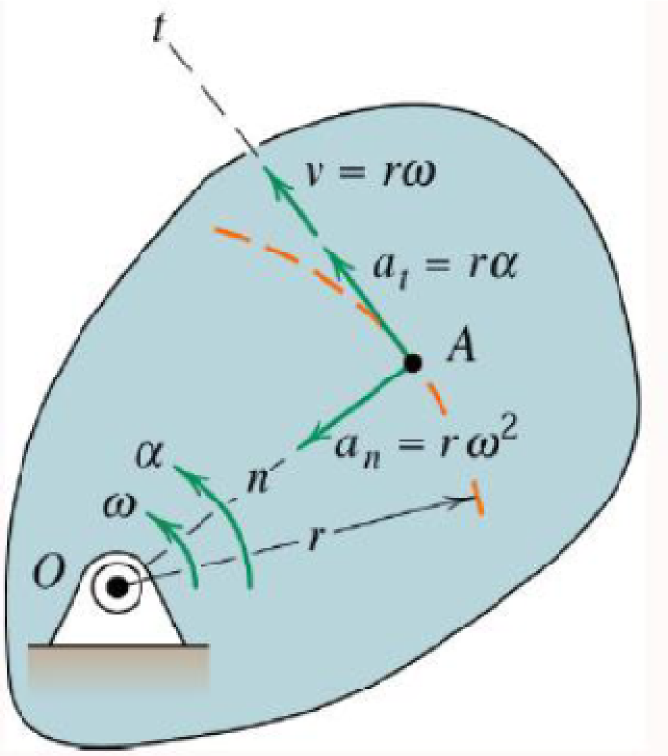
$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

5-1 Rotation

1. Rotation about a Fixed Axis



- Any point in the body moves in circular motion
- For Point A

Circular Motion

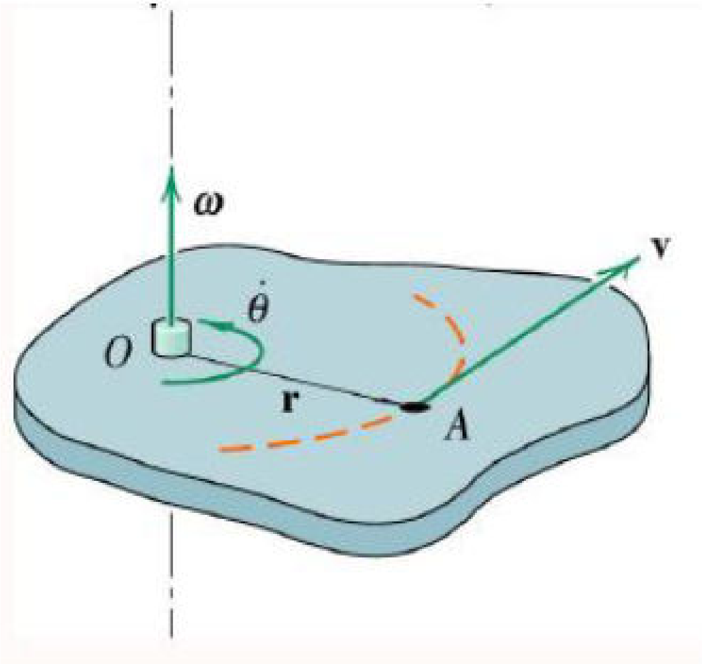
$$v = r\omega$$
$$a_n = r\omega^2 = v^2/r = v\omega$$
$$a_t = r\alpha$$

- Note: *v* and *a* of other points are different because of different *r* (ω and α are the same)

5-1 Rotation

1. Rotation about a Fixed Axis

Velocity



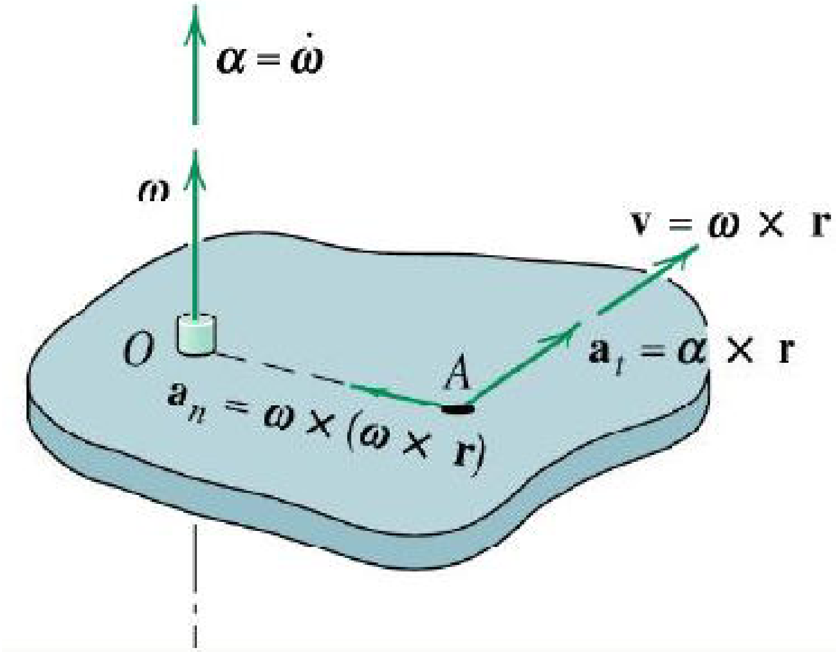
- The equations can be rewritten in a vector form (for plane motion)
- Direction of ω is given using the right-hand rule.

Velocity (Pure Rotation)
$\vec{V} = \vec{\omega} \times \vec{r}$

5-1 Rotation

1. Rotation about a Fixed Axis

Acceleration



- Direction of α is given using the right-hand rule.

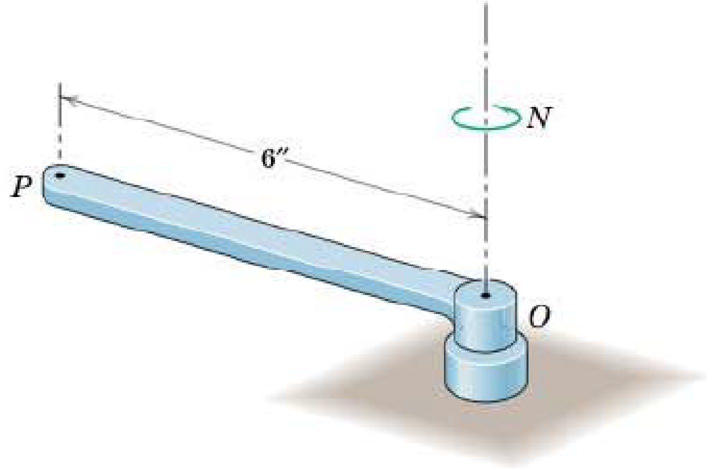
Acceleration (Pure Rotation)

$$\vec{a}_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$
$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

5-1 Rotation

Example 1: Rotating arm

The rotating arm starts from rest and acquires a rotational speed $N = 600\, rev/min$ in 2 seconds with constant angular acceleration. Find the time t after starting before the acceleration vector of end P makes an angle of 45^o with the arm OP.



Solution

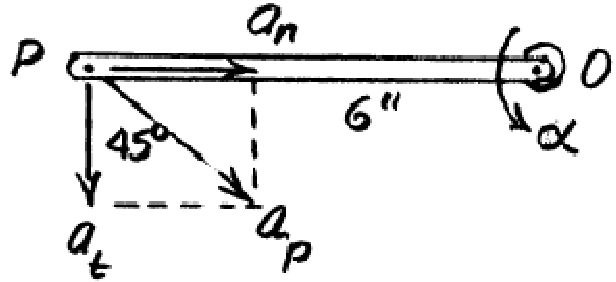
$$\alpha = \frac{600(2\pi)}{60}\frac{1}{2} = 10\pi\ rad/sec^2$$

$$a_t = r\alpha = 6(10\pi) = 60\pi\ in./sec^2$$

$$a_n = r\omega^2 = 60\pi\ in./sec^2 \text{ for } 45^o$$

$$\text{so } \omega^2 = 60\pi/6 = 10\pi, \quad \omega = 5.60\ rad/s$$

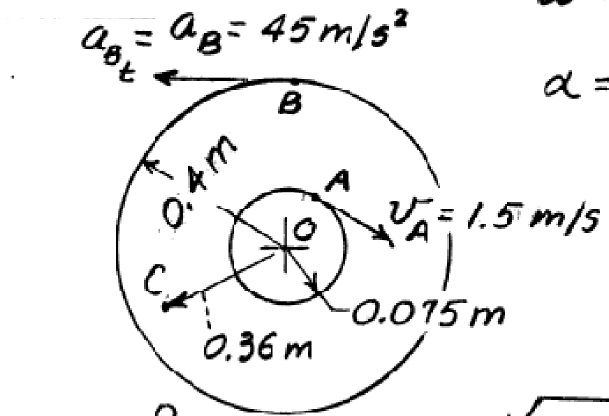
$$\omega = \omega_o + \alpha t : \ 5.60 = 0 + 10\pi t, \quad \underline{t = 0.1784\ sec}$$



Example 2: V-belt pulleys

The two V-belt pulleys form an integral unit and rotate about the fixed axis at O. At a certain instant, point A on the belt of smaller pulley has a velocity $v_A = 1.5\, m/s$, and point B on the belt of the larger pulley has an acceleration $a_B = 45\, m/s^2$ as shown. For this instant determine the magnitude of the acceleration a_C of point C and sketch the vector in your solution.

Solution



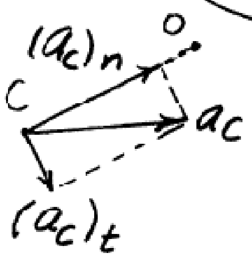
$$\omega = v/r = \frac{1.5}{0.075} = 20\ rad/s$$

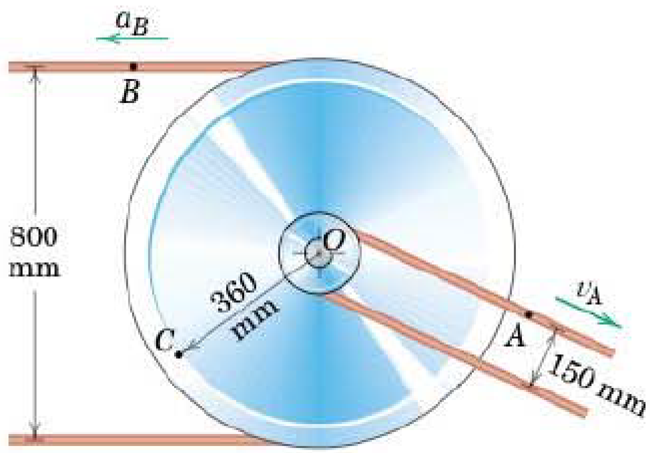
$$\alpha = a_t/r = \frac{45}{0.4} = 112.5\ rad/s^2$$

$$(a_c)_n = r\omega^2 = 0.36(20)^2 = 144\ m/s^2$$

$$(a_c)_t = r\alpha = 0.36(112.5) = 40.5\ m/s^2$$

$$a_c = \sqrt{(144)^2 + (40.5)^2} = \underline{149.6\ m/s^2}$$

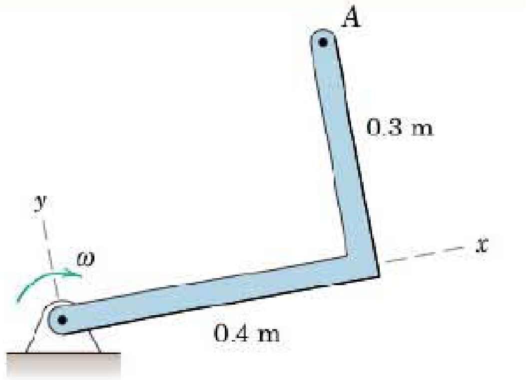




5-1 Rotation

Example 3: L-shaped bar

The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4 rad/s^2. Write the vector expression for the velocity and acceleration of point *A* when ω = 2 rad/s.



Solution. Using the right-hand rule gives

$$\boldsymbol{\omega} = -2\mathbf{k} \text{ rad/s} \qquad \text{and} \qquad \boldsymbol{\alpha} = +4\mathbf{k} \text{ rad/s}^2$$

The velocity and acceleration of *A* become

$$[\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}] \qquad \mathbf{v} = -2\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = 0.6\mathbf{i} - 0.8\mathbf{j} \text{ m/s} \qquad \textit{Ans.}$$

$$[\mathbf{a}_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})] \qquad \mathbf{a}_n = -2\mathbf{k} \times (0.6\mathbf{i} - 0.8\mathbf{j}) = -1.6\mathbf{i} - 1.2\mathbf{j} \text{ m/s}^2$$

$$[\mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}] \qquad \mathbf{a}_t = 4\mathbf{k} \times (0.4\mathbf{i} + 0.3\mathbf{j}) = -1.2\mathbf{i} + 1.6\mathbf{j} \text{ m/s}^2$$

$$[\mathbf{a} = \mathbf{a}_n + \mathbf{a}_t] \qquad \mathbf{a} = -2.8\mathbf{i} + 0.4\mathbf{j} \text{ m/s}^2 \qquad \textit{Ans.}$$

The magnitudes of **v** and **a** are

$$v = \sqrt{0.6^2 + 0.8^2} = 1 \text{ m/s} \qquad \text{and} \qquad a = \sqrt{2.8^2 + 0.4^2} = 2.83 \text{ m/s}^2$$

5-1 Rotation

Example 4: Right-angle bar

The right-angle bar rotates about the z-axis through O with an angular acceleration $\alpha = 3\,rad/s^2$ in the direction shown. Determine the velocity and acceleration of point P when the angular velocity reaches the value $\omega = 2\,rad/s$.

Solution

$$v_P = \omega \times r = 2k \times [0.5i + 0.2j + 0.050k]$$
$$= -0.4i + j \quad m/s$$

$$a_P = \alpha \times r + \omega \times (\omega \times r)$$
$$= -3k \times [0.5i + 0.2j + 0.050k]$$
$$+ 2k \times [2k \times (0.5i + 0.2j + 0.050k)]$$
$$= -1.4i - 2.3j \quad m/s^2$$

Note that r could have been taken as $0.5i + 0.2j$ m

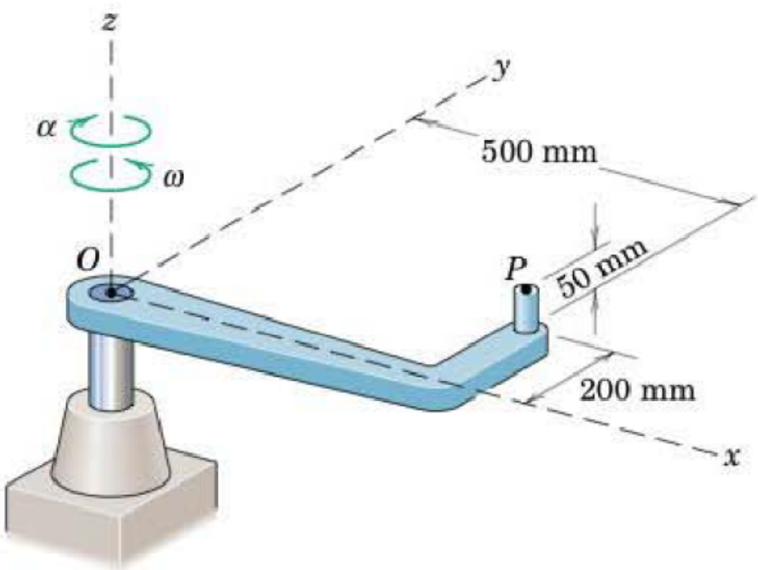
The magnitudes of the above results are

$v_P = 1.077$ m/s and $a_P = 2.69$ m/s^2.

These magnitudes check with

$$v_P = r_{xy}\,\omega = \sqrt{0.5^2 + 0.2^2}\,(2) = 1.077 \ m/s^2 \ ✓$$

and $$a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{(r_{xy}\alpha)^2 + (r_{xy}\omega^2)^2}$$
$$= \sqrt{0.5^2 + 0.2^2}\,\sqrt{3^2 + 2^4} = 2.69 \ m/s^2 \ ✓$$

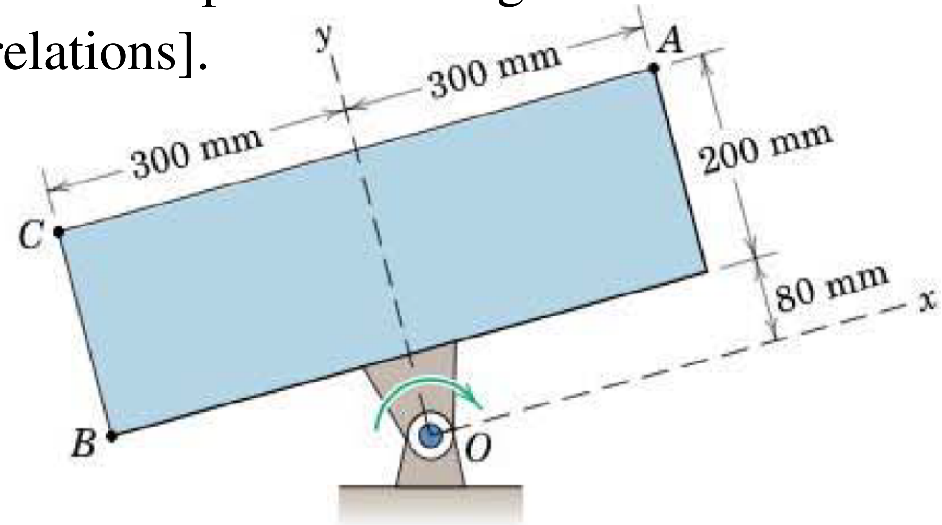


5-1 Rotation

Example 5: Rectangular plate

The rectangular plate rotates clockwise about its fixed bearing at O. If edge BC has a constant angular velocity of $6\, rad/s$, determine the vector and scalar expressions for the velocity and acceleration of point A using the coordinates given. [Check your solution using scalar relations].

$$Ans. \qquad V_A = 1.68i - 1.8j\ m/s$$
$$a_A = -10.8i - 10.08j\ m/s^2$$



H.W: Solve Problems: (5.9, 5.17 and 5.26). "Engineering Mechanics Dynamics, 6th edition, Meriam & Kraige".

5-2 Absolute Motion

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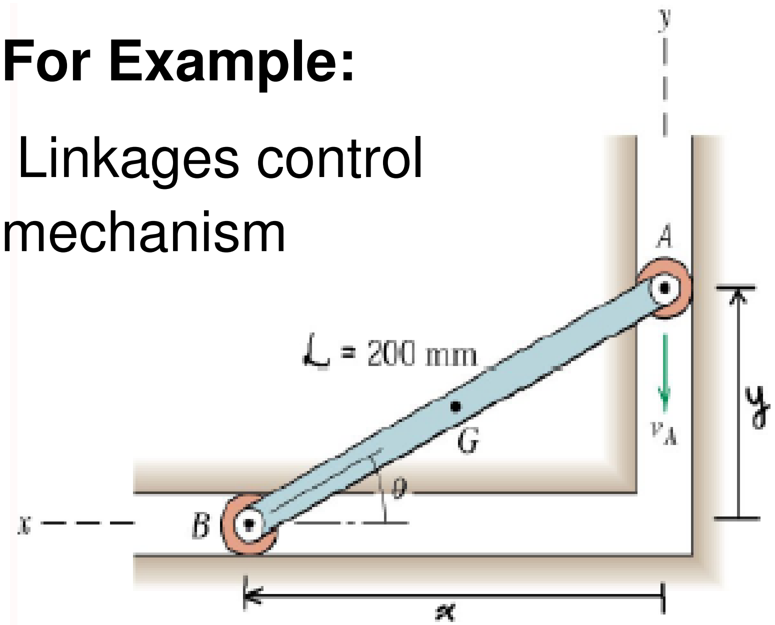
M.Sc. Mechanical Engineering

5.2 Absolute Motion

Absolute motion of rigid bodies is described by using the **geometric relations** which define the configuration of the body involved and then proceed to **take the time derivatives** of the defining the geometric relations to obtain **velocities** and **accelerations**.

For Example:

Linkages control mechanism



- Postion variables x and y are related as

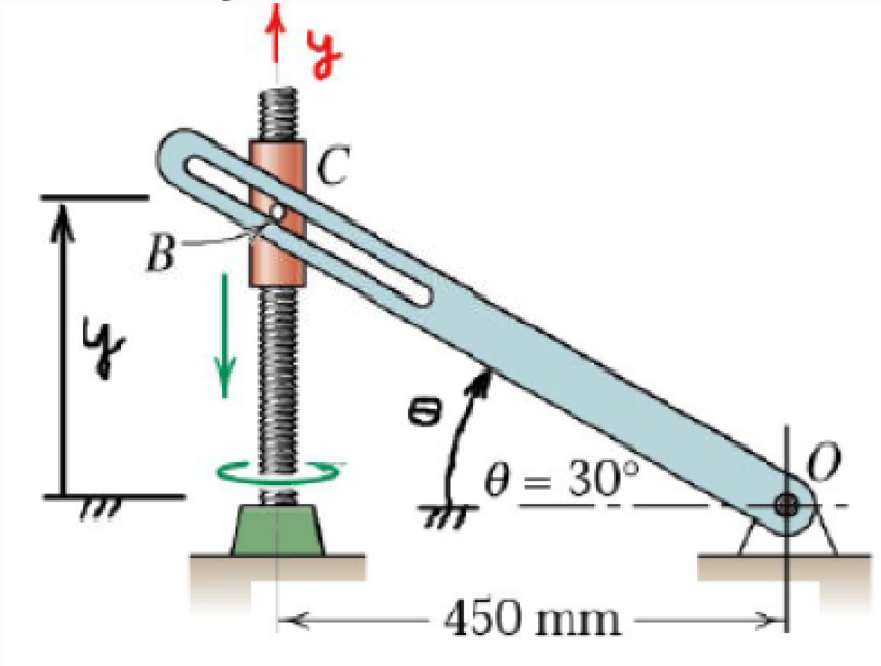
$$x^2 + y^2 = L^2$$

- Position x and angular position θ

$$L\cos\theta = x$$

5.2 Absolute Motion

- For simple mechanism, (absolute) positional relation is easy.



- Position y and angular position θ

$$y/0.45 = \tan\theta$$

- For complex mechanism, relative methods may be easier (5/4-5/7).
- Relative metods will be used again in Mechanics of Machinary.

5.2 Absolute Motion

<u>Steps</u> to find velocity and acceleration relation between two points (or a point and a line, or two points):

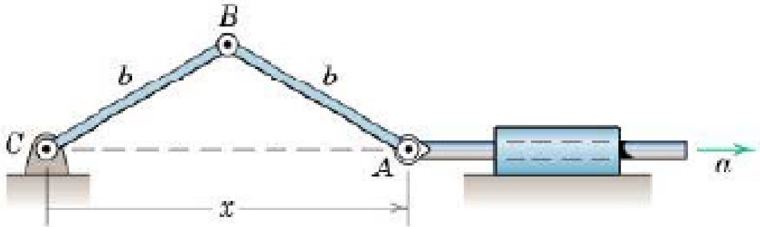
- Draw the diagram of the problem with all dimensions.
- Write the positional relations between the variables.
- This relation must **hold the duration of motion**.
  (Not at just the current position).
- Differentiate it to obtain velocity and acceleration relation.

JAAFAR MOHAMMED HAMZAH

5.2 Absolute Motion

Example 1: Link

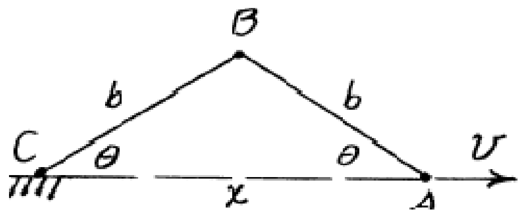
Point A is given a constant acceleration $\boldsymbol{a}$ to the right starting from rest with $\boldsymbol{x}$ essentially zero. Determine the angular velocity $\boldsymbol{\omega}$ of link AB in terms of $\boldsymbol{x}$ and $\boldsymbol{a}$.



Solution

$$x = 2b\cos\theta,$$

$$\dot{x} = -2b\dot{\theta}\sin\theta, \quad v = \dot{x}$$



$$\omega = \omega_{AB} = \dot{\theta} \qquad \text{so} \quad \omega = \frac{-v}{2b\sin\theta} \quad \text{CW}$$

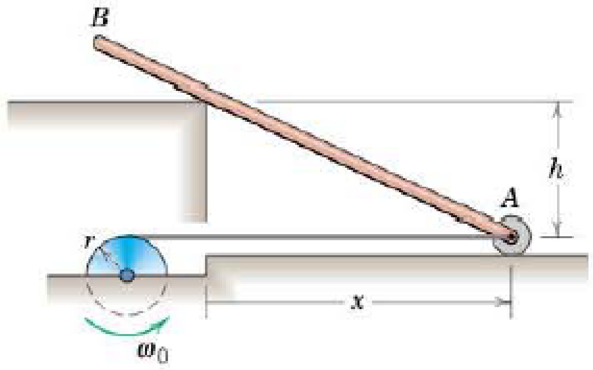
$$\text{For } a = \ddot{x} \text{ const.,} \qquad \dot{x}^2 = 2ax \qquad v = \sqrt{2ax}$$

$$\text{so} \quad \omega = \frac{\sqrt{2ax}}{2b\sqrt{1-\cos^2\theta}} = \frac{\sqrt{2ax}}{\sqrt{4b^2-x^2}}$$

5.2 Absolute Motion

Example 2: Thin bar

Calculate the angular velocity $\boldsymbol{\omega}$ of the slender bar AB as a function of the distance $\boldsymbol{x}$ and the constant angular velocity $\boldsymbol{\omega_o}$ of the drum.



Solution

$$h = x \tan\theta$$

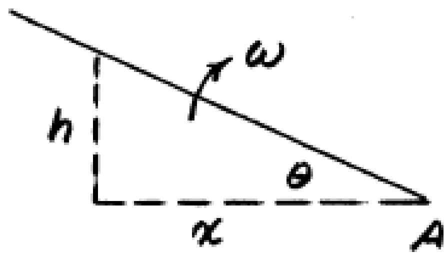
$$0 = \dot{x}\tan\theta + x\dot{\theta}\sec^2\theta$$

$$\omega = \dot{\theta} = -\frac{\dot{x}}{x}\sin\theta\cos\theta$$

$$= -\frac{\dot{x}}{x}\,\frac{hx}{x^2+h^2}$$

$$v_A = r\omega_0 = -\dot{x},$$

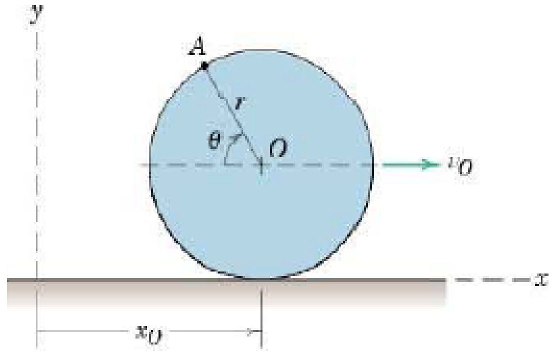
$$\omega = \frac{rh\omega_0}{x^2+h^2}$$



5.2 Absolute Motion

Example 3: Wheel

The wheel of radius r rolls without slipping, and its center O has a constant velocity v_o to the right. Determine expressions for the magnitudes of the velocity **v** and acceleration **a** of point A on the rim by differentiating its x- and y-coordinates. Represent your results graphically as vectors on your sketch and show that **v** is the vector sum of two vectors, each of which has a magnitude v_o.



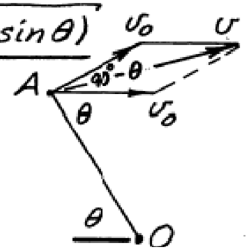
Solution

Coordinates of A are

$$x = x_o - r\cos\theta \qquad \dot{x} = \dot{x}_o + r\dot{\theta}\sin\theta = v_o(1+\sin\theta)$$

$$y = r + r\sin\theta \qquad \dot{y} = r\dot{\theta}\cos\theta = v_o\cos\theta$$

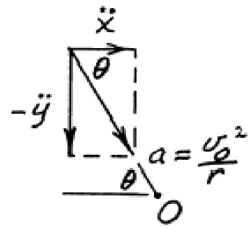
$$v = \sqrt{\dot{x}^2+\dot{y}^2} = v_o\sqrt{(1+\sin\theta)^2+\cos^2\theta} = v_o\sqrt{2(1+\sin\theta)}$$

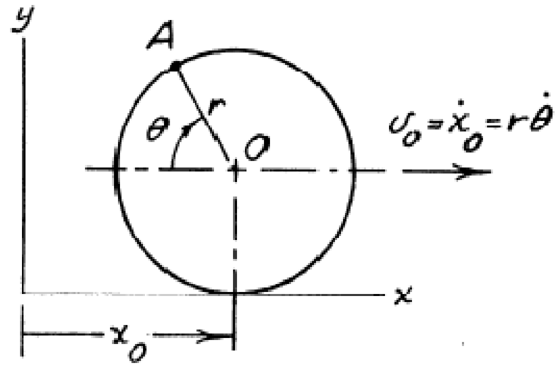


$$\ddot{x} = v_o\dot{\theta}\cos\theta = v_o\left(\frac{v_o}{r}\right)\cos\theta = \frac{v_o^2}{r}\cos\theta$$

$$\ddot{y} = -v_o\dot{\theta}\sin\theta = -v_o\left(\frac{v_o}{r}\right)\sin\theta = -\frac{v_o^2}{r}\sin\theta$$

$$a = \sqrt{\ddot{x}^2+\ddot{y}^2} = \frac{v_o^2}{r}\sqrt{\cos^2\theta+\sin^2\theta} = \frac{v_o^2}{r} \text{ toward } O$$

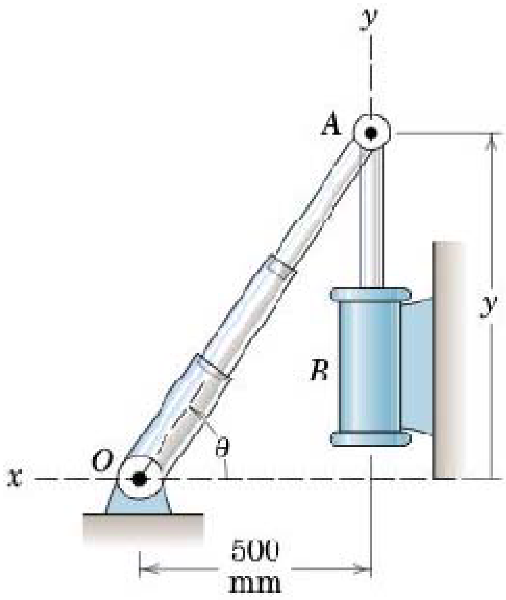




5.2 Absolute Motion

Example 4: Telescoping link

The telescoping link is hinged at O, and its end A is given a constant upward velocity of 200 mm/s by the piston rod of the fixed hydraulic cylinder B. Calculate the angular velocity $\dot{\boldsymbol{\theta}}$ and the angular acceleration $\ddot{\boldsymbol{\theta}}$ of link OA for instant when $y = 600mm$.



Solution

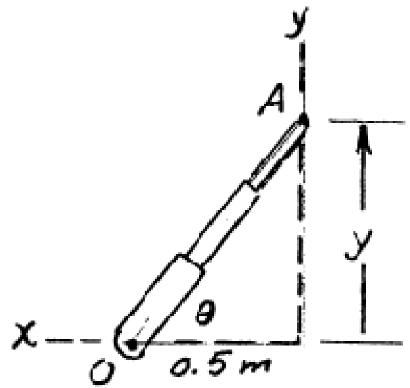
$y = 0.5 \tan\theta$

$\dot{y} = 0.5 \sec^2\theta\,\dot{\theta}$

$\ddot{y} = 0 = \sec\theta(\tan\theta \sec\theta)\dot{\theta}^2 + 0.5\sec^2\theta\,\ddot{\theta}$

$\dot{\theta} = 2\dot{y}/\sec^2\theta$

$\ddot{\theta} = -2\tan\theta\,\dot{\theta}^2$



For $y = 0.6\ m$, $\tan\theta = \frac{0.6}{0.5} = 1.2$, $\theta = 50.2°$

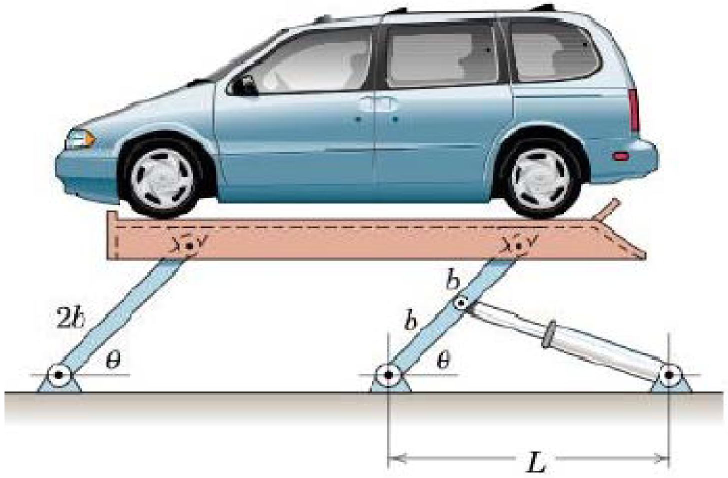
$\sec\theta = 1.562$

So for $\dot{y} = 0.2\ m/s$, $\dot{\theta} = \frac{2(0.2)}{(1.562)^2} = \underline{0.1639\ rad/s}$

$\ddot{\theta} = -2(1.2)(0.1639)^2 = -0.0645\ rad/s^2$

Example 5: Car hoist

Derive an expression for the upward velocity $\boldsymbol{v}$ of the car hoist in terms of θ. The piston rod of the hydraulic cylinder is extending at the rate $\dot{s}$.



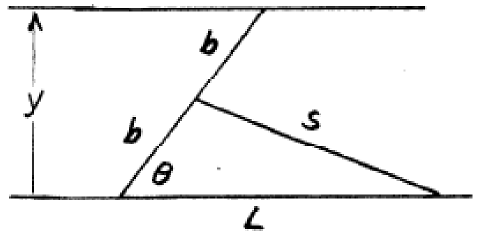
Solution

$$y = 2b \sin\theta$$
$$v = \dot{y} = 2b\dot{\theta}\cos\theta$$

$$s^2 = b^2 + L^2 - 2bL\cos\theta$$
$$2s\dot{s} = 0 + 0 + 2bL\dot{\theta}\sin\theta$$

$$\dot{\theta} = \frac{s\dot{s}}{bL\sin\theta}$$

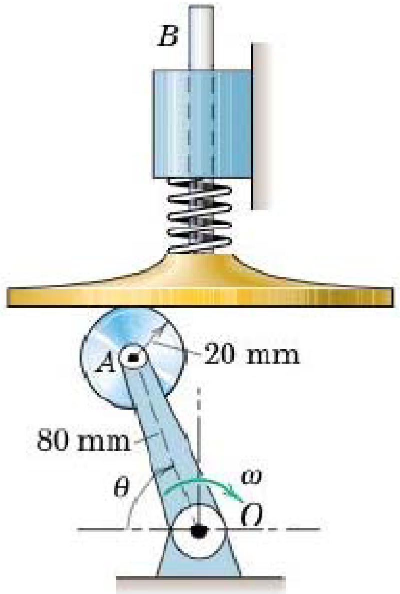
$$\text{so } v = 2b\frac{s\dot{s}}{bL\sin\theta}\cos\theta = 2\frac{\sqrt{b^2+L^2-2bL\cos\theta}}{L\tan\theta}\dot{s}$$



5.2 Absolute Motion

Example 6: Plunger and roller

Determine the acceleration of the shaft B for $\theta = 60^o$ if the crank OA has an angular acceleration $\ddot{\theta} = 8\, rad/s^2$ and an angular velocity $\dot{\theta} = 4\, rad/s$ at this position. The spring maintains contact between the roller and the surface of the plunger.



Solution

$y = 20 + 80 \sin\theta, \quad \dot{y} = 80\dot{\theta}\cos\theta$

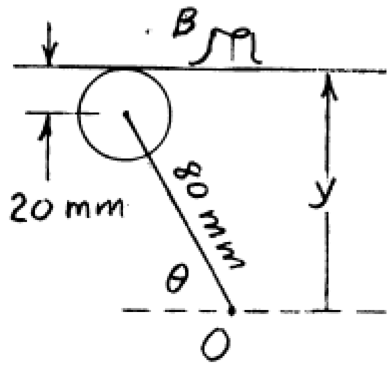
$\ddot{y} = 80\ddot{\theta}\cos\theta - 80\dot{\theta}^2\sin\theta$

For $\theta = 60^\circ$, $\dot{\theta} = 4\,\frac{rad}{s}$, $\ddot{\theta} = 8\,\frac{rad}{s^2}$,

$\ddot{y} = 80(8)(\frac{1}{2}) - 80(4)^2\frac{\sqrt{3}}{2}$

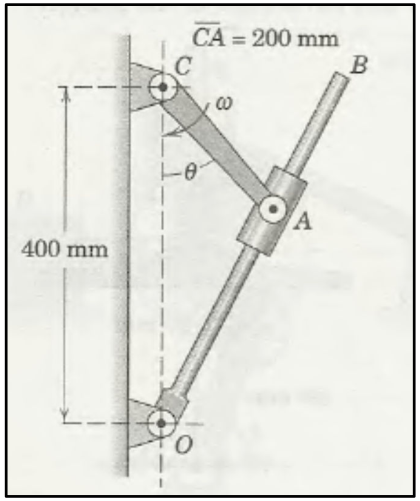
$= 320 - 1109 = -789\ mm/s^2$

Thus $a_B = \underline{789\ mm/s^2\ down}$



Example 7: Link

The rod OB slides through the collar pivoted to the link at A. If CA has an angular velocity $\omega = 3\ rad/s$ for an interval of motion, calculate the angular velocity of OB when $\theta = 45^o$.



Solution

$$\tan\beta = \frac{0.2\sin\theta}{0.4 - 0.2\cos\theta}, \quad \tan\beta\,(2-\cos\theta) = \sin\theta$$

$$\dot\beta \sec^2\beta\,(2-\cos\theta) + \tan\beta\,(\dot\theta \sin\theta) = \dot\theta\cos\theta$$

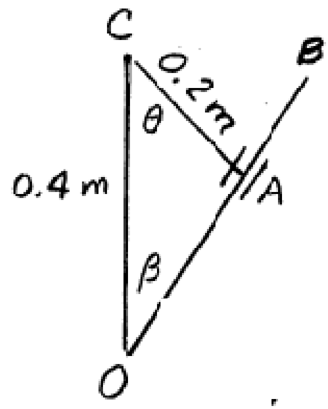
$$\dot\beta = \frac{\cos\theta - \sin\theta\tan\beta}{2-\cos\theta}\,\dot\theta\cos^2\beta$$

$$= \frac{2\cos\theta - 1}{(2-\cos\theta)^2}\,\dot\theta\cos^2\beta$$

For $\omega = -\dot\theta = 3\ \frac{rad}{s}$, $\theta = 45^\circ$, $\beta = \tan^{-1}\frac{1/\sqrt2}{2-1/\sqrt2} = 28.7^\circ$

$$\dot\beta = \frac{2/\sqrt2 - 1}{(2 - 1/\sqrt2)^2}(-3)\cos^2 28.7^\circ = -0.572\ rad/s$$

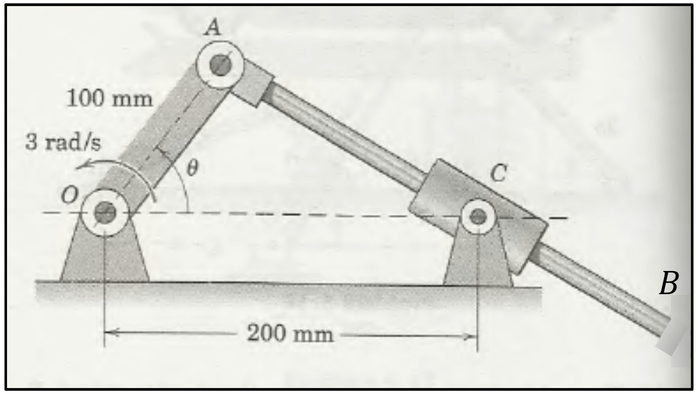
So $\omega_{OB} = 0.572\ rad/s$ CCW



5.2 Absolute Motion

Example 8: Link

Link OA revolves counterclockwise with an angular velocity of $3\,rad/s$. Link AB slides through piloted collar at C. Determine the angular velocity ω of AB when $\theta = 40^o$.



$Ans.$

$$\omega = \dot{\beta} = \frac{r\dot{\theta}\cos(\theta+\beta)}{\ell\cos\beta - r\cos(\theta+\beta)} \Rightarrow \omega = 0.825\ rad/sec$$

H.W: Solve Problems: (5.39, and 5.58). "Engineering Mechanics Dynamics, 6th edition, Meriam & Kraige".

5-3 Relative Velocity

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

5.3 Relative Velocity

1. Introduction

- We will apply concepts on relative motion from kinematics of a particle to a rigid body.

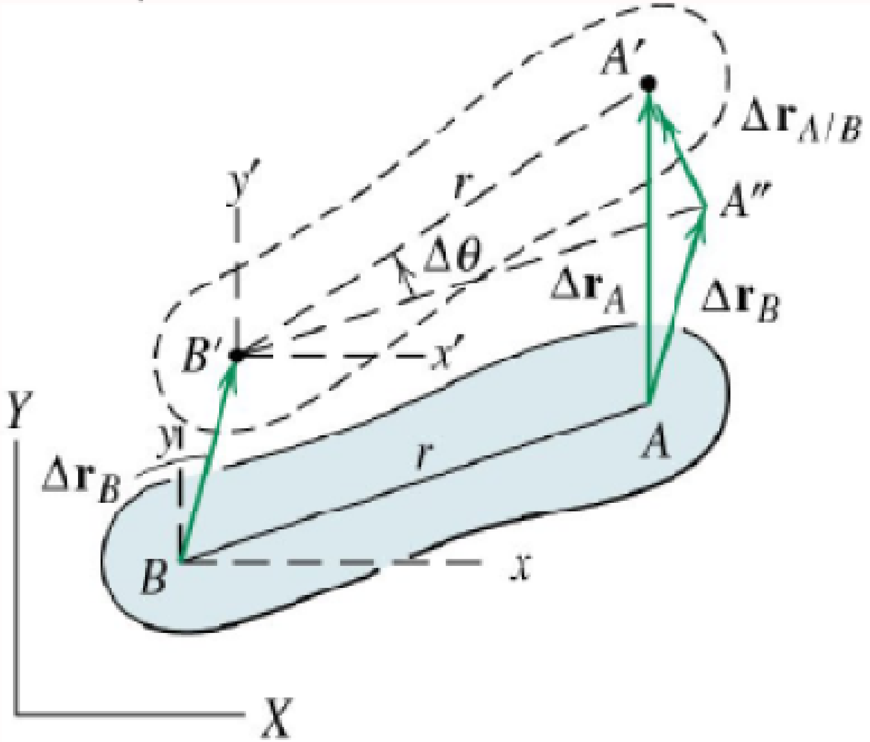
$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

where A and B are two points on the rigid body.

5.3 Relative Velocity

2. Relative Velocity due to Rotation

- Consider point *A* and *B* on a rigid body in a general plane motion,

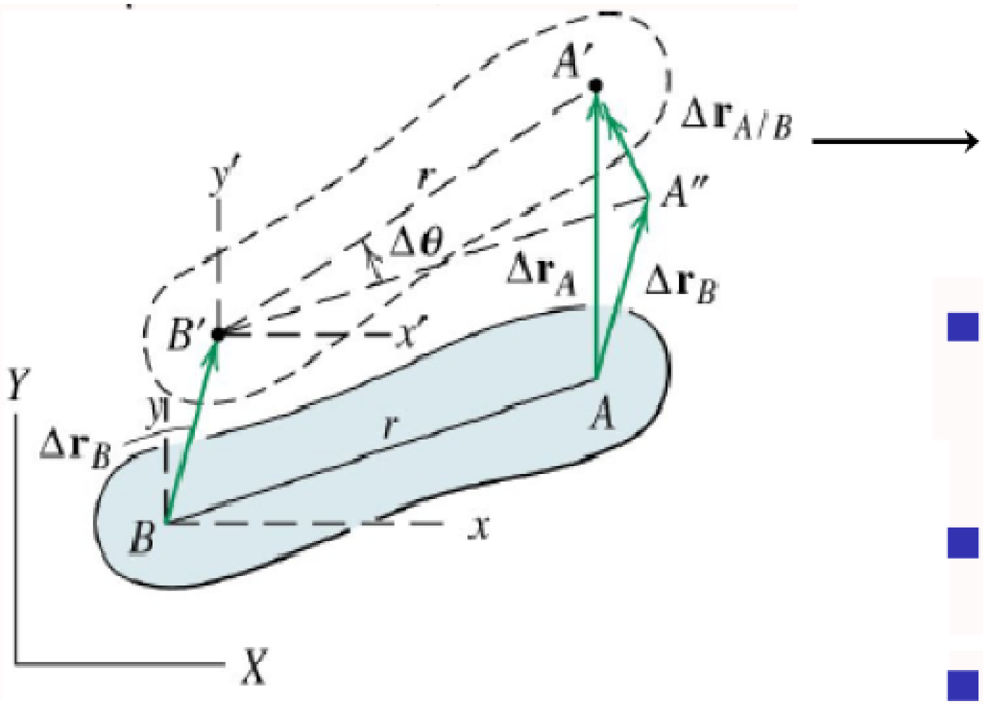


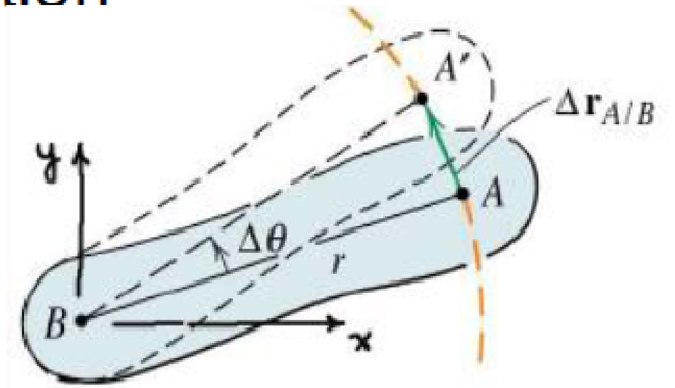
- Need $\vec{v}_{A/B}$ to apply $\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$
- Recall that in $\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$ the observer at *B* must be translating.
- *x*-*y* represents the reference frame of the observer at *B*

JAAFAR MOHAMMED HAMZAH

5.3 Relative Velocity

2. Relative Velocity due to Rotation





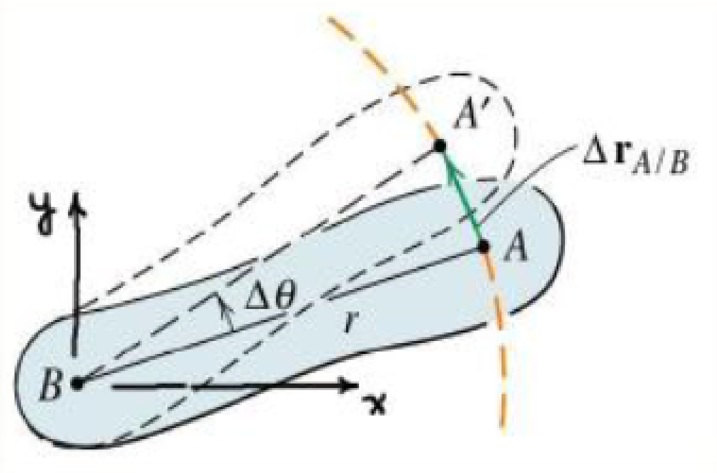
- Since the length $\overline{AB}$ is constant, B will see A moving in a circular arc.
- The displacement $\Delta\theta$ is relative to x-y
- $v_{A/B} = \omega_{AB}\overline{AB}$

Relative Velocity

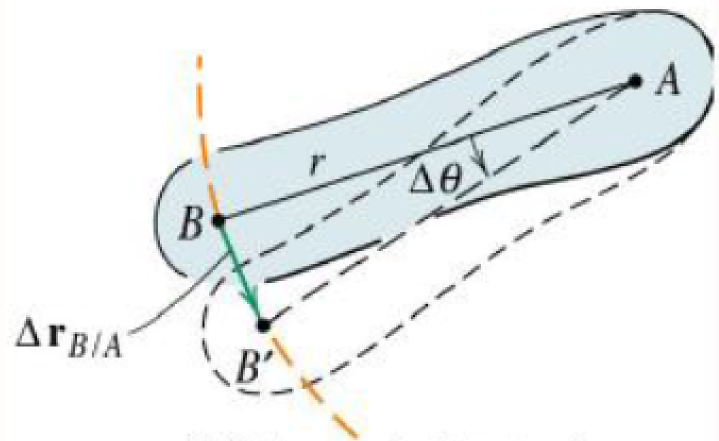
$$\vec{V}_{A/B} = \vec{\omega}_{AB} \times \vec{r}_{B\rightarrow A}$$

5.3 Relative Velocity

3. Choice of Observer and their ω



$$\vec{v}_{A/B} = \vec{\omega}_{AB} \times \vec{r}_{B\to A}$$



Motion relative to *A*

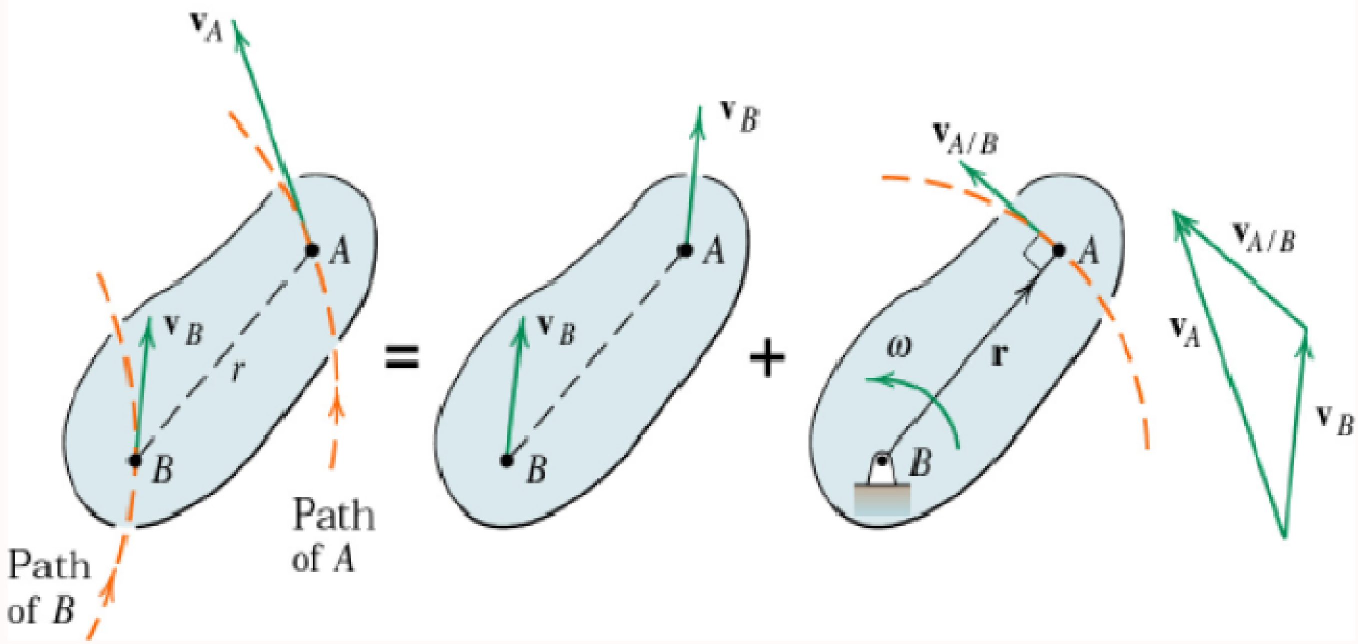
$$\vec{v}_{B/A} = \vec{\omega}_{BA} \times \vec{r}_{A\to B}$$

Notice that either line from *A* to *B* or from *B* to *A* can be used to define angular velocity and acceleration of the body; i.e., $\vec{\omega}_{AB} = \vec{\omega}_{BA}$. And, both are CCW.

5.3 Relative Velocity

4. Visualization of Relative Velocity

- Actual motion = Translation + Rotation

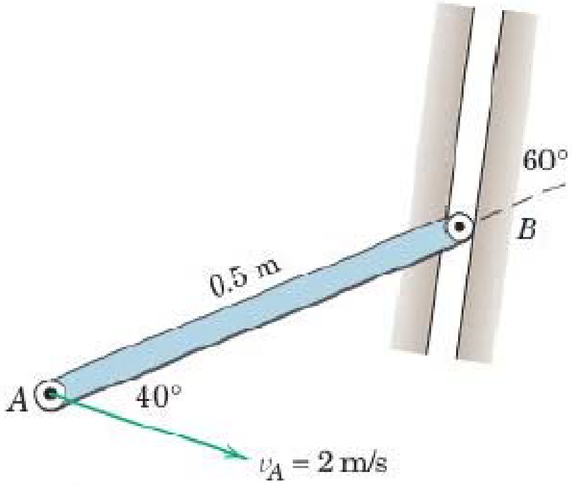


- Pick any point B, the body translate with velocity $\vec{v}_B$
- And rotate about B with ang. velocity $\vec{\omega}$
- For any point A, $\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{B \to A}$

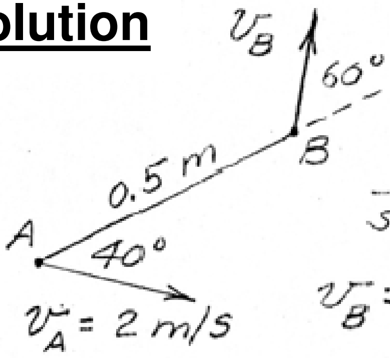
5.3 Relative Velocity

Example 1: Link and Slot

End A of the link has the velocity shown at the instant depicted. End B is confined to move in the slot. For this instant calculate the velocity of B and the angular velocity of AB.



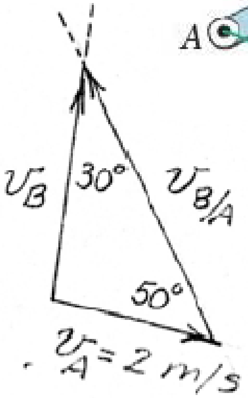
Solution



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$$

$$\frac{v_B}{\sin 50^\circ} = \frac{2}{\sin 30^\circ}$$

$$v_B = \frac{2 \sin 50^\circ}{\sin 30^\circ} = \underline{3.06\ m/s}$$



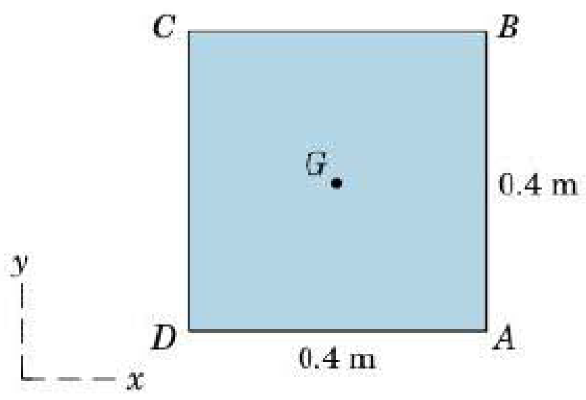
$$v_{B/A} = v_B \cos 30^\circ + v_A \cos 50^\circ$$
$$= 3.06 \cos 30^\circ + 2 \cos 50^\circ = 3.94\ m/s$$

$$\omega_{AB} = \frac{v_{B/A}}{AB} = \frac{3.94}{0.5} = \underline{7.88\ rad/s\ ccw}$$

5.3 Relative Velocity

Example 2: Square

The uniform square plate moves in the x-y plane and has a clockwise angular velocity. At the instant represented, point A has a velocity of 2 m/s to the right, and the velocity of C relative to a nonrotating observer at B has the magnitude of 1.2 m/s. Determine the vector expressions for the angular velocity of the plate and the velocity of its center G.



Solution

$$v_{C/B} = \overline{CB}\,\omega \,, \quad \omega = \frac{1.2}{0.4} = 3 \text{ rad/s CW}$$

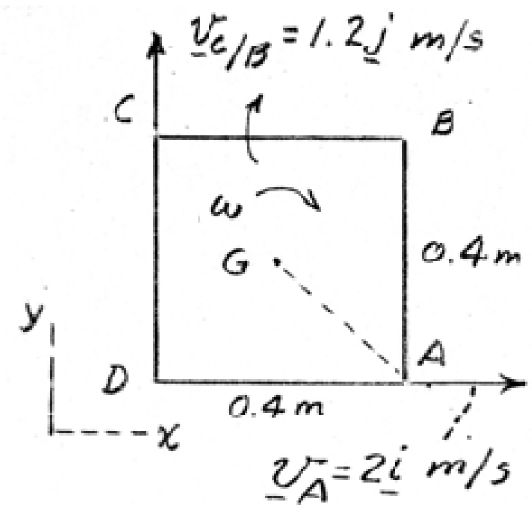
$$\underline{\omega} = -3\underline{k} \text{ rad/s}$$

$$\underline{v}_G = \underline{v}_A + \underline{\omega} \times \underline{r}_{AG}$$

$$= 2\underline{i} - 3\underline{k} \times (-0.2\underline{i} + 0.2\underline{j})$$

$$= 2\underline{i} + 0.6\underline{j} + 0.6\underline{i}$$

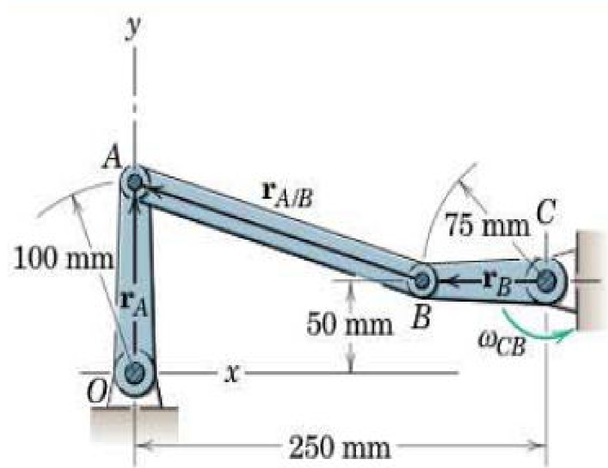
$$\underline{v}_G = 2.6\underline{i} + 0.6\underline{j} \quad \text{m/s}$$



5.3 Relative Velocity

Example 3: Crank

Crank CB oscillates about C through a limited arc, causing crank OA to oscillate about O. When the linkage passes the position shown with CB horizontal and OA vertical, the angular velocity of CB is $2\ rad/s$ counterclockwise. For this instant, determine the angular velocities of OA and AB.



Solution I (Vector). The relative-velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ is rewritten as

$$\boldsymbol{\omega}_{OA} \times \mathbf{r}_A = \boldsymbol{\omega}_{CB} \times \mathbf{r}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

where

$$\boldsymbol{\omega}_{OA} = \omega_{OA}\mathbf{k} \qquad \boldsymbol{\omega}_{CB} = 2\mathbf{k}\ \text{rad/s} \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$$

$$\mathbf{r}_A = 100\mathbf{j}\ \text{mm} \qquad \mathbf{r}_B = -75\mathbf{i}\ \text{mm} \qquad \mathbf{r}_{A/B} = -175\mathbf{i} + 50\mathbf{j}\ \text{mm}$$

Substitution gives

$$\omega_{OA}\mathbf{k} \times 100\mathbf{j} = 2\mathbf{k} \times (-75\mathbf{i}) + \omega_{AB}\mathbf{k} \times (-175\mathbf{i} + 50\mathbf{j})$$

$$-100\omega_{OA}\mathbf{i} = -150\mathbf{j} - 175\omega_{AB}\mathbf{j} - 50\omega_{AB}\mathbf{i}$$

Matching coefficients of the respective **i**- and **j**-terms gives

$$-100\omega_{OA} + 50\omega_{AB} = 0 \qquad 25(6 + 7\omega_{AB}) = 0$$

then; $\omega_{AB} = -6/7$ rad/s and $\omega_{OA} = -3/7$ rad/s *Ans.*

5.3 Relative Velocity

Solution II (Scalar-Geometric). Solution by the scalar geometry of the vector triangle is particularly simple here since $\mathbf{v}_A$ and $\mathbf{v}_B$ are at right angles for this special position of the linkages. First, we compute v_B, which is

$[v = r\omega]$ $$v_B = 0.075(2) = 0.150 \text{ m/s}$$

and represent it in its correct direction as shown. The vector $\mathbf{v}_{A/B}$ must be perpendicular to AB, and the angle θ between $\mathbf{v}_{A/B}$ and $\mathbf{v}_B$ is also the angle made by AB with the horizontal direction. This angle is given by

$$\tan\theta = \frac{100 - 50}{250 - 75} = \frac{2}{7}$$

The horizontal vector $\mathbf{v}_A$ completes the triangle for which we have

$$v_{A/B} = v_B/\cos\theta = 0.150/\cos\theta$$

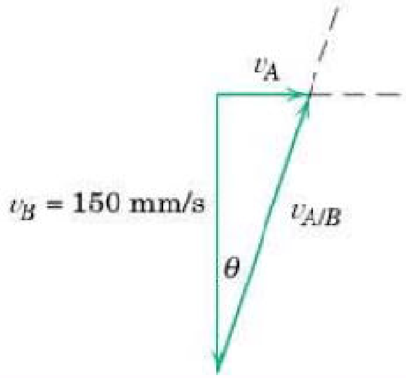
$$v_A = v_B \tan\theta = 0.150(2/7) = 0.30/7 \text{ m/s}$$

The angular velocities become

$[\omega = v/r]$ $$\omega_{AB} = \frac{v_{A/B}}{AB} = \frac{0.150}{\cos\theta}\,\frac{\cos\theta}{0.250 - 0.075}$$

$$= 6/7 \text{ rad/s CW}$$ *Ans.*

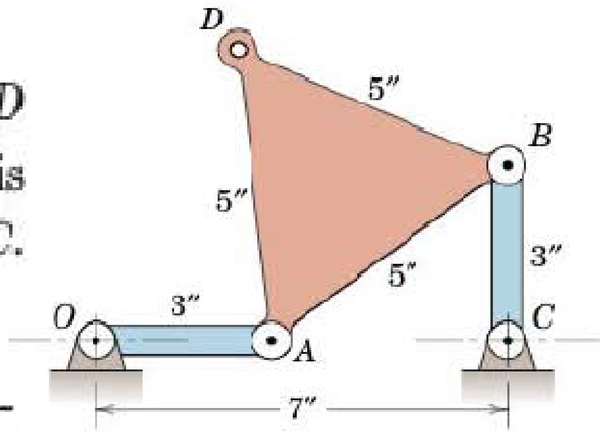
$[\omega = v/r]$ $$\omega_{OA} = \frac{v_A}{OA} = \frac{0.30}{7}\,\frac{1}{0.100} = 3/7 \text{ rad/s CW}$$ *Ans.*



5.3 Relative Velocity

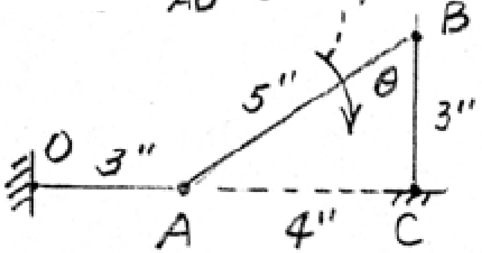
Example 4: Triangle

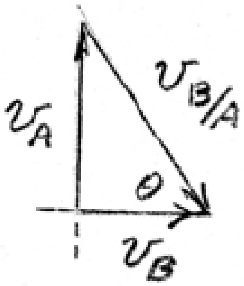
At the instant represented the triangular plate *ABD* has a clockwise angular velocity of 3 rad/sec. For this instant determine the angular velocity ω_{BC} of link *BC*.



Solution

$\omega_{AB} = 3\ rad/sec$





$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}\ ,\quad \omega_{BC} = \frac{v_B}{\overline{BC}}$$

$$v_{B/A} = \overline{AB}\,\omega_{AB}$$

$$= 5(3) = 15\ in./sec$$

$$\theta = \cos^{-1}\frac{3}{5}$$

$$v_B = v_{B/A}\cos\theta$$

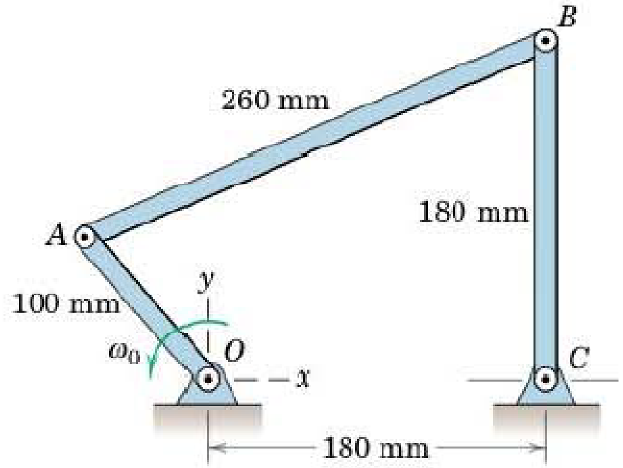
$$= 15(3/5) = 9\ in./sec$$

$$\omega_{BC} = 9/3 = \underline{3\ rad/sec\ CW}$$

5.3 Relative Velocity

Example 5: Linkages

In the four-bar linkage shown, control link OA has a counterclockwise angular velocity $\omega_0 = 10$ rad/s during a short interval of motion. When link CB passes the vertical position shown, point A has coordinates $x = -60$ mm and $y = 80$ mm. By means of vector algebra determine the angular velocities of AB and BC.



Solution $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$$\underline{v}_A = \underline{\omega}_{AO} \times \underline{r}_{AO} = 10\underline{k} \times (-0.06\underline{i} + 0.08\underline{j}) = -0.6j - 0.8\underline{i} \text{ m/s}$$

$$\underline{v}_B = \underline{\omega}_{BC} \times \underline{r}_{BC} = \omega_{BC}\underline{k} \times 0.18j = -0.18\omega_{BC}\underline{i}$$

$$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{A/B}$$

$$= \omega_{AB}\underline{k} \times (-0.24\underline{i} - 0.1\underline{j})$$

$$= -0.24\omega_{AB}\underline{j} + 0.1\omega_{AB}\underline{i} \text{ m/s}$$

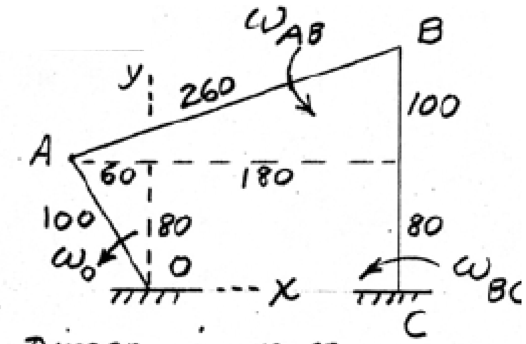
Thus,

$$-0.6\underline{j} - 0.8\underline{i} = -0.18\omega_{BC}\underline{i} - 0.24\omega_{AB}\underline{j} + 0.1\omega_{AB}\underline{i}$$

Equate j terms & set $\omega_{AB} = \dfrac{0.6}{0.24} = 2.5$ rad/s

$\underline{\omega}_{BC} = 5.83\underline{k}$ rad/s

$\underline{\omega}_{AB} = 2.5\underline{k}$ rad/s

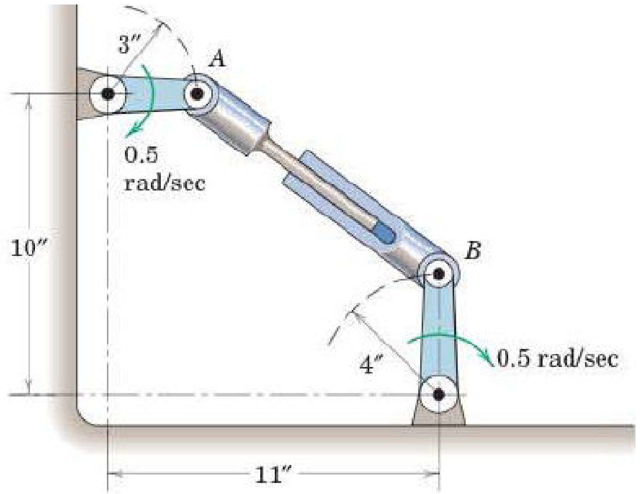


Dimen. in mm

$\omega_{AO} = \omega_0 = 10$ rad/s

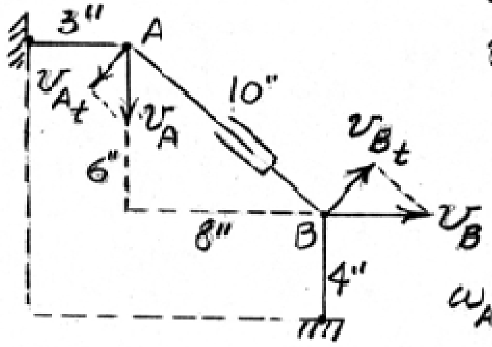
5.3 Relative Velocity

Example 6: Telescoping Link



Determine the angular velocity ω of the telescoping link AB at the instant represented. The angular velocity of each of the driving links is shown.

Solution



$$v_A = 3(0.5) = 1.5 \text{ in./sec}$$

$$v_{A_t} = 1.5\left(\frac{4}{5}\right) = 1.2 \text{ in./sec}$$

$$v_B = 4(0.5) = 2.0 \text{ in./sec}$$

$$v_{B_t} = 2.0\left(\frac{3}{5}\right) = 1.2 \text{ in./sec}$$

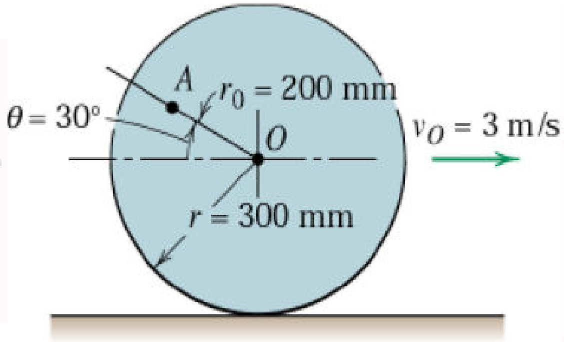
$$\omega_{AB} = \omega = \frac{v_{A_t} + v_{B_t}}{\overline{AB}}$$

$$\omega = \frac{1.2 + 1.2}{10} = 0.24 \text{ rad/sec CCW}$$

5.3 Relative Velocity

Example 7: Wheel *

The wheel of radius $r = 300$ mm rolls to the right without slipping and has an angular velocity $\vec{\omega} = 10$ rad/s. Calculate the velocity of point A on the wheel for the instant represented.

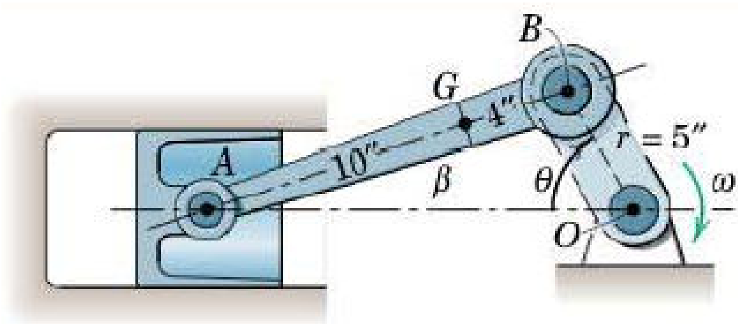


Page (359) “Engineering Mechanics Dynamics, 6th edition, Meriam & Kraige”.

5.3 Relative Velocity

Example 8: Reciprocating engine *

The common configuration of a reciprocating engine is that of the slider-crank mechanism shown. If the crank OB has a clockwise rotational speed of 1500 rev/min, determine for the position where $\theta = 60°$ the velocity of the piston A, the velocity of point G on the connecting rod, and the angular velocity of the connecting rod.

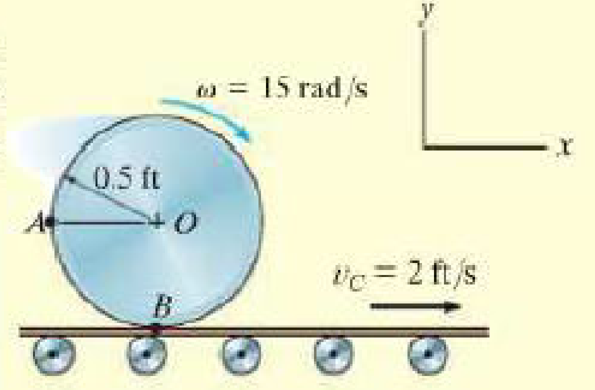


Page (361) “Engineering Mechanics Dynamics, 6th edition, Meriam & Kraige”.

5.3 Relative Velocity

Example 9: Cylinder

The cylinder shown in Fig. 16–15*a* rolls without slipping on the surface of a conveyor belt which is moving at 2 ft/s. Determine the velocity of point *A*. The cylinder has a clockwise angular velocity $\omega = 15$ rad/s at the instant shown.



Ans.

$$v_A = \sqrt{(9.50)^2 + (7.50)^2} = 12.1 \text{ ft/s} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{7.50}{9.50} = 38.3^\circ \qquad \textit{Ans.}$$

H.W: Solve Problems: (5.75^{5th} or 5.81^{6th} and 5.81^{5th} or 5.87^{6th}). “Engineering Mechanics Dynamics, 6th edition, Meriam & Kraige”.

5-4 Relative Acceleration

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

5.3 Relative Acceleration

1. Introduction

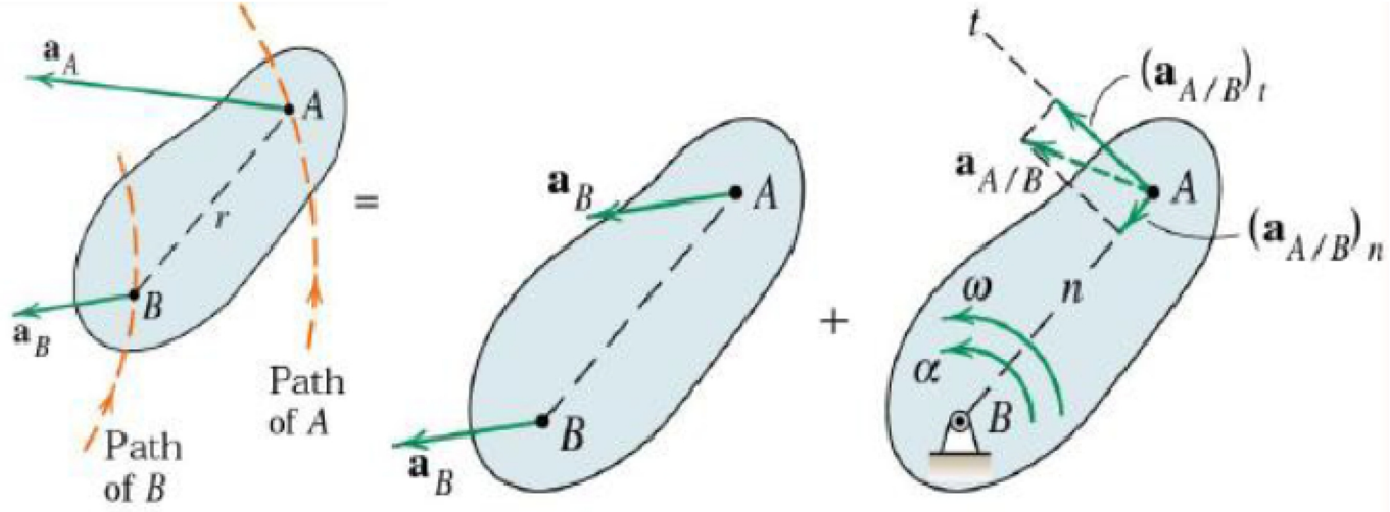
- We will apply concepts on relative motion from kinematics of a particle to a rigid body.

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

where *A* and *B* are two points on the rigid body.

5.3 Relative Acceleration

2. Relative Acceleration due to Rotation

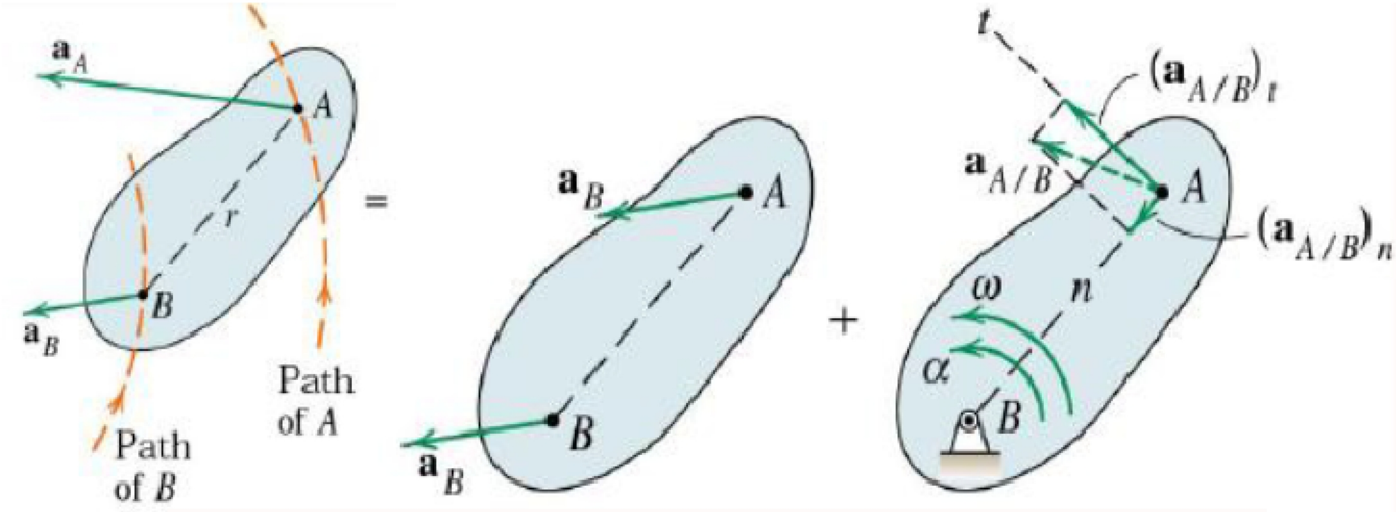


- Acceleration of A equals to acc. of B plus the acc. of *A* relative to *B*

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

- A general motion can be thought of a two separate motion:
  - Translating with acc. of point *B*
  - Rotation of point A about point *B*

5.3 Relative Acceleration



- Relative equation

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

- The relative term $\vec{a}_{A/B} = (\vec{a}_{A/B})_n + (\vec{a}_{A/B})_t$

Rel. Acc. (Tran. Axis)

$$(a_{A/B})_n = v_{A/B}^2/r = r\omega^2$$
$$(a_{A/B})_t = \dot{v}_{A/B} = r\alpha$$

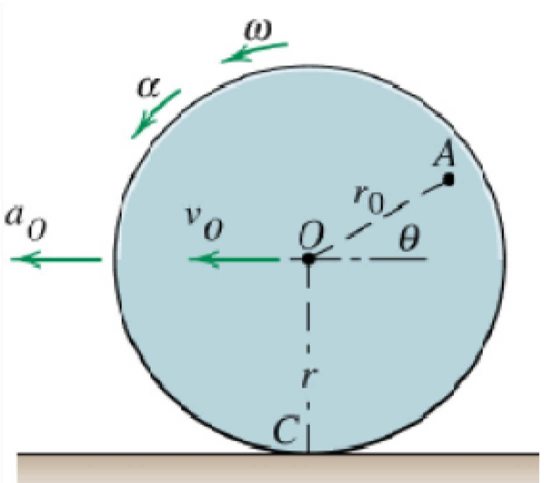
Rel. Acc. (Tran. Axis)

$$(\vec{a}_{A/B})_n = \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B\to A})$$
$$(\vec{a}_{A/B})_t = \vec{\alpha}_{AB} \times \vec{r}_{B\to A}$$

5.3 Relative Acceleration

Example 1: Rolling Wheel

The wheel of radius r rolls to the left without slipping and, at the instant considered, the center O has a velocity $\vec{v}_O$ and an acceleration $\vec{a}_O$ to the left. Determine the acceleration of points A and C on the wheel for the instant considered.



Solution

From our previous analysis of Sample Problem we know that the angular velocity and angular acceleration of the wheel are

$$\omega = v_O/r \qquad \text{and} \qquad \alpha = a_O/r$$

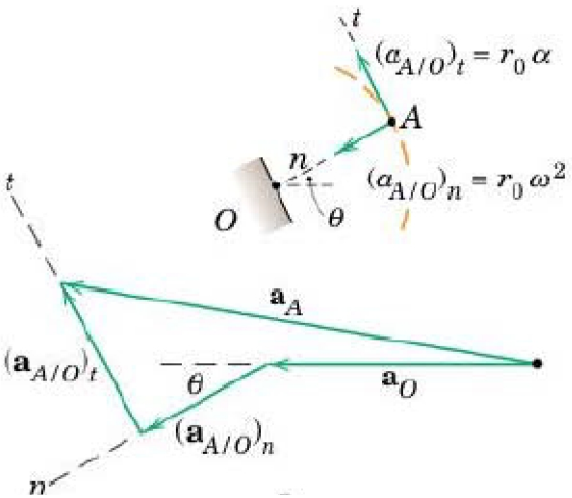
The acceleration of A is written in terms of the given acceleration of O. Thus,

$$\mathbf{a}_A = \mathbf{a}_O + \mathbf{a}_{A/O} = \mathbf{a}_O + (\mathbf{a}_{A/O})_n + (\mathbf{a}_{A/O})_t$$

The relative-acceleration terms are viewed as though O were fixed, and for this relative circular motion they have the magnitudes

$$(a_{A/O})_n = r_0\omega^2 = r_0\left(\frac{v_O}{r}\right)^2$$

$$(a_{A/O})_t = r_0\alpha = r_0\left(\frac{a_O}{r}\right)$$



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5.3 Relative Acceleration

and the directions shown.

Adding the vectors head-to-tail gives $\mathbf{a}_A$ as shown. In a numerical problem, we may obtain the combination algebraically or graphically. The algebraic expression for the magnitude of $\mathbf{a}_A$ is found from the square root of the sum of the squares of its components. If we use n- and t-directions, we have

$$a_A = \sqrt{(a_A)_n{}^2 + (a_A)_t{}^2}$$
$$= \sqrt{[a_O \cos\theta + (a_{A/O})_n]^2 + [a_O \sin\theta + (a_{A/O})_t]^2}$$
$$= \sqrt{(r\alpha \cos\theta + r_0\omega^2)^2 + (r\alpha \sin\theta + r_0\alpha)^2} \qquad \textit{Ans.}$$

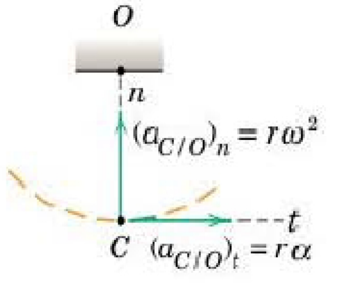
The direction of $\mathbf{a}_A$ can be computed if desired.

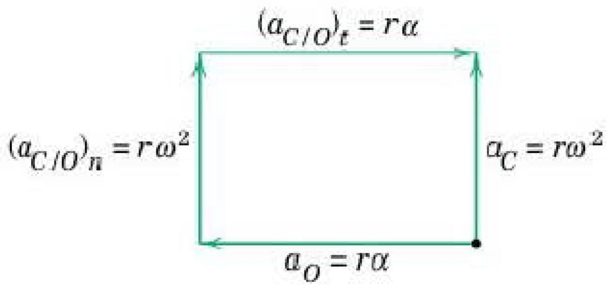
The acceleration of the instantaneous center C of zero velocity, considered a point on the wheel, is obtained from the expression

$$\mathbf{a}_C = \mathbf{a}_O + \mathbf{a}_{C/O}$$

where the components of the relative-acceleration term are $(a_{C/O})_n = r\omega^2$ directed from C to O and $(a_{C/O})_t = r\alpha$ directed to the right because of the counterclockwise angular acceleration of line CO about O. The terms are added together in the lower diagram and it is seen that

$$a_C = r\omega^2 \qquad \textit{Ans.}$$

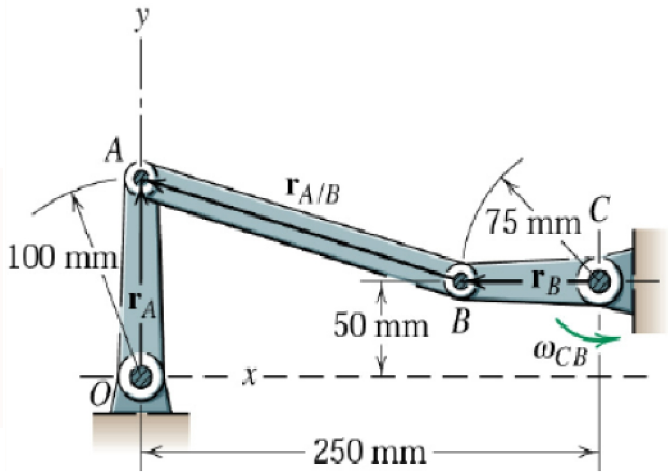




5.3 Relative Acceleration

Example 2: Crank

Crank *CB* has a constant counterclockwise angular velocity of 2 rad/s in the position shown during a short interval of its motion. Determine the angular acceleration of links *AB* and *OA* for this position.



Solution

We first solve for the velocities which were obtained in Sample Problem They are

$$\omega_{AB} = -6/7 \text{ rad/s} \qquad \text{and} \qquad \omega_{OA} = -3/7 \text{ rad/s}$$

where the counterclockwise direction (+**k**-direction) is taken as positive. The acceleration equation is

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_n + (\mathbf{a}_{A/B})_t$$

where, from Eqs. 5/3 and 5/9a, we may write

$$\mathbf{a}_A = \boldsymbol{\alpha}_{OA} \times \mathbf{r}_A + \boldsymbol{\omega}_{OA} \times (\boldsymbol{\omega}_{OA} \times \mathbf{r}_A)$$

$$= \alpha_{OA}\mathbf{k} \times 100\mathbf{j} + (-\tfrac{3}{7}\mathbf{k}) \times (-\tfrac{3}{7}\mathbf{k} \times 100\mathbf{j}) = -100\alpha_{OA}\mathbf{i} - 100(\tfrac{3}{7})^2\mathbf{j} \text{ mm/s}^2$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{CB} \times \mathbf{r}_B + \boldsymbol{\omega}_{CB} \times (\boldsymbol{\omega}_{CB} \times \mathbf{r}_B)$$

$$= \mathbf{0} + 2\mathbf{k} \times (2\mathbf{k} \times [-75\mathbf{i}]) = 300\mathbf{i} \text{ mm/s}^2$$

5.3 Relative Acceleration

$$(\mathbf{a}_{A/B})_n = \boldsymbol{\omega}_{AB} \times (\boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B})$$

$$= -\tfrac{6}{7}\mathbf{k} \times [(-\tfrac{6}{7}\mathbf{k}) \times (-175\mathbf{i} + 50\mathbf{j})]$$

$$= (\tfrac{6}{7})^2(175\mathbf{i} - 50\mathbf{j}) \text{ mm/s}^2$$

$$(\mathbf{a}_{A/B})_t = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$$

$$= \alpha_{AB}\mathbf{k} \times (-175\mathbf{i} + 50\mathbf{j})$$

$$= -50\alpha_{AB}\mathbf{i} - 175\alpha_{AB}\mathbf{j} \text{ mm/s}^2$$

We now substitute these results into the relative-acceleration equation and equate separately the coefficients of the **i**-terms and the coefficients of the **j**-terms to give

$$-100\alpha_{OA} = 429 - 50\alpha_{AB}$$

$$-18.37 = -36.7 - 175\alpha_{AB}$$

The solutions are

$$\alpha_{AB} = -0.1050 \text{ rad/s}^2 \qquad \text{and} \qquad \alpha_{OA} = -4.34 \text{ rad/s}^2 \qquad \textit{Ans.}$$

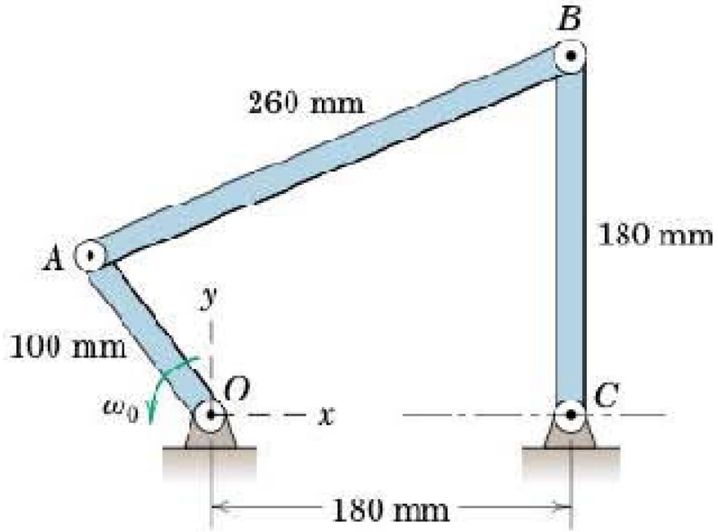
Since the unit vector **k** points out from the paper in the positive z-direction, we see that the angular accelerations of AB and OA are both clockwise (negative).

It is recommended that the student sketch each of the acceleration vectors in its proper geometric relationship according to the relative-acceleration equation to help clarify the meaning of the solution.

5.3 Relative Acceleration

Example 3:

For the Linkage, if OA has a constant CCW angular velocity $\omega_o = 10\, rad/s$, calculate the α of link AB for the position where the coordinates of A are $x = 60\, mm$ and $y = 80\, mm$. Link BC is vertical for this position. Solve by using vector algebra. $\omega_{BC} = 5.83k\ \ rad/s$ and $\omega_{AB} = 2.5k\ \ rad/s$.



Solution: $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} \Rightarrow \underline{a}_B = \underline{\omega}_{BC} \times (\underline{\omega}_{BC} \times \underline{r}_{B/C}) + \underline{\alpha}_{BC} \times \underline{r}_{B/C}$

$= 5.83\underline{k} \times (5.83\underline{k} \times 0.18\underline{j}) + \alpha_{BC}\underline{k} \times 0.18\underline{j}\ \ m/s^2 = -6.125\underline{j} - 0.18\alpha_{BC}\underline{i}\ \ m/s^2$

$\underline{a}_A = \underline{\omega}_o \times (\underline{\omega}_o \times \underline{r}_{A/O}) = 10\underline{k} \times (10\underline{k} \times [-0.06\underline{i} + 0.08\underline{j}]) = 6\underline{i} - 8\underline{j}\ m/s^2 \quad (\alpha_{OA} = 0)$

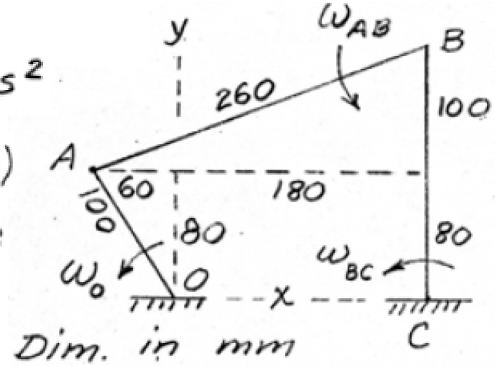
$(a_{B/A})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{B/A}) = 2.5\underline{k} \times (2.5\underline{k} \times [0.24\underline{i} + 0.1\underline{j}]) = -1.5\underline{i} - 0.625j\ m/s^2$

$(a_{B/A})_t = \alpha_{AB}\underline{k} \times (0.24\underline{i} + 0.1\underline{j}) = -0.1\alpha_{AB}\underline{i} + 0.24\alpha_{AB}\underline{j}$

Substitute in accel. equation & equate coefficients & get

$$\left.\begin{aligned} -0.18\alpha_{BC} &= 6 - 1.5 - 0.1\alpha_{AB} \\ -6.125 &= -8 - 0.625 + 0.24\alpha_{AB} \end{aligned}\right\}$$

Sol. is $\underline{\alpha}_{AB} = \underline{10.42\underline{k}\ rad/s^2}$ $\quad (\underline{\alpha}_{BC} = -19.21\underline{k}\ rad/s^2)$



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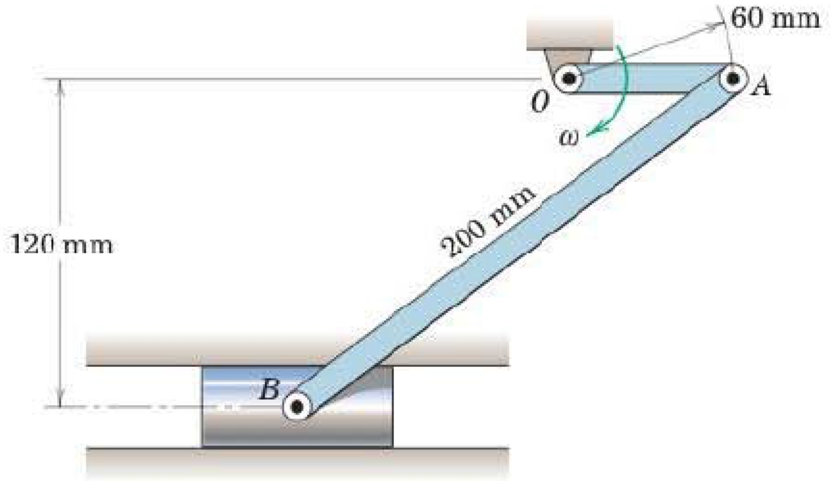
5.3 Relative Acceleration

Example 4:

For a short interval of motion, link OA has a constant angular velocity $\omega = 4$ rad/s. Determine the angular acceleration α_{AB} of link AB for the instant when OA is parallel to the horizontal axis through B.



Solution

$$(60+s)^2 + 120^2 = 200^2$$

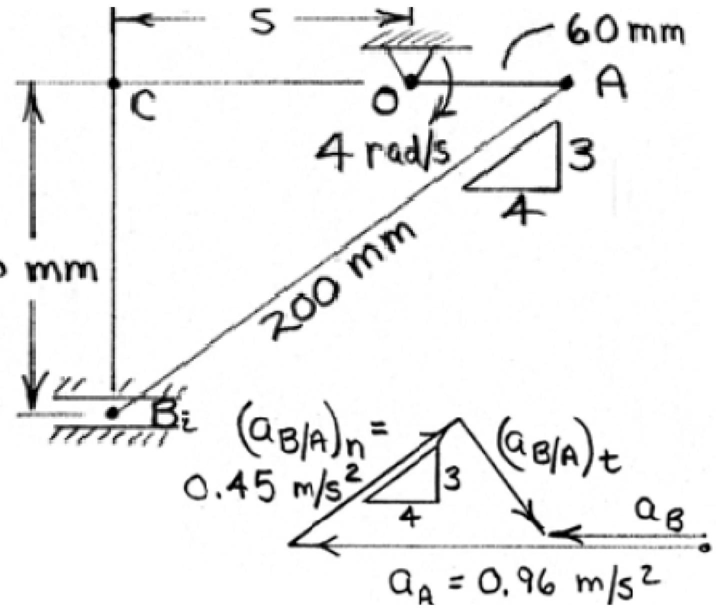
$$s = 100 \text{ mm}$$

$$v_A = 0.06(4) = 0.24 \text{ m/s}$$

$$\omega_{AB} = \frac{v_A}{AC} = \frac{0.24}{0.160} = 1.5 \text{ rad/s}$$

$$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t \; ; \quad a_A = (a_A)_n = 0.06(4)^2 = 0.96 \frac{m}{s^2} \leftarrow$$

$$(a_{B/A})_n = 0.2(1.5)^2 = 0.45 \text{ m/s}^2 \; \angle 45°$$



From the diagram,

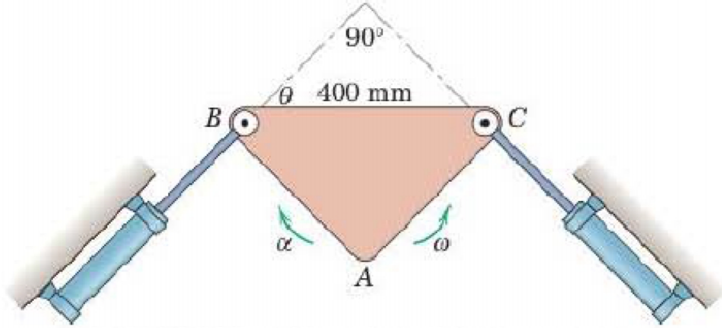
$$(a_{B/A})_t = \frac{3}{4}(0.45) = 0.338 \frac{m}{s^2}$$

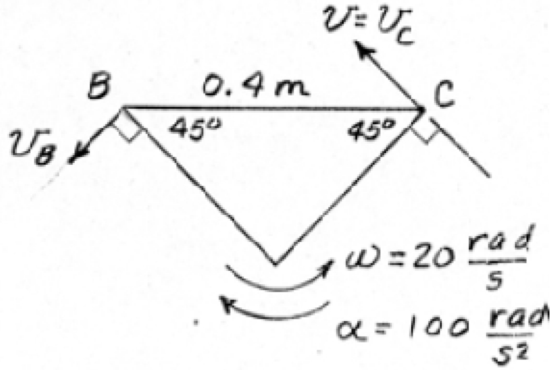
$$\alpha_{AB} = (a_{B/A})_t / \overline{AB} = \frac{0.338}{0.2} = \underline{1.688 \text{ rad/s}^2 \text{ ccw}}$$

5.3 Relative Acceleration

Example 5:

At the instant represented $\theta = 45°$ and the triangular plate ABC has a counterclockwise angular velocity of 20 rad/s and a clockwise angular acceleration of 100 rad/s^2. Determine the magnitudes of the corresponding velocity **v** and acceleration **a** of the piston rod of the hydraulic cylinder attached to C.





Solution

$\underline{a} = \underline{a}_C = \underline{a}_B + (\underline{a}_{C/B})_n + (\underline{a}_{C/B})_t$

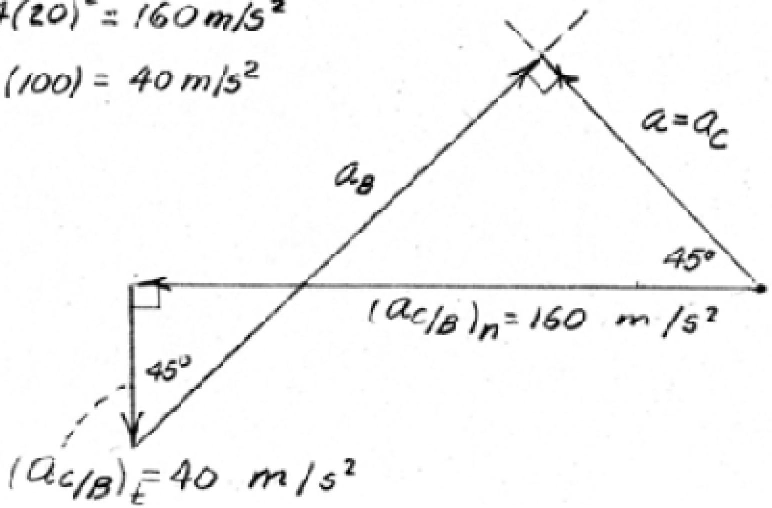
$(a_{C/B})_n = \overline{CB}\omega^2 = 0.4(20)^2 = 160\ m/s^2$

$(a_{C/B})_t = \overline{CB}\alpha = 0.4(100) = 40\ m/s^2$

From diagram

$a = 160/\sqrt{2} - 40/\sqrt{2}$

$= \underline{84.9\ m/s^2}$



$\underline{v} = \underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$

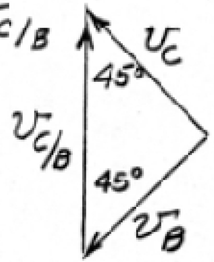
$v_{C/B} = \overline{CB}\omega$

$= 0.4(20)$

$= 8\ m/s$

$v = v_C = 8/\sqrt{2}$

$= \underline{5.66\ m/s}$

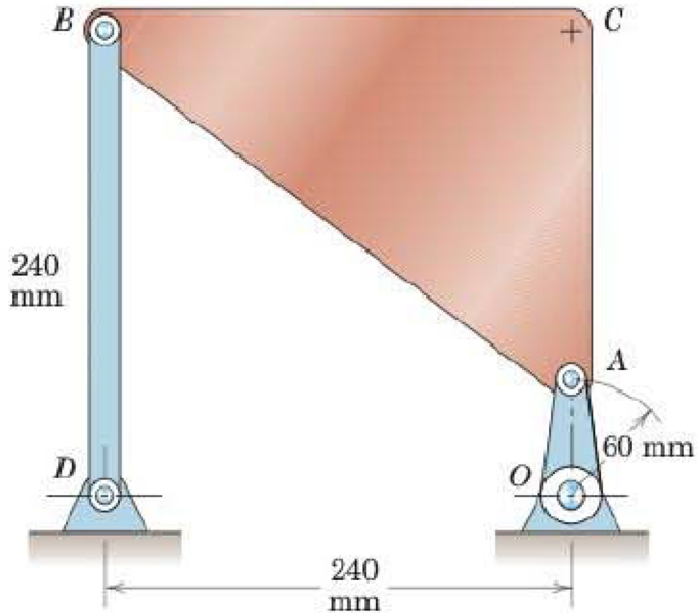


5.3 Relative Acceleration

Example 6:

Plane motion of the triangular plate *ABC* is controlled by crank *OA* and link *DB*. For the instant represented, when *OA* and *DB* are vertical, *OA* has a clockwise angular velocity of 3 rad/s and a counterclockwise angular acceleration of 10 rad/s^2. Determine the angular acceleration of *DB* for this instant.

Ans. $\alpha_{DB} = 1.234$ rad/s^2 CCW

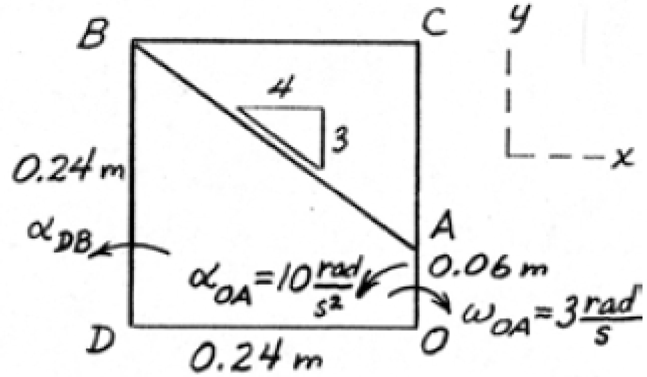


Hint:

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

$$\underline{a}_{B_n} + \underline{a}_{B_t} = \underline{a}_{A_n} + \underline{a}_{A_t} + \underline{a}_{B/A_n} + \underline{a}_{B/A_t}$$

$$-0.135\underline{j} - 0.24\alpha_{DB}\underline{i} = -0.54\underline{j} - 0.6\underline{i} + \underline{0} - 0.24\alpha_{AB}\underline{j} - 0.18\alpha_{AB}\underline{i}$$



5-5 Instantaneous Center of Zero Velocity

BY: JAAFAR MOHAMMED HAMZAH

MSc. Mechanical Engineering

5.5 ICZV

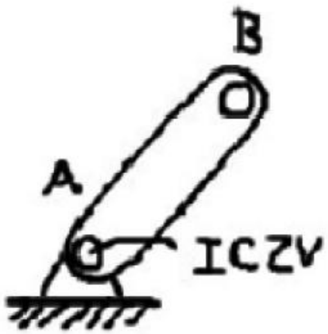
1. Introduction

- For a moving body, *at each instant of time*, there is always a point with zero velocity.

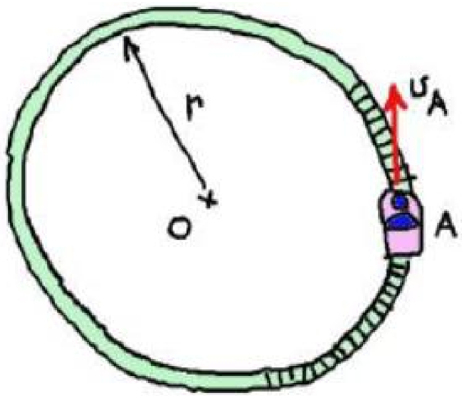
- This point is called the Instantaneous Center of Zero Velocity or ICZV.

Examples:

A rotating link



A train on a circular track



5.5 ICZV

1. Introduction

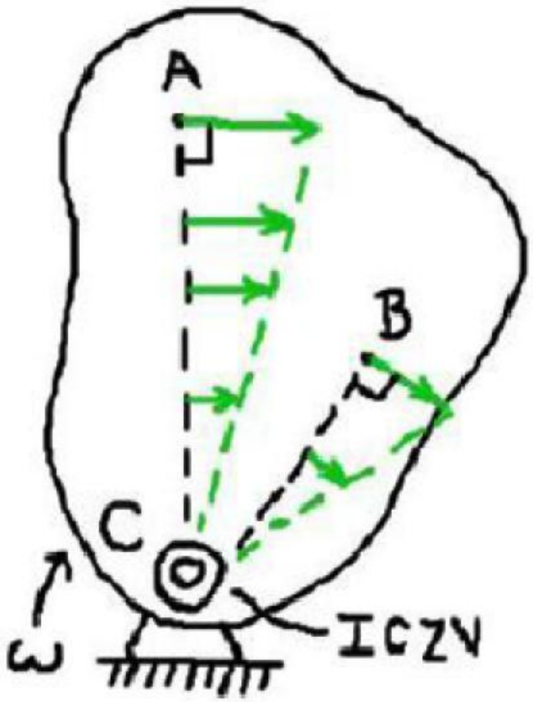
NOTES:

- ICZV is a point on the body that, at that instant, has zero velocity.
- ICZV may be on the body or anywhere else
- ICZV may be located at infinity.
- ICZV will usually not be the same point on the body all the time
- ICZV can be used to calculate velocity only.
- The body will appear to rotate about ICZV.
- Acceleration of the ICZV will not be zero.

5.5 ICZV

2. Locating ICZV

- 1. Fixed Axis Rotation



- Pure rotation, C = ICZV
- $\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}$
- $\vec{v}_C = 0$
- $\vec{v}_{A/C} = \vec{\omega} \times \vec{r}_{C \to A}$
- $\vec{v}_A = \vec{\omega} \times \vec{r}_{C \to A}$

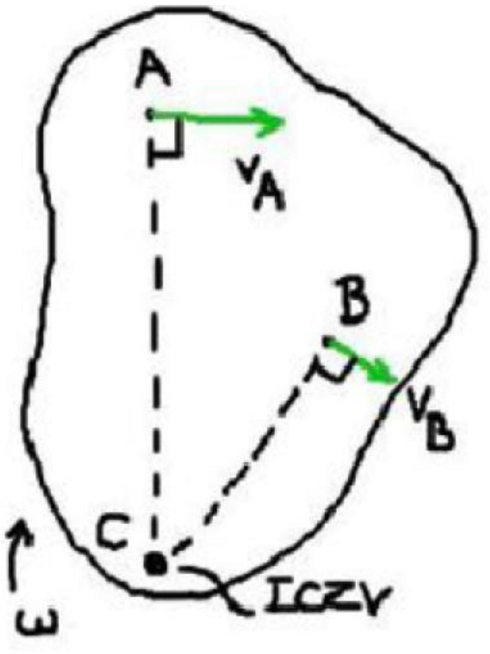
★ See that $\vec{v}_A$ must be $\perp$ to CA

★ See that v_A must be propotional to its distance from C

5.5 ICZV

2. Locating ICZV

- 1. Fixed Axis Rotation



- Suppose, $\vec{v}_A$ and $\vec{v}_B$ are known
- Draw a line $\perp$ to $\vec{v}_A$ and $\vec{v}_B$ passing through A and B
- The intersection is ICZV.
- **At this instant**, the body is rotating around the ICZV.
- In general, $a_C \neq 0$
- C = ICZV
- $\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}$
- $\vec{v}_C = 0$
- $\vec{v}_{A/C} = \vec{\omega} \times \vec{r}_{C \to A}$

★ $\vec{v}_A = \vec{\omega} \times \vec{r}_{C \to A}$

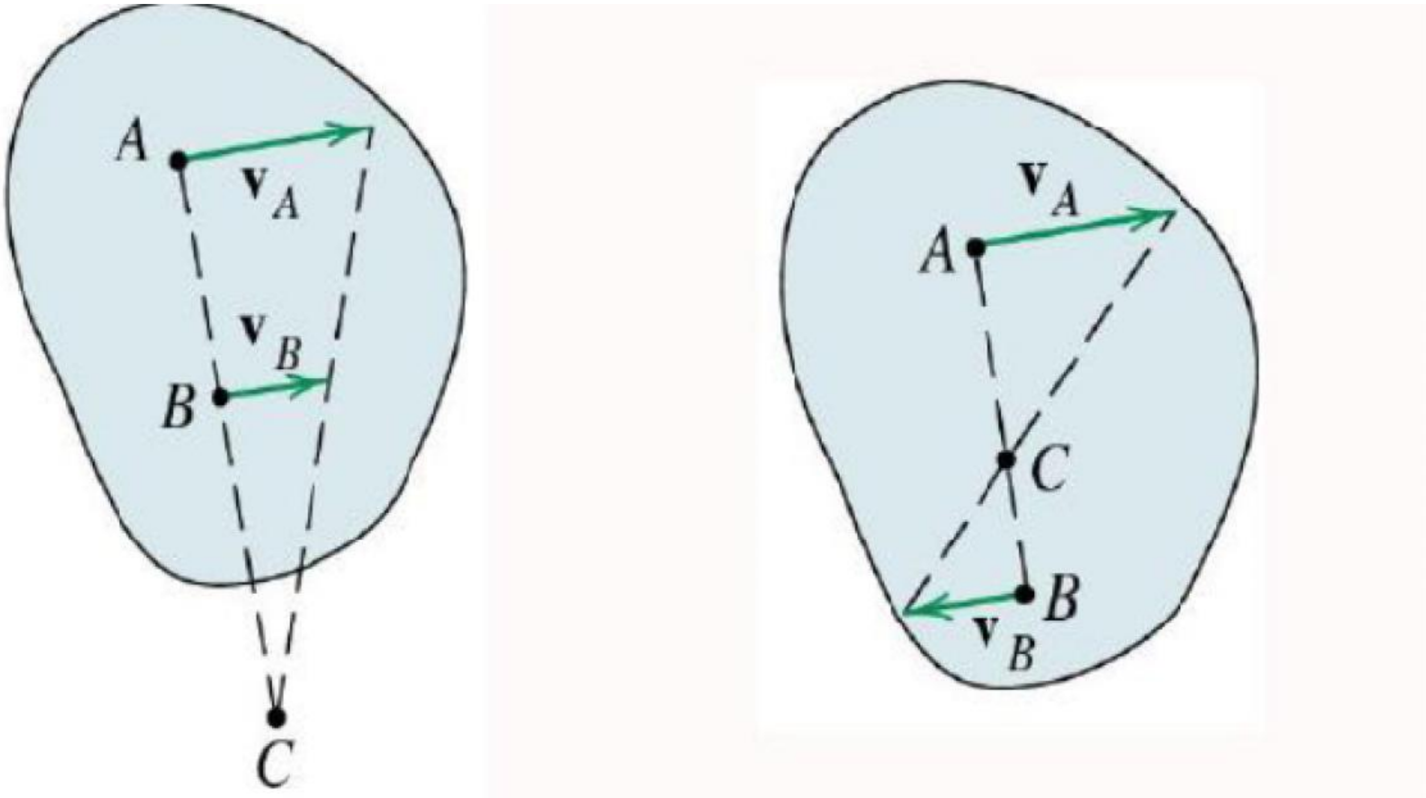
★ See that $\vec{v}_A$ must be $\perp$ to CA

★ See that v_A must be propotional to its distance from C

5.5 ICZV

2. Locating ICZV

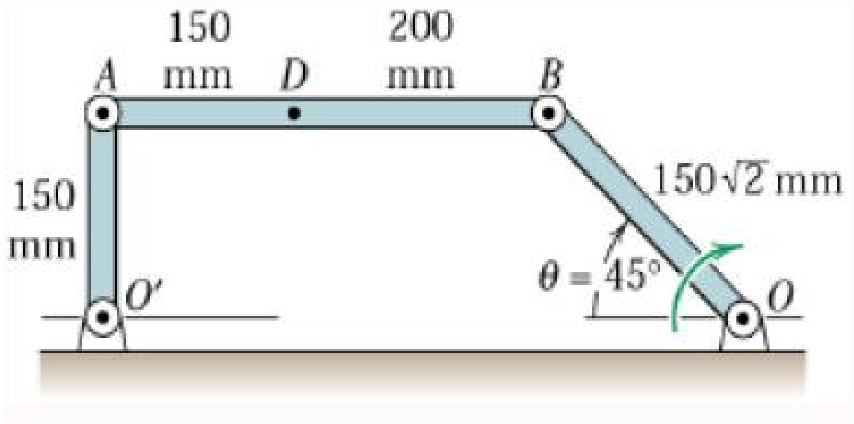
- 1. Fixed Axis Rotation



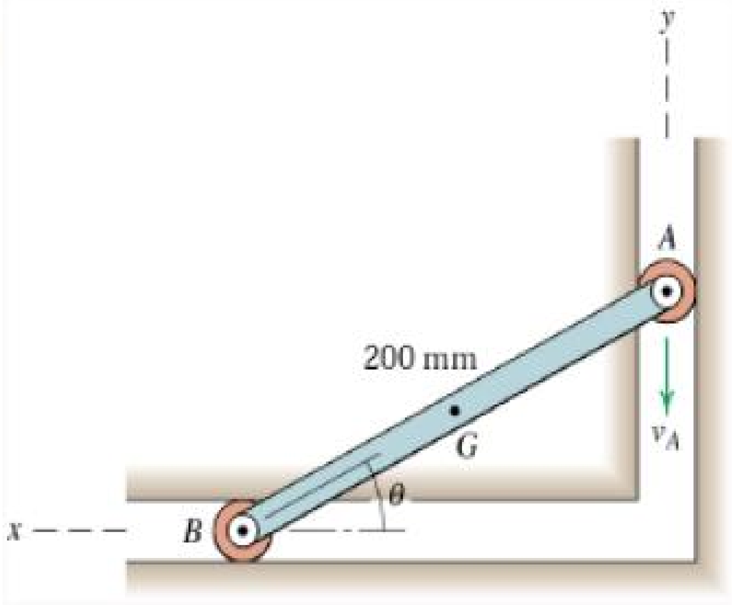
- If $\vec{v}_A = \vec{v}_B$ then ICZV is at ∞ (i.e., body in translation).

5.5 ICZV

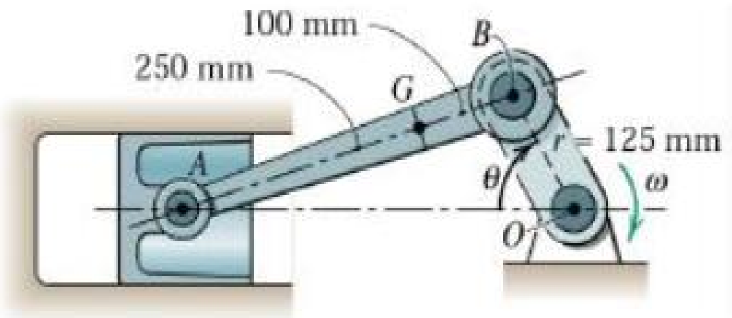
Example 1: Find the ICZV's



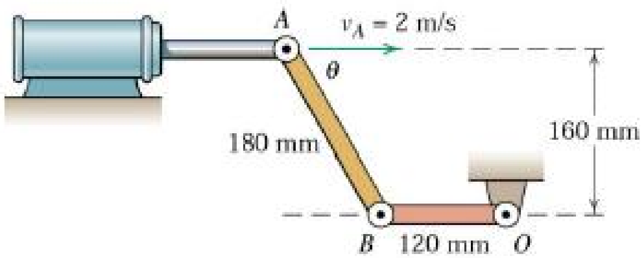
a)



b)



c)

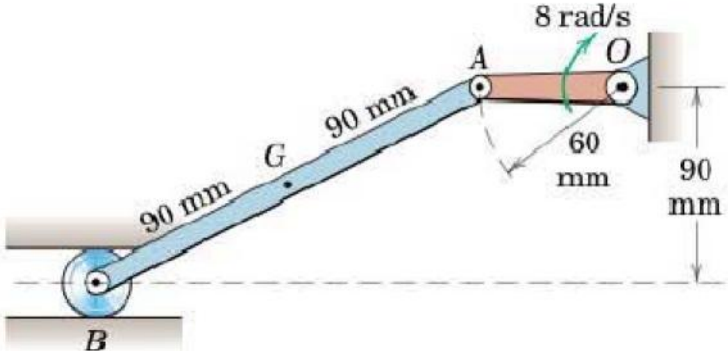


d)

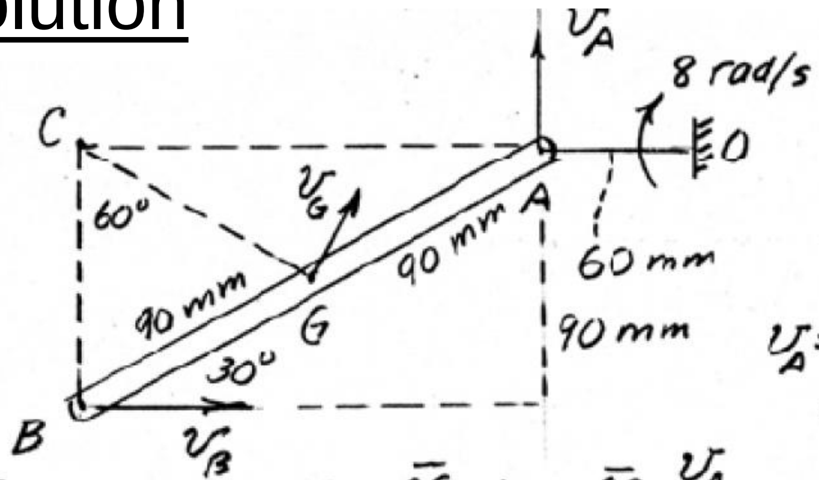
5.5 ICZV

Example 2:

For the instant represented, when crank *OA* passes the horizontal position, determine the velocity of the center *G* of link *AB* by the method of this article.



Solution



$$v_A = \overline{OA}\,\omega_{OA}$$

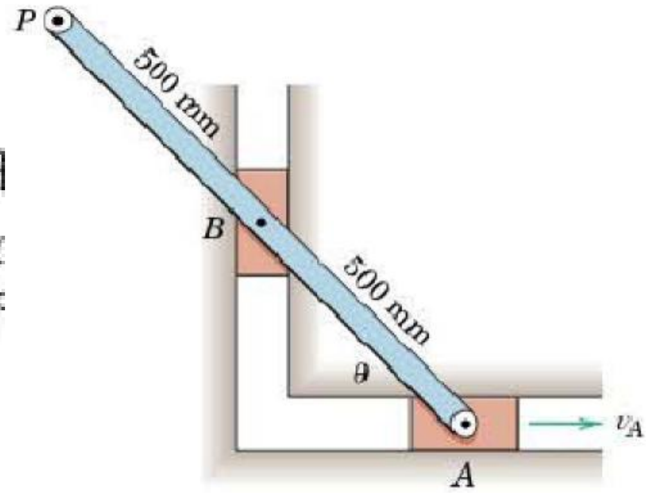
$$= 60(8) = 480 \text{ mm/s}$$

$\overline{CG} = 90$ mm, $\quad v_G = \overline{CG}\,\omega_{CG} = \overline{CG}\,\dfrac{v_A}{\overline{AC}} = \dfrac{90}{180\cos 30^\circ}\,480$

$$v_G = 277 \text{ mm/s}$$

5.5 ICZV

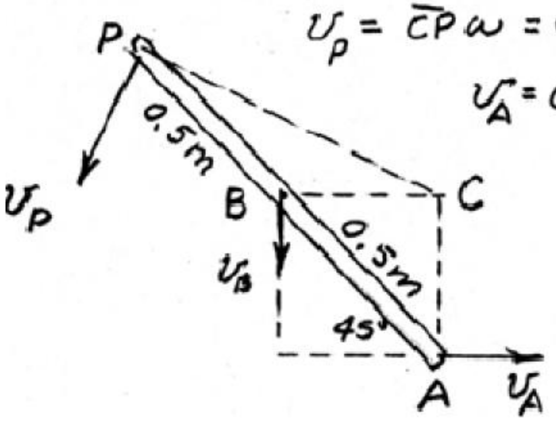
Example 3:



Motion of the bar is controlled by the constrained paths of A and B. If the angular velocity of the bar is 2 rad/s counterclockwise as the position $\theta = 45°$ is passed, determine the speeds of points A and P.

Solution

$$\overline{CP} = \sqrt{(1 \times \cos 45°)^2 + (0.5 \sin 45°)^2} = 0.791 \text{ m}$$



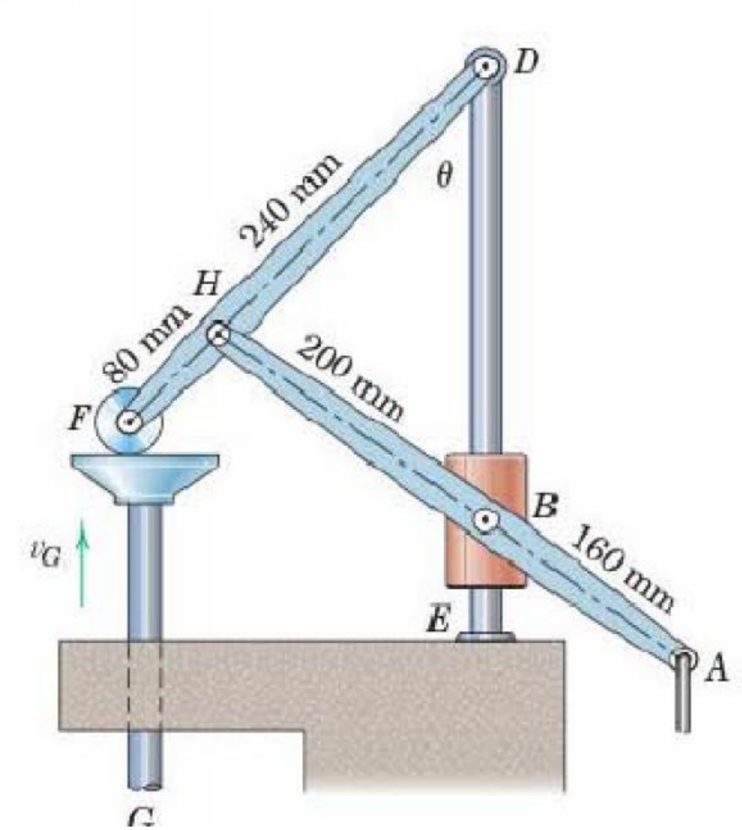
$$v_P = \overline{CP}\,\omega = 0.791(2) = \underline{1.581 \text{ m/s}}$$

$$v_A = \overline{CA}\,\omega = 0.5 \sin 45° (2)$$

$$\underline{v_A = 0.707 \text{ m/s}}$$

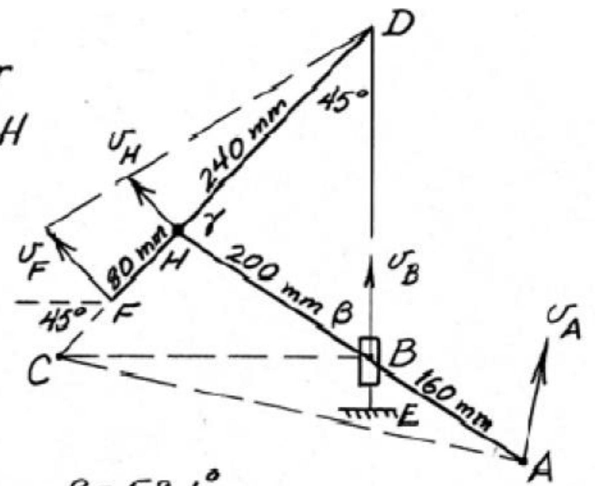
Example 4:

In the design of this mechanism, upward motion of the plunger G controls the motion of a control rod attached at A. Point B of link AH is confined to move with the sliding collar on the fixed vertical shaft ED. If G has a velocity $v_G = 2$ m/s for a short interval, determine the velocity of A for the position $\theta = 45°$.



Solution

C = instantaneous center of zero velocity of ABH



$v_F \cos 45° = v_G = 2$ m/s

so $v_F = 2.83$ m/s

& $v_H = \frac{240}{80+240} 2.83 = 2.12$ m/s

Law of sines, $\frac{240}{\sin\beta} = \frac{200}{\sin 45°}$, $\beta = 58.1°$

$\gamma = 180° - (58.1° + 45°) = 76.9°$

$\frac{\overline{BD}}{\sin 76.9°} = \frac{200}{\sin 45°}$, $\overline{BD} = 276$ mm & $\overline{DC} = \frac{276}{\cos 45°} = 390$ mm

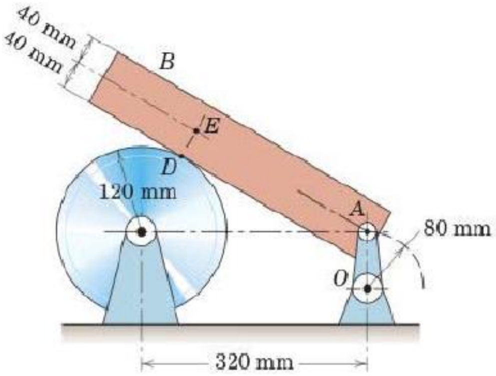
$\overline{CA}^2 = 276^2 + 160^2 - 2(276)(160)\cos(90° + 58.1°)$, $\overline{CA} = 420$ mm

$\overline{CH} = \overline{CD} - 240 = 390 - 240 = 149.7$ mm

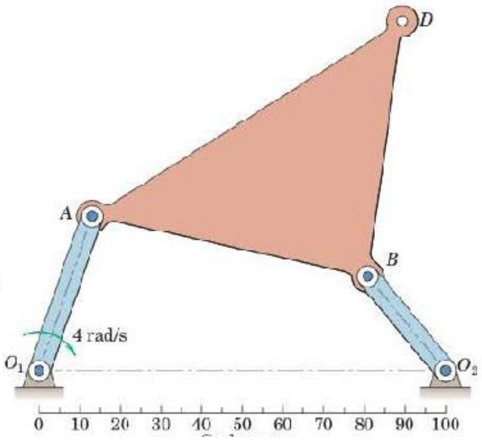
$v_A / \overline{AC} = v_H / \overline{CH}$, $v_A = 2.12 \frac{420}{149.7} = \underline{5.95 \text{ m/s}}$

5.5 ICZV

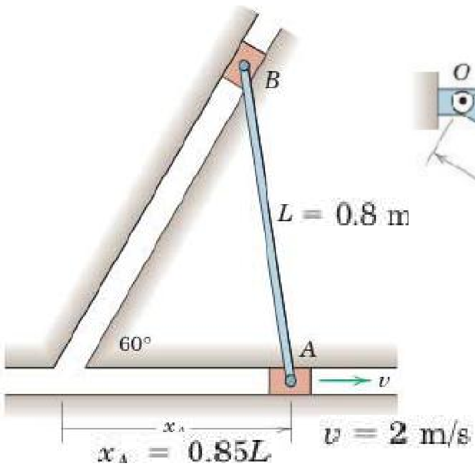
Example5: Find the ICZV's:



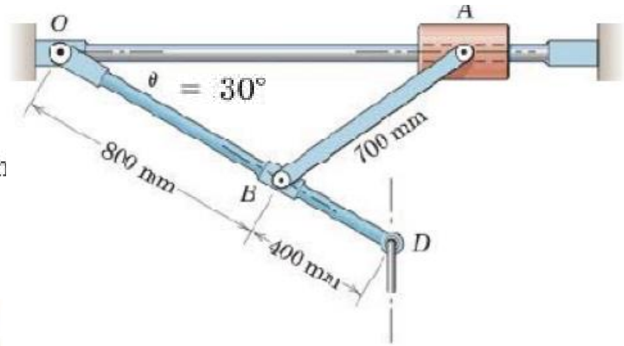
(a)



(b)

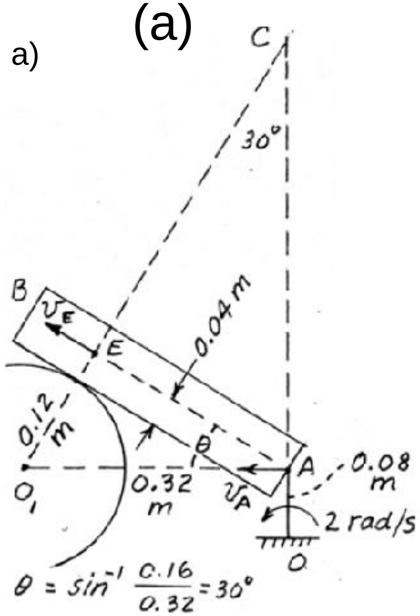


(c) H.W



(d) H.W

a)



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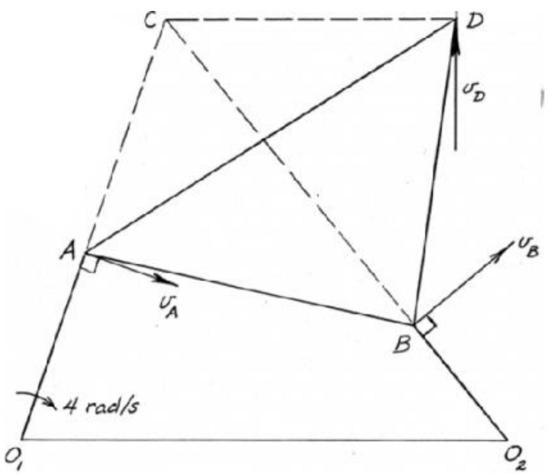
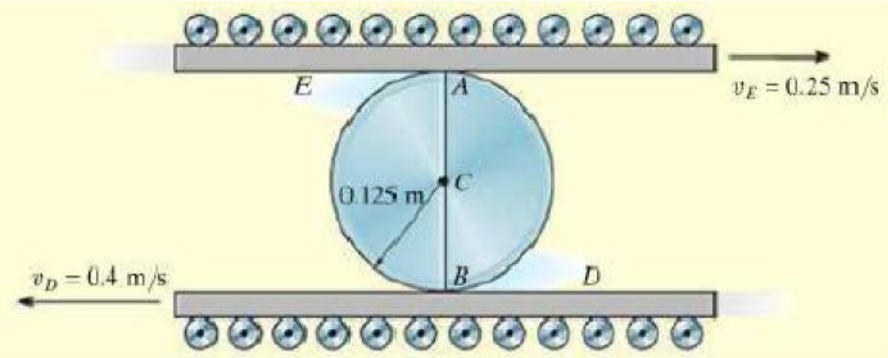


Example6:

The cylinder shown in Fig. 16–23a rolls without slipping between the two moving plates E and D. Determine the angular velocity of the cylinder and the velocity of its center C.



$$Ans. \quad v_c = 0.0750\, m/s \leftarrow$$

H.W: Solve Problems in "Engineering Mechanics Dynamics, Meriam & Karaige", (5.119&5.121), 6th Edition; or (5.81&5.105), 5th Edition.

Chapter 6 Plane Kinetics of Rigid Bodies

BY: JAAFAR MOHAMMED HAMZAH

M.Sc. Mechanical Engineering

6.1 Force, Mass, and Acceleration

- 1. Introduction
- 2. Force Equation
- 3. Moment Equation (about G)
- 4. Kinetic Diagram
- 5. Moment Equation about Other Point
- 6. Translation
  - Rectilinear
  - Curvilinear
- 7. Fixed Axis Rotation
- 8. General Plane Motion

6.1 Force, Mass, and Acceleration

1. Introduction

- A free body diagram is required.
- Three Newton's laws of Motion are used.
- The second law has two equations,
  - force equation
  - moment equation

  both applies simultaneously.

  Proofs are in Chapter 4: Systems of Particles.

$$\Sigma \vec{F} = m\vec{a}_G$$
$$\Sigma \vec{M}_G = \dot{\vec{H}}_G$$

6.1 Force, Mass, and Acceleration

2. Force Equation

Newton's Second Law (Rigid Body)

$$\Sigma \vec{F} = m\vec{a}_G$$

- $\vec{F}$ = forces acting on the rigid body,
- m = mass of the body,
- $\vec{a}_G$ = acceleration of the center of mass, G

3. Moment Equation (about G)

The Moment Equation (Rigid Body)

$$\Sigma M_G = I_G \alpha$$

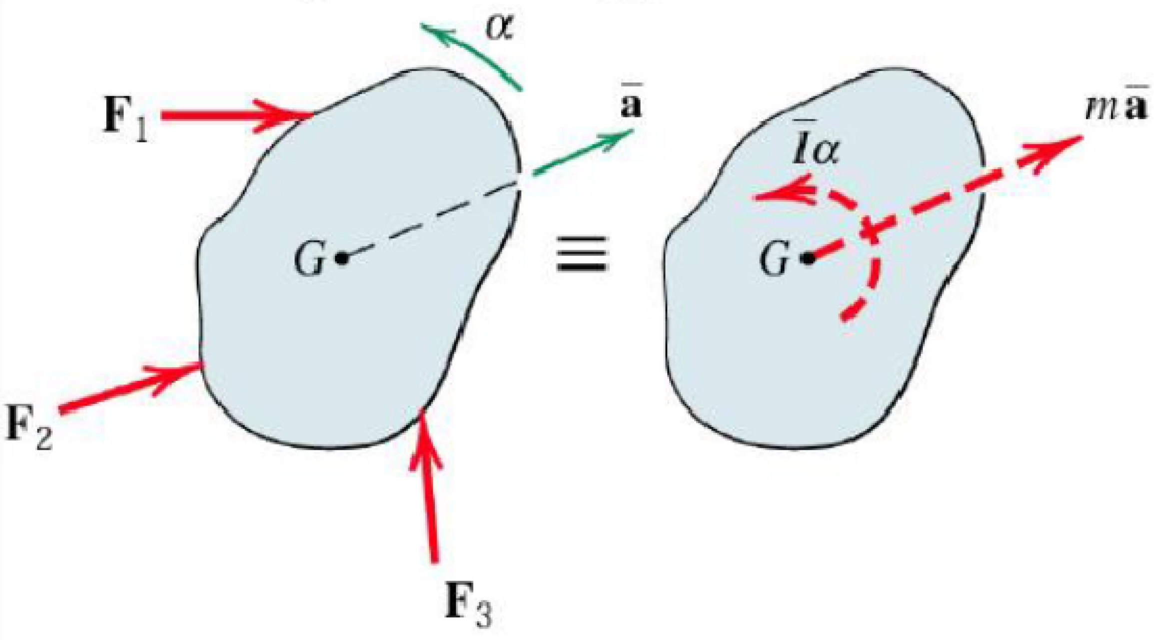
- $\vec{M}_G$ = moment (of external force) about G
- $\vec{H}_G$ = Angular momentum of the system about G

- Hence, for plane motion $H_G = \bar{I}\omega = I_G\omega$
- Then, (since I_G is a constant) $\dot{H}_G = \bar{I}\dot{\omega} = I_G\dot{\omega}$

6.1 Force, Mass, and Acceleration

4. Kinetic Diagram

- A Kinetic Diagram is strongly recommended.



Free-Body Diagram Kinetic Diagram

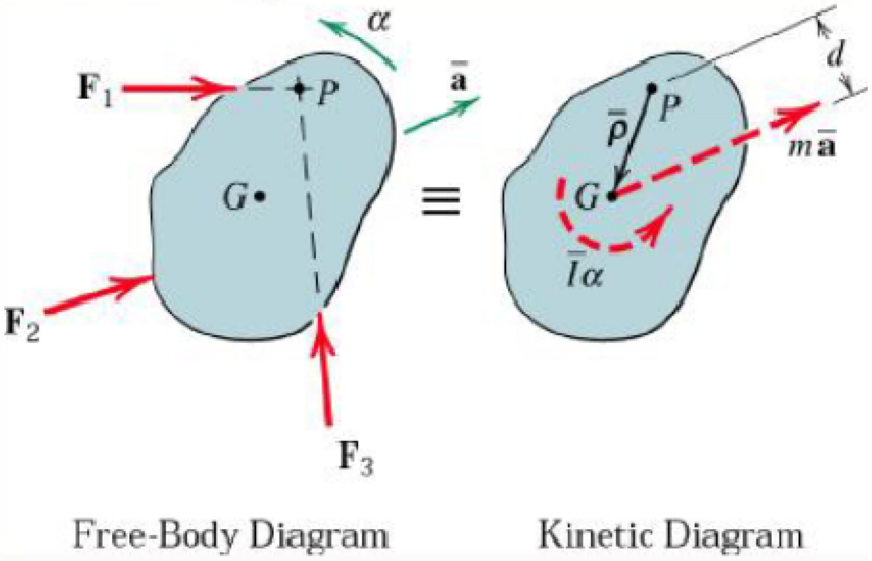
6.1 Force, Mass, and Acceleration

5. Moment Equation about Other Point

Alternative Moment Equation

$$\Sigma \vec{M}_P = I_G \vec{\alpha} + \vec{\rho}_G \times m\vec{a}_G$$

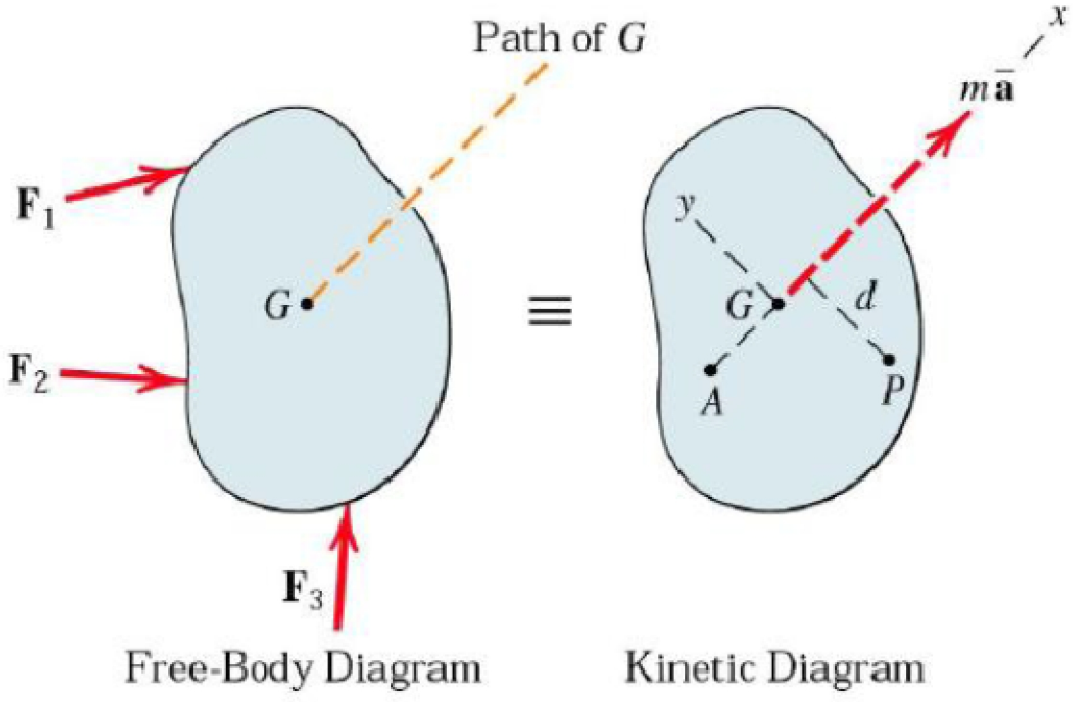
- M_P = moment about some point P
- ρ_G = vector from P to the mass center G



$$\Sigma M_P = I_G \alpha + d m a_G$$

6.1 Force, Mass, and Acceleration

6.1 Rectilinear Translation

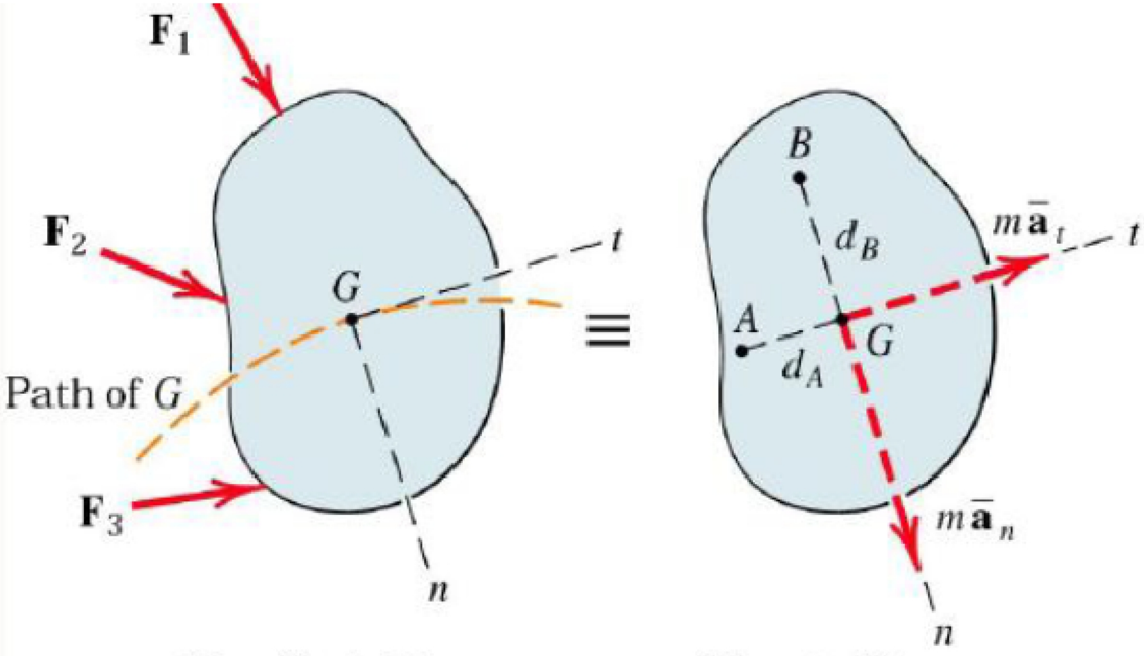


(a) Rectilinear Translation
$(\alpha = 0,\ \omega = 0)$

- $\Sigma \vec{F} = m\vec{a}_G,\ \Sigma \vec{M}_G = 0$

6.1 Force, Mass, and Acceleration

6.2 Curvilinear Translation



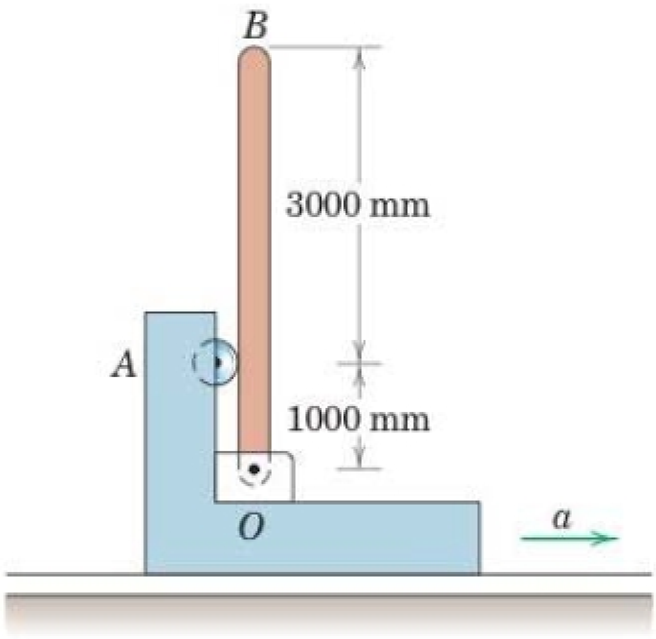
(b) Curvilinear Translation
($\alpha = 0,\ \omega = 0$)

- $\Sigma\vec{F} = m\vec{a}_G,\ \Sigma\vec{M}_G = 0$

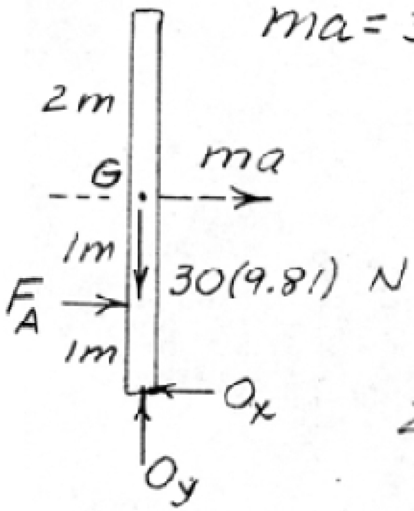
6.1 Force, Mass, and Acceleration

Example 1:

The uniform 30-kg bar OB is secured in the vertical position to the accelerating frame by the hinge at O and the roller at A. If the horizontal acceleration of the frame is $a = 20\ \text{m/s}^2$, compute the force F_A on the roller and the horizontal component of the force supported by the pin at O.



Solution:



$$ma = 30(20) = 600\ \text{N}$$

$$\Sigma M_A = m a d_A$$

$$F_A(1) = 600(2)$$

$$\underline{F_A = 1200\ \text{N}}$$

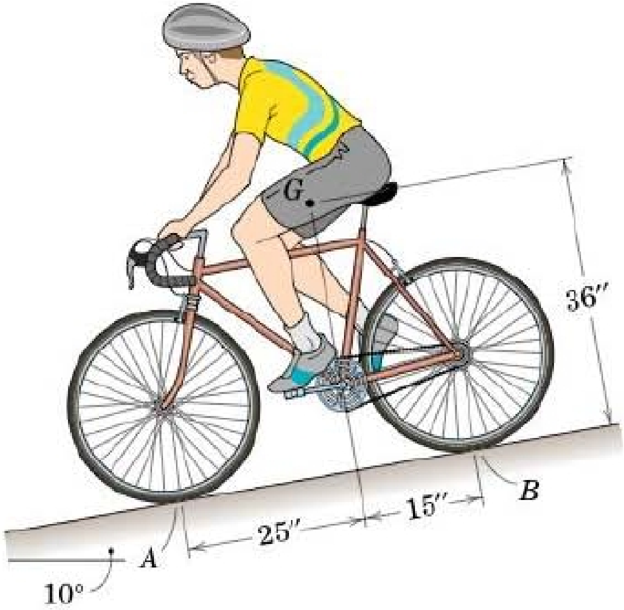
$$\Sigma F_x = m a_x ;\quad 1200 - O_x = 600$$

$$\underline{O_x = 600\ \text{N}}$$

6.1 Force, Mass, and Acceleration

Example 2:

The bicyclist applies the brakes as he descends the 10° incline. What deceleration a would cause the dangerous condition of tipping about the front wheel A? The combined center of mass of the rider and bicycle is at G.

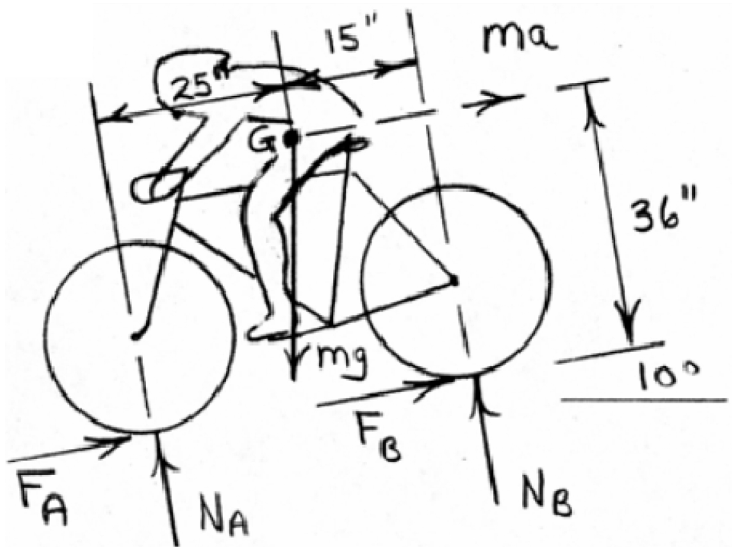


Solution:

Tipping at front wheel : $N_B, F_B \to 0$

$+\circlearrowleft \Sigma M_A = m a d : \quad mg\,(25\cos 10° - 36 \sin 10°) = ma\,(36)$

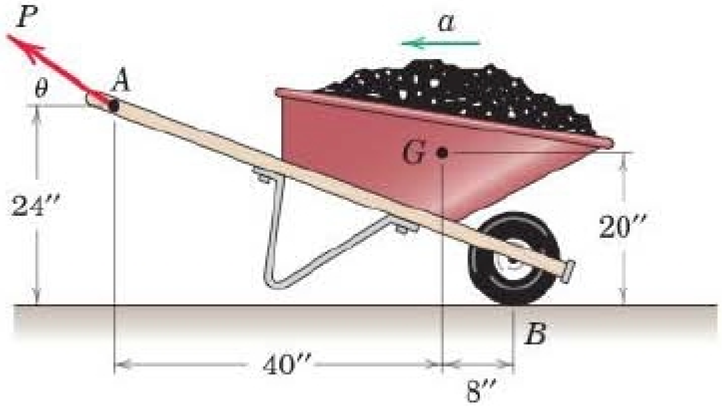
Solve to obtain $a = \underline{0.510g}$ $(\underline{16.43\ ft/sec^2})$



6.1 Force, Mass, and Acceleration

Example 3:

Determine the magnitude P and direction θ of the force required to impart a rearward acceleration $a = 5\ \text{ft/sec}^2$ to the loaded wheelbarrow with no rotation from the position shown. The combined weight of the wheelbarrow and its load is 500 lb with center of gravity at G. Compare the normal force at B under acceleration with that for static equilibrium in the position shown. Neglect the friction and mass of the wheel.



Solution:

Static equilibrium : $P_x = ma = 0$

$\circlearrowleft + \; \Sigma M_A = 0 : \; 500(40) - B(48) = 0, \quad B_{st} = 417\ \text{lb}$

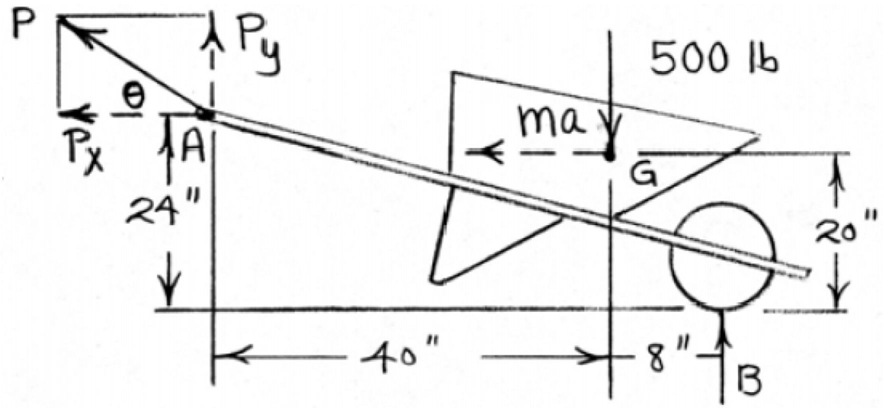
Dynamic : $\circlearrowleft + \; \Sigma M_A = mad$:

$500(40) - B(48) = \frac{500}{32.2}(5)(4), \quad B = 410\ \text{lb}$

$\xleftarrow{+} \; \Sigma F_x = ma : \; P_x = \frac{500}{32.2}(5) = 77.6\ \text{lb}$

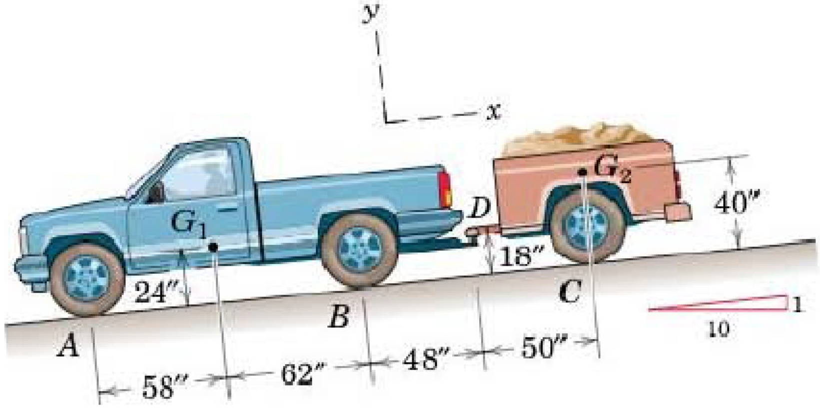
$+\uparrow \; \Sigma F_y = 0 : \; B - 500 + P_y = 0, \; P_y = 89.8\ \text{lb}$

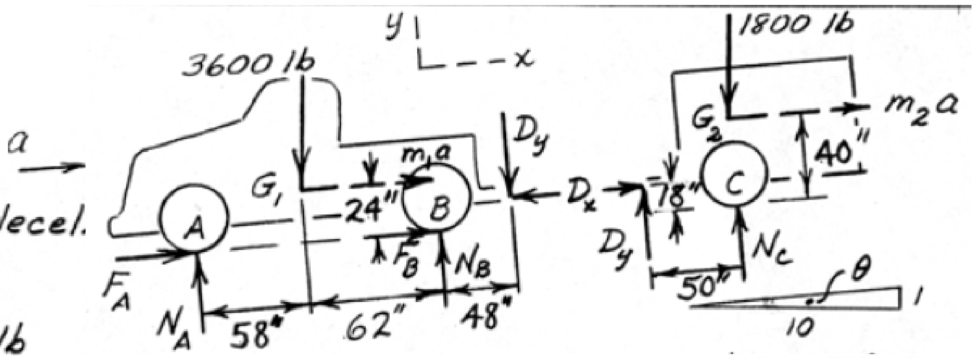
$\therefore \; P = 118.7\ \text{lb}, \quad \theta = 49.2^\circ$



Example 4:

The loaded pickup truck, which weighs 3600 lb with mass center at G_1, is hauling the 1800-lb trailer with mass center at G_2. While going down a 10-percent grade, the driver applies his brakes and slows down from 60 mi/hr to 30 mi/hr in a distance of 360 ft. For this interval, compute the x- and y-components of the force exerted on the trailer hitch at D by the truck. Also find the corresponding normal force under each pair of wheels at B and C. Neglect the rotational effect of the wheels.





Solution

For const. accel.,

$v^2 = v_0^2 + 2as$: $44^2 = 88^2 - 2a(360)$, $a = 8.07$ ft/sec^2 decel.

$m_1 a = \frac{3600}{32.2} \times 8.07 = 902$ lb, $m_2 a = \frac{1800}{32.2} \times 8.07 = 451$ lb

Trailer: $\Sigma F_x = ma_x$: $D_x - 1800 \sin 5.71° = 451$, $\underline{D_x = 630 \text{ lb}}$

$\circlearrowleft \Sigma M_C = mad$: $50D_y + 630(18) - 1800 \sin 5.71°(40) = 451(40)$, $\underline{D_y = 277 \text{ lb}}$

$\Sigma F_y = 0$: $N_C - 1800 \cos 5.71° + 277 = 0$, $\underline{N_C = 1514 \text{ lb}}$

$\theta = \tan^{-1}\frac{1}{10} = 5.71°$

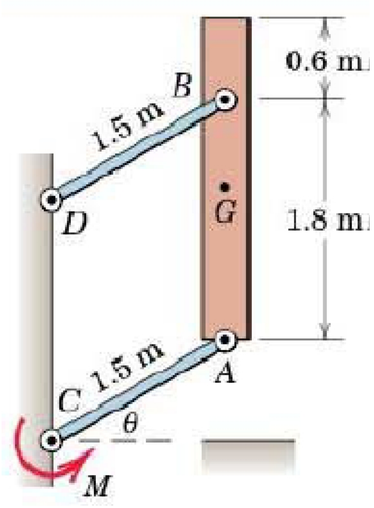
Truck: $\circlearrowleft \Sigma M_A = mad$: $3600 \cos 5.71° \times 58 - 3600 \sin 5.71° \times 24 - 120 N_B + 277(168) - 630(18) = 902(24)$

$\underline{N_B = 1773 \text{ lb}}$

6.1 Force, Mass, and Acceleration

Example 5:

The vertical bar AB has a mass of 150 kg with center of mass G midway between the ends. The bar is elevated from rest at $\theta = 0$ by means of the parallel links of negligible mass, with a constant couple $M = 5$ kN·m applied to the lower link at C. Determine the angular acceleration α of the links as a function of θ and find the force B in the link DB at the instant when $\theta = 30°$.



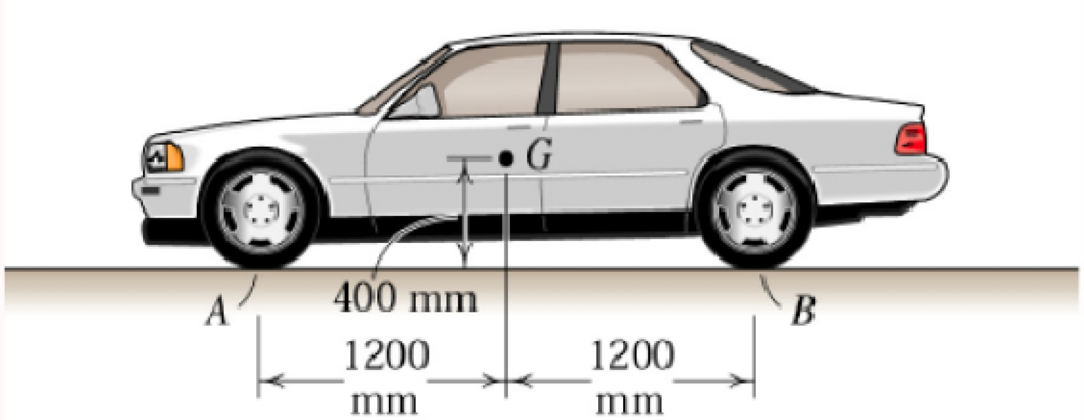
Solution:

"Mechanical Engineering Dynamics", 6th Edition, Meriam, Page 431.

6.1 Force, Mass, and Acceleration

Example: H.W

The 1600-kg car has its mass center at G. Calculate the normal force N_A and N_B between the road and the front and rear pairs of wheels when acceleration of the car is 2 m/s^2. The mass of the wheels are small compared to the mass of the car.



Ans. $N_A = 6.85$ kN, $N_B = 9.34$ kN

- Do this: If the coefficient of **static** friction between the tire and the ground is 0.8, what is the maximum possible acceleration of this car if
  - it is a front wheel drive car,
  - it is a rear wheel drive car.

6-2 Work and Energy

6-2 Work and Energy

- 1. Introduction
- 2. Work
- 3. Kinetic Energy
- 4. Work - Energy Equation
- 5. Conservation of Energy

6-2 Work and Energy

1. Introduction

- Useful when forces involved are function of displacement/position of the system; i.e., configuration of the rigid body.
- To get changes in velocity/angular velocity between the starting point and the end point of a motion (or configuration of the system)
- Very easy for an interconnected rigid body.

6-2 Work and Energy

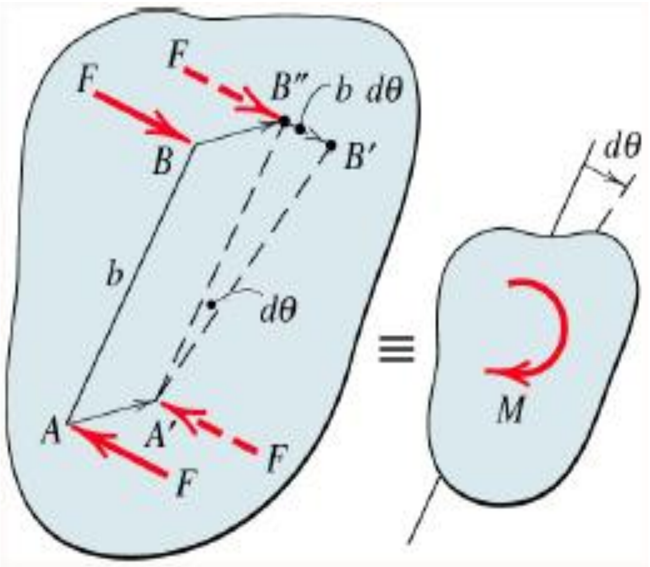
2.1 Work of a Force

$$U_{1-2} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} \quad \text{or} \quad U_{1-2} = \int_{s_1}^{s_2} (F\cos\alpha)\,ds$$

2.2 Work of a Couple (a kind of Moment)

Work of a Couple

$$U = \int M d\theta$$



6-2 Work and Energy

3. Kinetic Energy

- 1. Translation Only

$$T = \frac{1}{2} m v_G^2$$

- 2. Fixed Axis Rotation

$$T = \frac{1}{2} I_O \omega^2$$

or

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

- 3. General Plane

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

- Note: Rotation about ICZV

$$T = \frac{1}{2} I_C \omega^2$$

6-2 Work and Energy

4. Work – Energy Equation

- Elastic potential is the same as in the particle case.
- Gravitational potential, use location of the mass center, G
- Work-energy relation also applied to a rigid body and interconnected rigid bodies

Work-Energy Equation

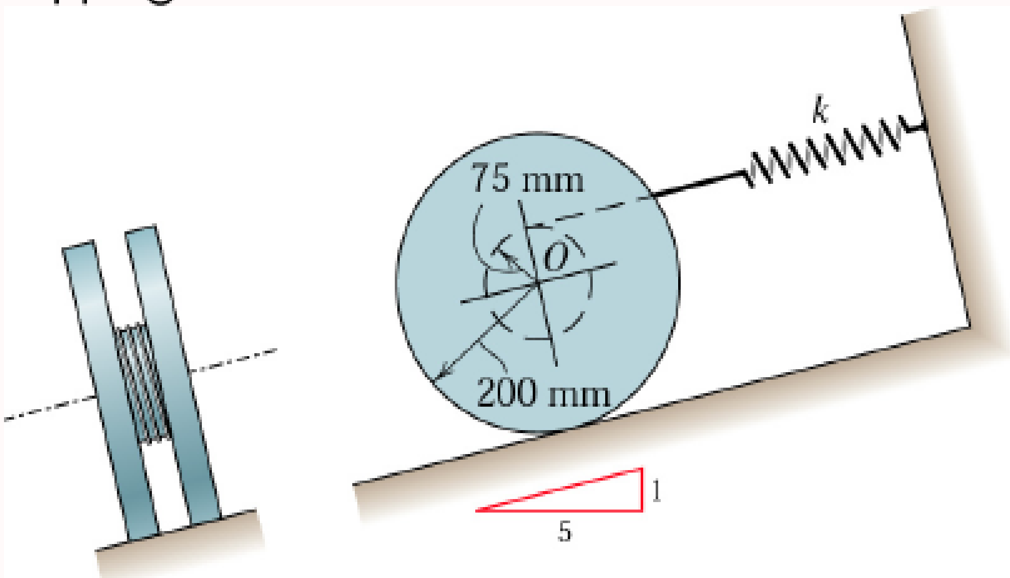
$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

- U'_{1-2} = Work of **external** force on the system, not including gravitational and elastic force.

6-2 Work and Energy

Example 1:

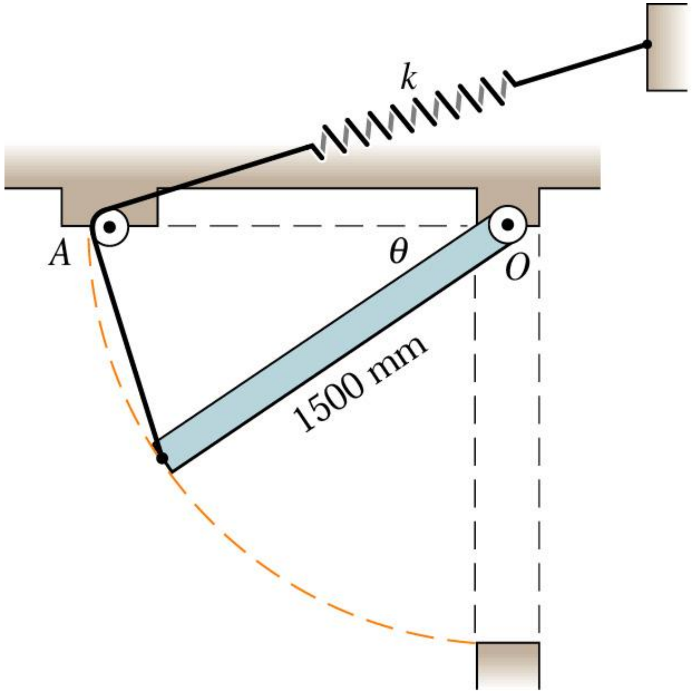
The 10-kg double wheel with radius of gyration of 125 mm about O is connected to the spring of stiffness $k = 600$ N/m by a cord which is wrapped securely around the inner hub. If the wheel is released from rest on the incline with the spring stretched 225 mm, calculate the maximum velocity v of its center O during the ensuing motion. The wheel rolls without slipping.



6-2 Work and Energy

Example 2: Ventilator Door

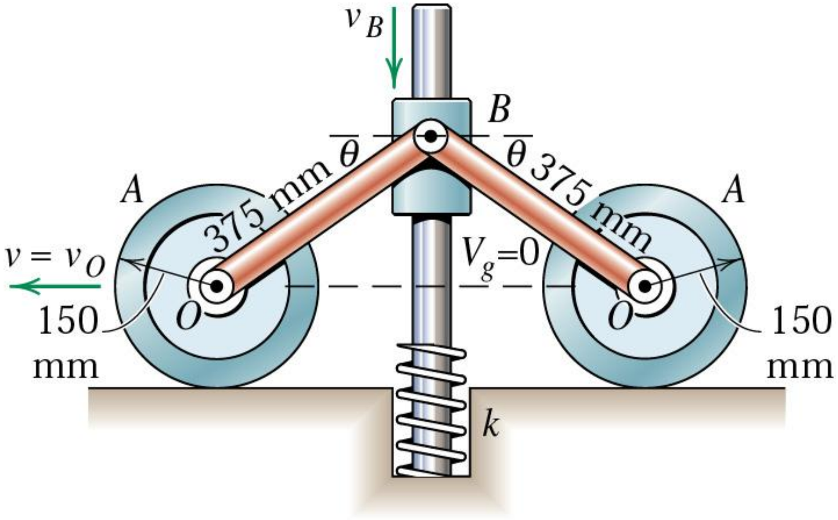
The figure shows the cross section of a uniform 100-kg ventilator door hinged about its upper horizontal edge at O. The door is controlled by a spring-loaded cable which passes over the pulley at A. The spring has a stiffness of 200 N/m and is undeformed when θ = 0. If the door is released from rest in the horizontal position, determine the door's maximum ω and the corresponding angle θ.



6-2 Work and Energy

Example 3: Rolling Wheels

Each of two wheels has a mass of 30 kg and a radius of gyration of 100 mm. Each link OB has a mass of 10 kg. The 7-kg collar at B slides on the fixed vertical shaft with no friction. The spring has the stiffness k = 30 kN/m and is contacted by the collar when θ = 0º. If the collar is released from rest at θ = 45º and the wheels do not slip, determine a) v_B when θ = 0º and b) maximum deflection of the spring.



6-3 Impulse and Momentum

6-3 Impulse and Momentum

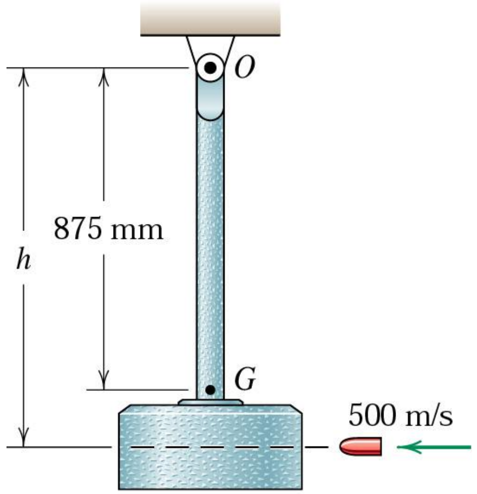
- 1. Introduction
- 2. Linear Impulse and Momentum
- 3. Angular Impulse and Momentum
- 4. Conservation of Momentum

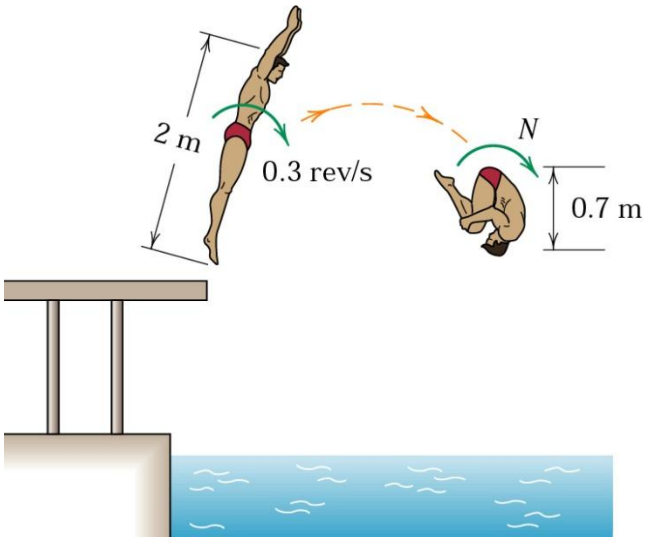
6-3 Impulse and Momentum

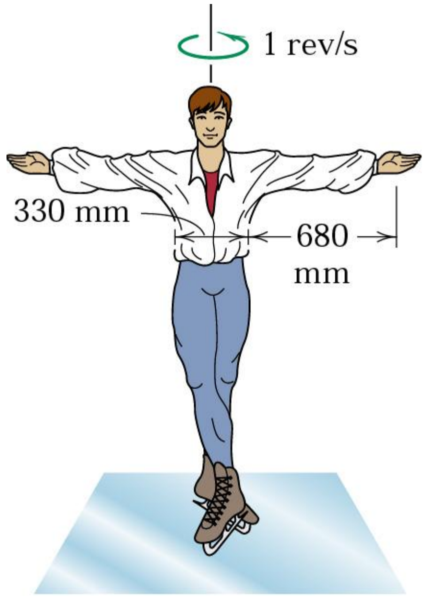
1. Introduction

- Advantages
  - Good when force is a function of time
  - Good if interaction of bodies occurs during a short time; e.g., impact problems
  - Good when momentum is conserved (obviously).

Applications:







6-3 Impulse and Momentum

2. Linear Impulse and Momentum

Linear Momentum

$$\vec{G} = m\vec{v}_G$$

Newton's generalized second law

$$\Sigma\vec{F} = \dot{\vec{G}}$$

$$\Sigma F_x = \dot{G}_x$$
$$\Sigma F_y = \dot{G}_y$$

Impulse-Moment Equation

$$\int_{t_1}^{t_2} \Sigma\vec{F}\,dt = \vec{G}_2 - \vec{G}_1$$

$$\int_{t_1}^{t_2} \Sigma F_x\,dt = G_{x_2} - G_{x_1}$$
$$\int_{t_1}^{t_2} \Sigma F_y\,dt = G_{y_2} - G_{y_1}$$

6-3 Impulse and Momentum

2. Linear Impulse and Momentum

Notes:

- Even when the wheel is rolling without slipping, the friction will have impulse!
- However, recall that friction have no work if the wheel is rolling without slipping.

6-3 Impulse and Momentum

3. Angular Impulse and Momentum

3.1 About G

The moment equation

$$\Sigma M_G = \dot{H}_G$$

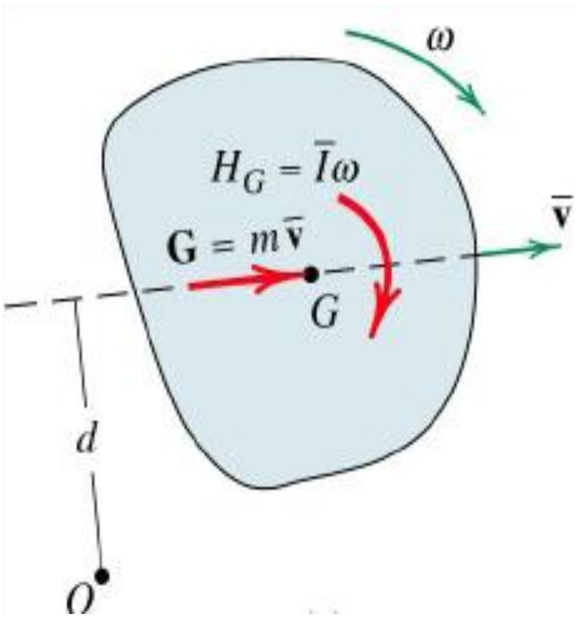
The Impulse and Momentum Equation

$$\int_{t_1}^{t_2} \Sigma M_G \, dt = H_{G_2} - H_{G_1}$$

6-3 Impulse and Momentum

3. Angular Impulse and Momentum

3.2 About Any Fixed Point



- The moment equation:

$$\Sigma M_O = \dot{H}_O$$

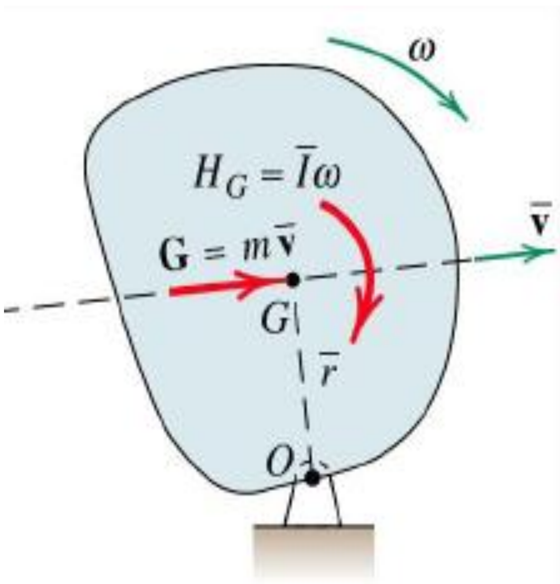
where,

$$H_O = I_G\omega + mv_G d$$

6-3 Impulse and Momentum

3. Angular Impulse and Momentum

3.3 About the Fixed Axis



Fixed-Axis Rotation

$$H_O = I_O\omega \rightarrow \Sigma M_O = I_O\dot{\omega}$$

Fixed-Axis Rotation

$$\int_{t_1}^{t_2} \Sigma M_O \, dt = I_O(\omega_2 - \omega_1)$$

6-3 Impulse and Momentum

4. Conservation of Momentum

For system of particle,

- When no net external force

$$\Delta\vec{G} = 0$$

- When no net moment (external forces and couples only) about a **fixed** point O

$$\Delta H_O = 0$$

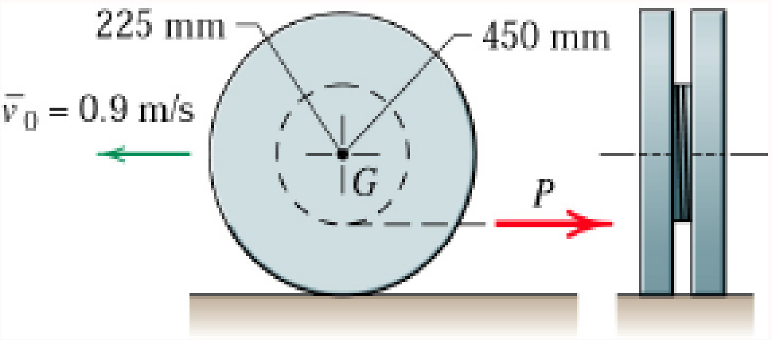
- Or about the system's center of mass G

$$\Delta H_G = 0$$

6-3 Impulse and Momentum

Example 1: Rolling Wheel

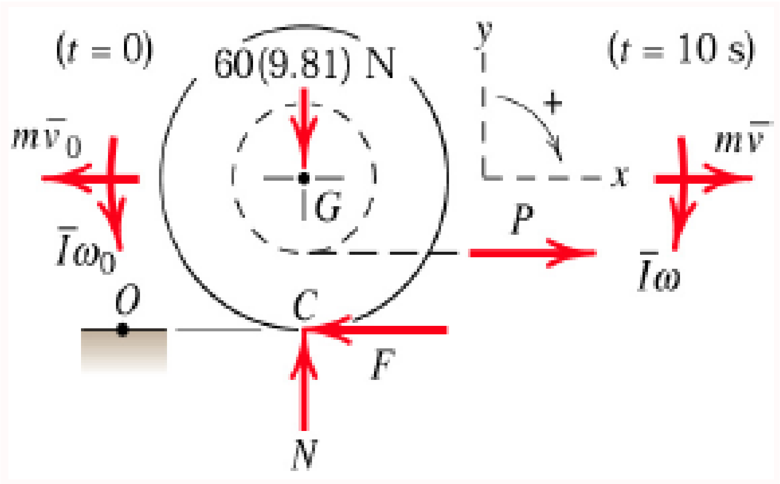
The force P, which is applied to the cable wrapped around the central hub of the symmetrical wheel, is increased slowly according to $P = 6.5t$, where P is in newtons and t is the time in second after P is first applied.



Determine the angular velocity ω of the wheel 10 s after P is applied if the wheel is rolling to the left with a velocity of its center of 0.9 m/s at time $t = 0$. The wheel, which has a mass of 60 kg and a radius of gyration about its center of 250 mm, rolls without slipping.

6-3 Impulse and Momentum

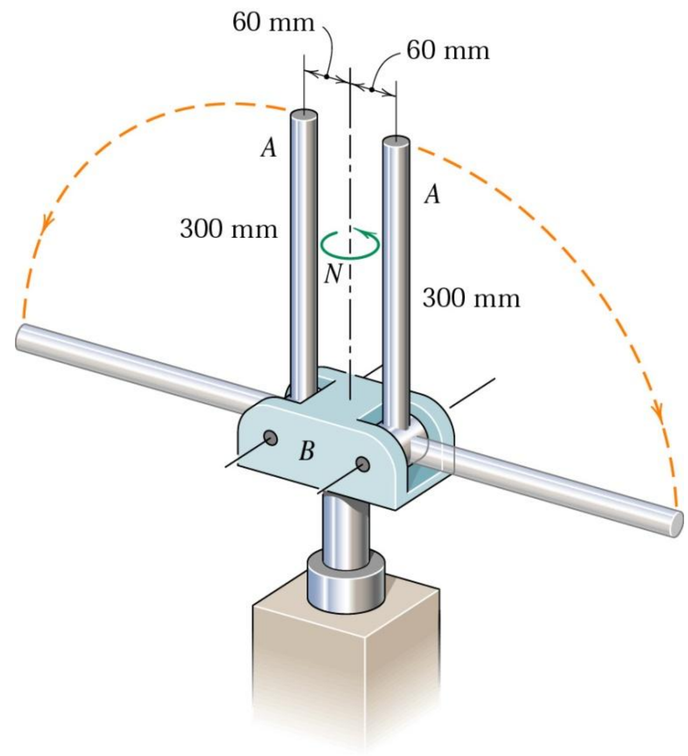
Solution: Example 1



6-3 Impulse and Momentum

Example 2:

Each of the two 300-mm rods A has a mass of 1.5 kg and is hinged at its end to the rotating base B. The 4-kg base has a radius of gyration of 40 mm and is initially rotating with a speed of 300 rev/min. If the rods are released to fall down to the horizontal positions, calculate the new rotational speed.



6/4 Fixed Axis Rotation

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M.Sc. Mechanical Engineering

6.4 Fixed Axis Rotation

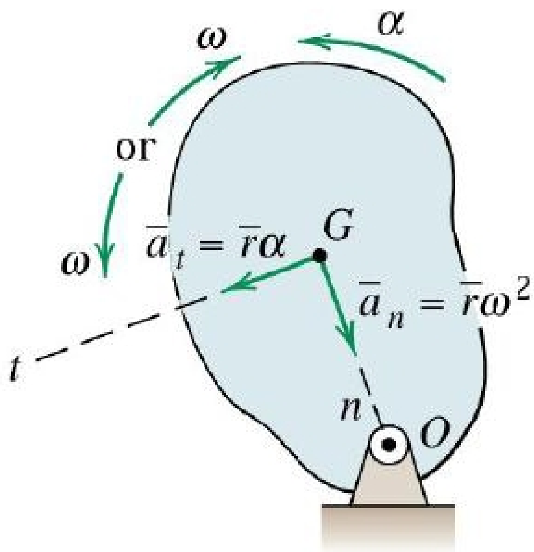
- Fixed Axis Rotation:

EOM: General Motion

$$\Sigma \vec{F} = m\vec{a}_G$$
$$\Sigma M_G = I_G \alpha$$

and

$$\Sigma M_O = I_O \alpha$$



Kinematics:

- $(\vec{a}_G)_t = \vec{\alpha}_{OG} \times \vec{r}_{O \to G}$
- $(\vec{a}_G)_n = \vec{\omega}_{OG} \times (\vec{\omega}_{OG} \times \vec{r}_{O \to G})$

$$\sum F_n = m(a_G)_n = m\omega^2 r$$

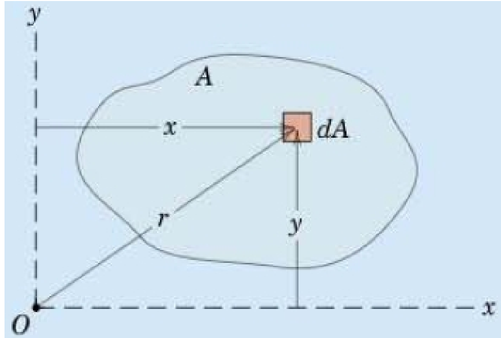
$$\sum F_t = m(a_G)_t = mr\alpha$$

- Rigid body rotates about O
- M_O = moment of forces about O
- I_O = mass moment of inertia about point O

6.4 Fixed Axis Rotation

- Area Moment of Inertia (Ix, Iy, Iz):

The moments of inertia of a plane area A about x- and y- axes in its plane and about z- axis normal to its plane are defined by:

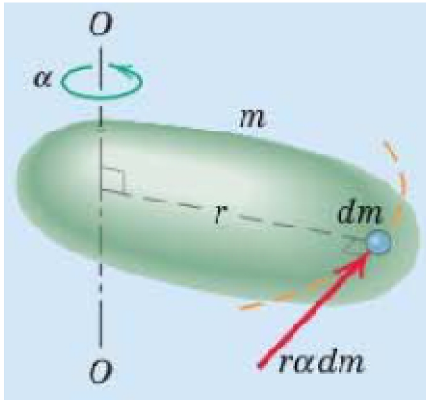


$$I_x = \int y^2\, dA \qquad I_y = \int x^2\, dA \qquad I_z = \int r^2\, dA$$

- Mass Moment of Inertia (I):

It is the force ($dm \times r\alpha$) multiplied by the radius of the rotation. Thus, it is:

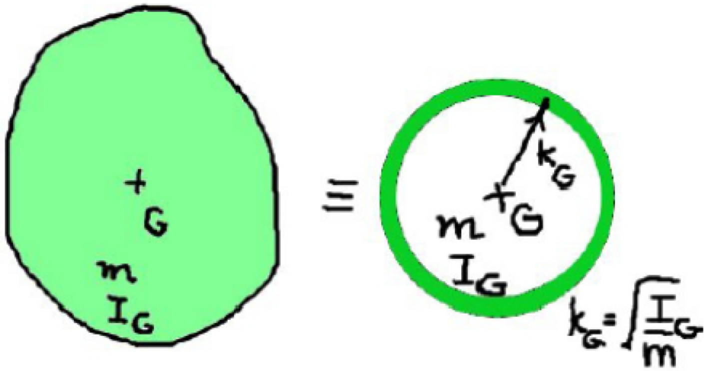
$$I = \int r^2\, dm \qquad I = \rho \int r^2\, dV$$



$$I = \Sigma r_i^2 m_i$$

6.4 Fixed Axis Rotation

- Radius of Gyration (k):



  - Radius of gyration is often used to specify the mass moment of inertia of a rigid body.
  - Given mass m and the radius of gyration k_P (about point P), we have

$$I_P = mk_P^2$$

  - Usually point P is G or the fixed point O
  - Imagine mass concentrated at radius k_G

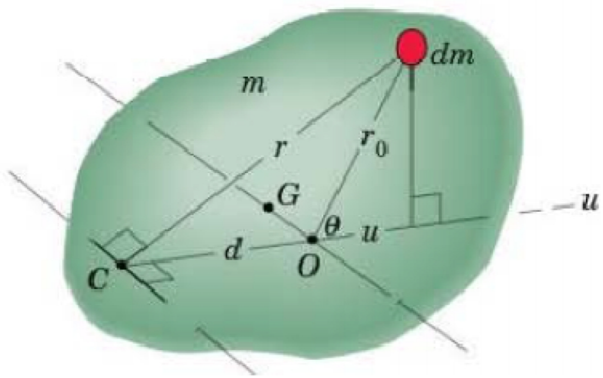
$$k = \sqrt{\frac{I}{m}} \qquad \text{or} \qquad I = k^2 m$$

- Parallel Axis Theorem:

It's easily to determine moment of inertia about any axis parallel to the mass center, as:

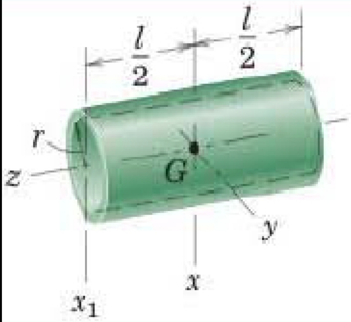
$$I = \bar{I}_G + md^2$$

I_G : Moment of inertia about G
d : Perpendicular distance to C



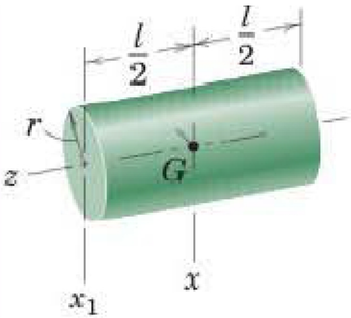
6.4 Fixed Axis Rotation

- Mass Moment of Inertia ($I_{about\ axis}$): (Look up TABLE D/4 in the Book)



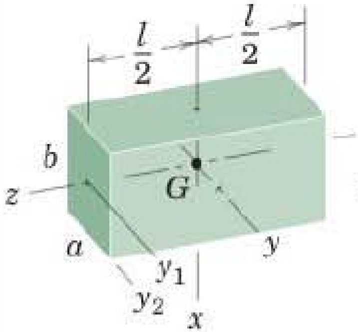
Circular Cylindrical Shell

$I_{zz} = mr^2$



Circular Cylinder

$I_{zz} = \frac{1}{2}mr^2$



Rectangular Parallelepiped

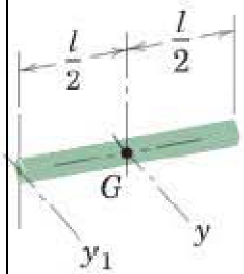
$I_{zz} = \frac{1}{12}m(a^2 + b^2)$

Spherical Shell

$I_{zz} = \frac{2}{3}mr^2$

Sphere

$I_{zz} = \frac{2}{5}mr^2$



Uniform Slender Rod

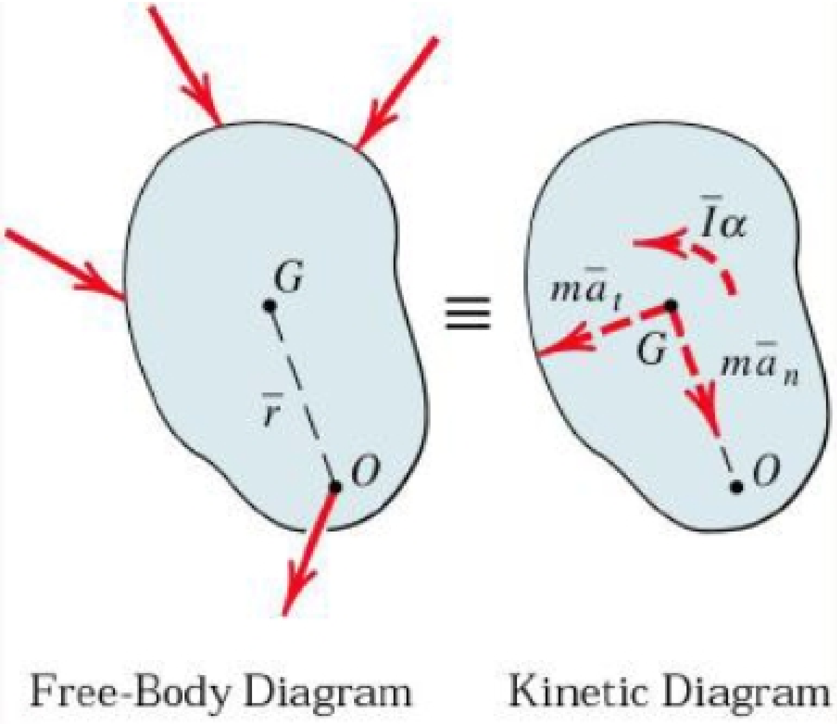
$I_{yy} = \frac{1}{12}ml^2$

JAAFAR MOHAMMED HAMZAH

6.4 Fixed Axis Rotation

Proof:

Moment Equation about point O
$\Sigma M_O = I_O \alpha$



- $\Sigma M_O = I_G \alpha + m a_{Gt} \bar{r}$
- But, $a_{Gt} = \alpha \bar{r}$
- Then,

$$\Sigma M_O = I_G \alpha + m\bar{r}^2 \alpha$$
$$\Sigma M_O = (I_G + m\bar{r}^2)\alpha$$

- We know that

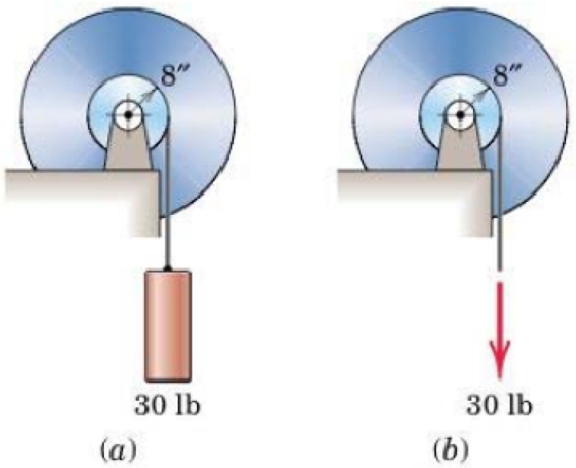
$$I_O = I_G + m\bar{r}^2$$

↑ Parallel Axis Theorem

6.4 Fixed Axis Rotation

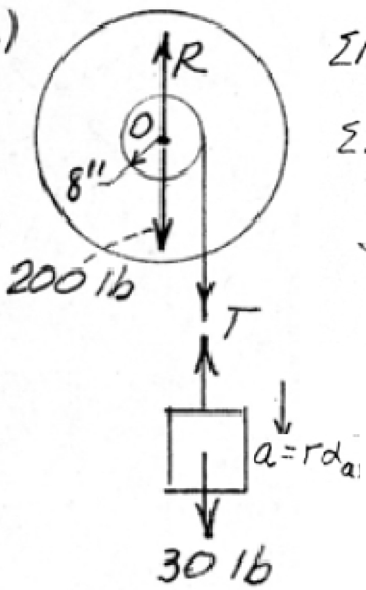
Example 1:

Each of the two drums and connected hubs of 8-in. radius weighs 200 lb and has a radius of gyration about its center of 15 in. Calculate the angular acceleration of each drum. Friction in each bearing is negligible.



Solution:

(a)



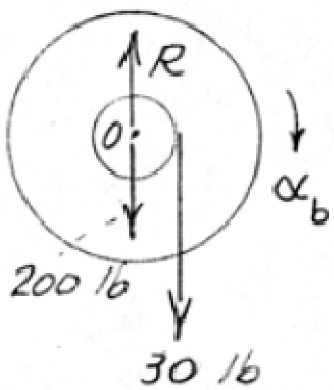
$\Sigma M_O = I_O\alpha$; $T\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2\alpha_a$

$\Sigma F = ma$; $30 - T = \frac{30}{32.2}\left(\frac{8}{12}\alpha_a\right)$

Solve simultaneously & get

$T = 28.77$ lb $\alpha_a = 1.976\ rad/sec^2$

(b)



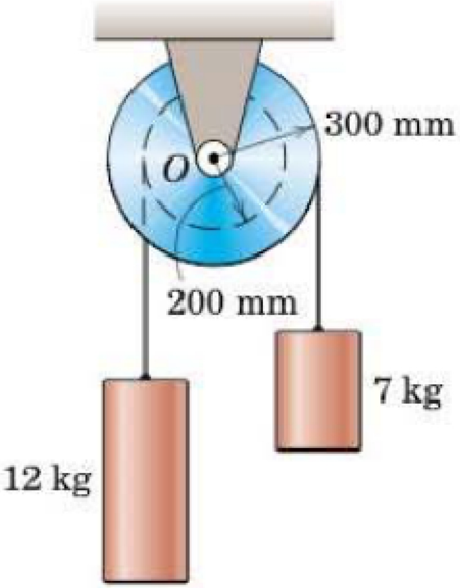
$\Sigma M_O = I_O\alpha$; $30\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2\alpha_b$

$\alpha_b = 2.06\ rad/sec^2$

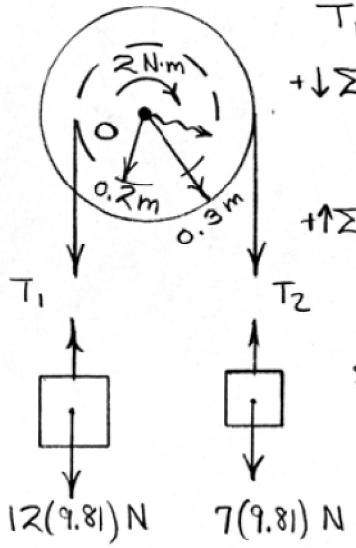
6.4 Fixed Axis Rotation

Example 2:

If the frictional moment at the pivot O is 2 N·m, determine the angular acceleration of the grooved drum, which has a mass of 8 kg and a radius of gyration $k_O = 225$ mm.



Solution:



$\circlearrowright + \Sigma M_O = I_O \alpha$ for drum:

$$T_1(0.2) - T_2(0.3) - 2 = 8(0.225)^2\alpha \quad (1)$$

$+\downarrow \Sigma F = ma$ for 12-kg cylinder:

$$12(9.81) - T_1 = 12(0.2\alpha) \quad (2)$$

$+\uparrow \Sigma F = ma$ for 7-kg cylinder:

$$T_2 - 7(9.81) = 7(0.3\alpha) \quad (3)$$

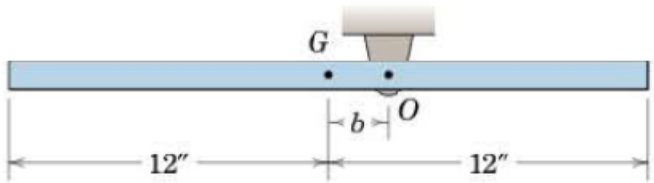
Solution of Eqs. (1)-(3):

$$\begin{cases} T_1 = 116.2 \text{ N} \\ T_2 = 70.0 \text{ N} \\ \alpha = 0.622 \text{ rad/s}^2 \end{cases}$$

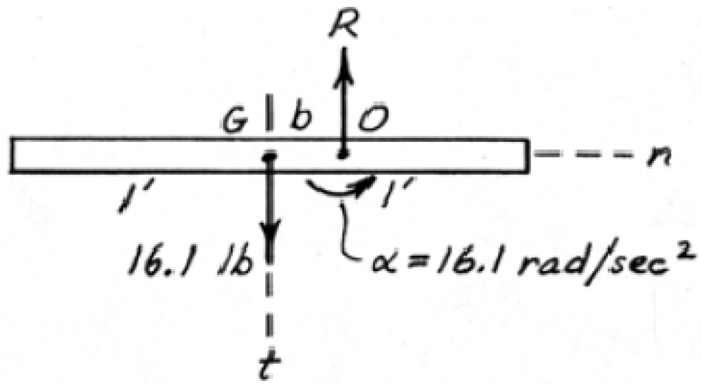
6.4 Fixed Axis Rotation

Example 3:

The uniform 16.1-lb slender bar is hinged about a horizontal axis through O and released from rest in the horizontal position. Determine the distance b from the mass center to O which will result in an initial angular acceleration of 16.1 rad/sec^2, and find the force R on the bar at O just after release.



Solution:



$$I_O = \frac{1}{12} mL^2 + mb^2 = \frac{16.1}{32.2}\left(\frac{2^2}{12} + b^2\right) = \frac{1}{6} + \frac{b^2}{2} \text{ lb-ft-sec}^2$$

$$\Sigma M_O = I_O \alpha: \quad 16.1 b = \left(\frac{1}{6} + \frac{b^2}{2}\right) 16.1, \quad 3b^2 - 6b + 1 = 0$$

$$b = 1 \pm \sqrt{24}/6, \quad b = 0.1835 \text{ ft } (1.817 \text{ ft}),$$

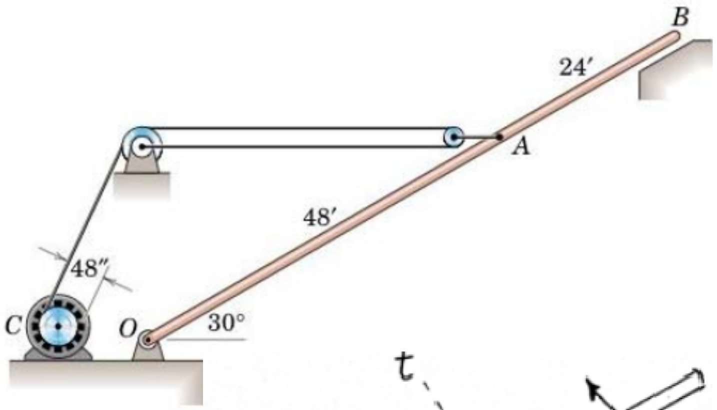
$$\underline{b = 2.20 \text{ in.}}$$

$$\Sigma F_t = m\bar{r}\alpha: \quad 16.1 - R = \frac{16.1}{32.2} 0.1835(16.1), \quad \underline{R = 14.62 \text{ lb}}$$

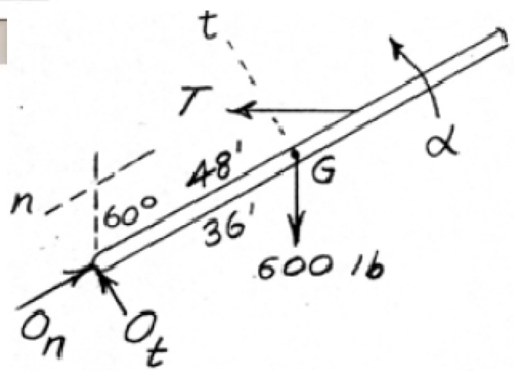
6.4 Fixed Axis Rotation

Example 4:

The uniform 72-ft mast weighs 600 lb and is hinged at its lower end to a fixed support at *O*. If the winch *C* develops a starting torque of 900 lb-ft, calculate the total force supported by the pin at *O* as the mast begins to lift off its support at *B*. Also find the corresponding angular acceleration α of the mast. The cable at *A* is horizontal, and the mass of the pulleys and winch is negligible.



Solution:



$M = \frac{T}{2} r$, $T = \frac{2(900)}{2} = 900$ lb

$\Sigma M_O = I_O \alpha$;

$900(48\cos 60°) - 600(36 \sin 60°) = \frac{1}{3}\frac{600}{32.2} 72^2 \alpha$

$\alpha = 0.0899$ rad/sec^2

$\Sigma F_t = m\bar{a}_t$; $O_t + 900\cos 60° - 600 \sin 60° = \frac{600}{32.2}(36)(0.0899)$

$O_t = 129.9$ lb

$\Sigma F_n = m\bar{a}_n = 0$; $900 \sin 60° + 600 \cos 60° - O_n = 0$

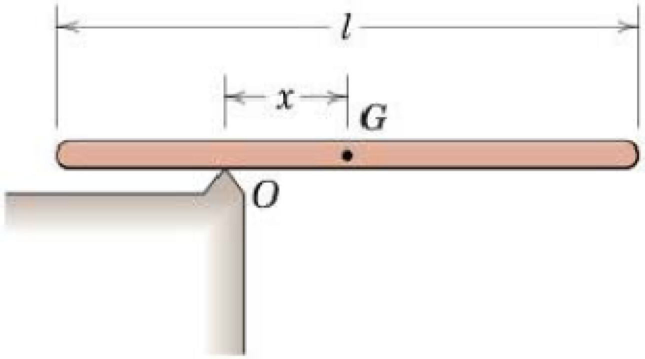
$O_n = 1079.4$ lb

$O = \sqrt{129.9^2 + 1079.4^2} = 1087$ lb

6.4 Fixed Axis Rotation

Example 5:

The uniform slender bar is released from rest in the horizontal position shown. Determine the value of x for which the angular acceleration is a maximum, and determine the corresponding angular acceleration α.



Solution:

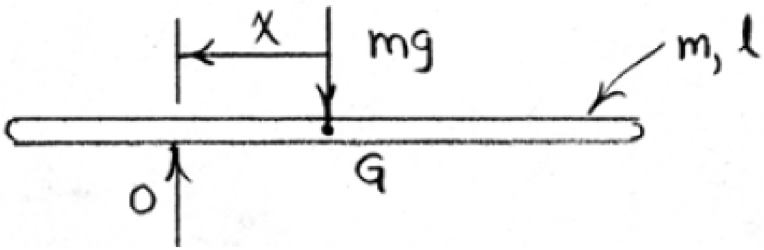
$$I_O = I_G + mx^2 = \frac{1}{12}ml^2 + mx^2 = m\left(\frac{l^2}{12} + x^2\right)$$

$$\circlearrowleft \Sigma M_O = I_O \alpha \; : \quad mgx = m\left(\frac{l^2}{12} + x^2\right)\alpha$$

$$\alpha = \frac{gx}{\frac{1}{12}l^2 + x^2}$$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{1}{12}l^2 + x^2\right)g - gx(2x)}{\left(\frac{1}{12}l^2 + x^2\right)^2} = 0 \Rightarrow \underline{x = \frac{l}{2\sqrt{3}}}$$

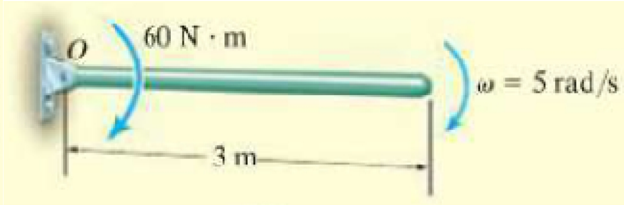
$$\alpha = \frac{g\frac{l}{\sqrt{12}}}{\frac{1}{12}l^2 + \frac{1}{12}l^2} = \underline{\sqrt{3}\,\frac{g}{l}}$$



6.4 Fixed Axis Rotation

Example: H.W1

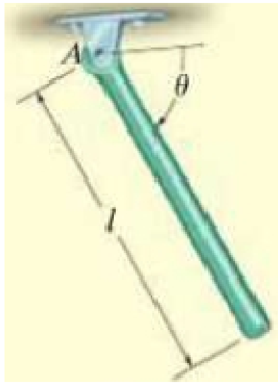
At the instant shown in Fig. 17–16*a*, the 20-kg slender rod has an angular velocity of $\omega = 5$ rad/s. Determine the angular acceleration and the horizontal and vertical components of reaction of the pin on the rod at this instant.



Ans. $O_n = 750$ N $\quad O_t = 19.05$ N $\quad \alpha = 5.90$ rad/s^2

Example: H.W2

The slender rod shown in Fig. 17–18*a* has a mass m and length l and is released from rest when $\theta = 0°$. Determine the horizontal and vertical components of force which the pin at A exerts on the rod at the instant $\theta = 90°$.

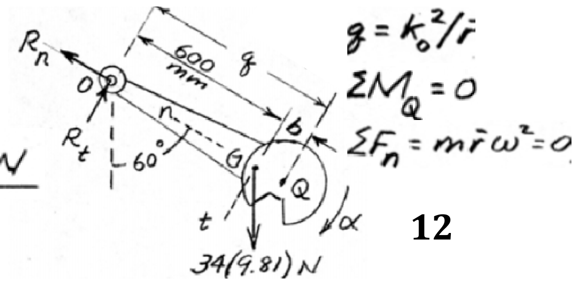


Ans. $\alpha = 0 \quad A_t = 0 \; A_n = 2.5mg$

Example: H.W3:

Solve Problem 6.54, Page 448. "Mechanical Engineering Dynamics", 6[th] Edition, Meriam. *Ans.:*

$R = \sqrt{(166.8)^2 + (18.35)^2} = \underline{167.8\ N}$



6/5 General Plane Motion

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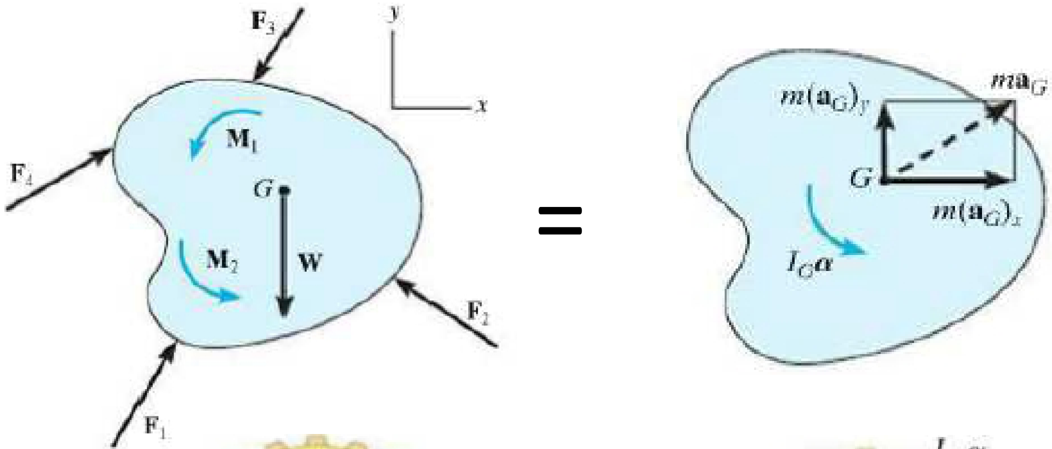
M.Sc. Mechanical Engineering

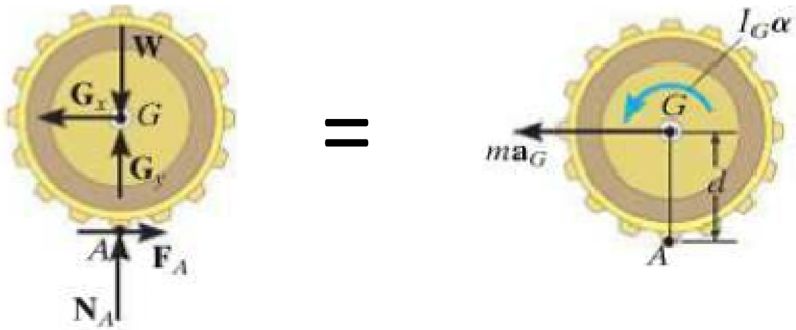
6.5 General Plane Motion

- **General Plane Motion:**

The rigid body is subjected to general plane motion caused by external applied force and couple-moment system. The three equations of motion may be written as:

$$\Sigma F_x = m(a_G)_x$$
$$\Sigma F_y = m(a_G)_y$$
$$\Sigma M_G = I_G\alpha$$

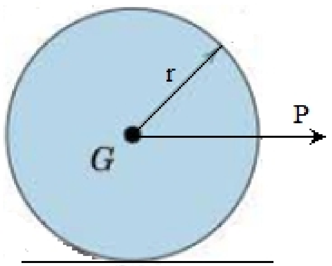


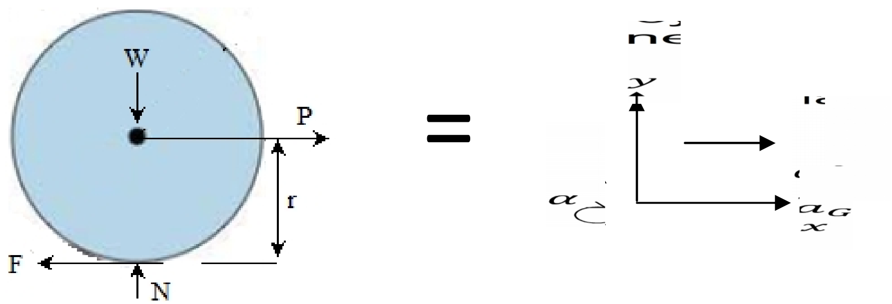


6.5 General Plane Motion

■ Friction Rolling Problems

There is a class of planer kinematics problems which deserves special motion. These problems involve wheels, cylinder, disk, or bodies of similar shape, which roll on a rough plane surface.





- Rolls without slipping

$\xrightarrow{+} [\sum F_x = m(a_G)_x \quad ; \quad P - F = ma_G]$ ... (1)

$+\uparrow [\sum F_y = m(a_G)_y \quad ; \quad N - mg = 0]$ ... (2)

$\curvearrowright [\sum M_G = I_G\alpha \quad ; \quad F \cdot r = I_G\alpha]$ ... (3)

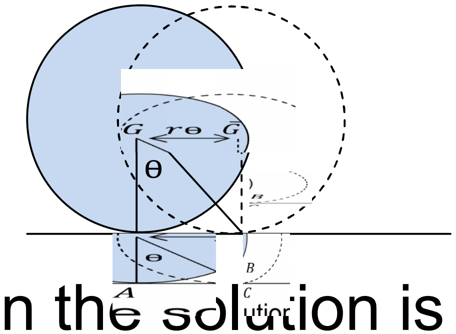
Three equations with four unknown variables: (F, a_G, N, α) we need to another equation.

6.5 General Plane Motion

- **No slipping:**

If the friction force F is greater enough to allow the disk to roll without slipping

$$[a_G = \alpha r] \quad \dots \quad (4)$$



$$s_G = r\theta$$
$$v_G = r\omega$$
$$a_G = r\alpha$$

Only for cylinder rolling without slipping.

When the solution is obtained, the assumption of no slipping must be checked.

Recall that no slipping occurs provided must be reworked, then the disk slips as it rolls.

إذا تحققت المعادلة $[F \leq \mu_s \cdot N]$ فالفرضية صحيحة وإلا فالفرضية خاطئة.

6.5 General Plane Motion

- **Slipping:**

If the case of slipping, a_G and α , are independent of one other so that eq.(1) doesn't apply. Instead, the magnitude of the fractional force is related to the magnitude of the natural force using μ_k is :

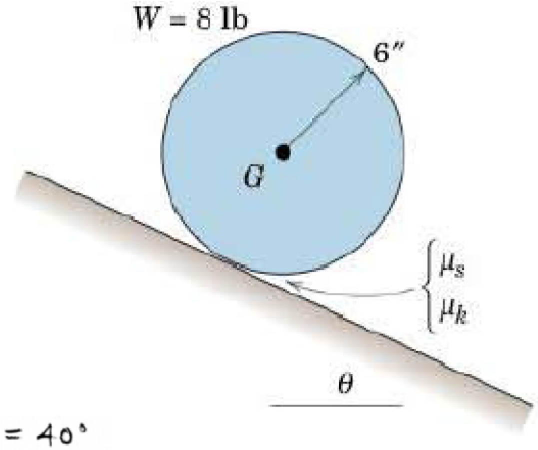
$$[F = \mu_k \cdot N \qquad \dots \qquad (5)]$$

- μ_s معامل الاحتكاك السكوني : يستخدم في حالة **No Slipping** .
- μ_k معامل الاحتكاك الكينماتيكي: يستخدم في حالة **Slipping** .

6.5 General Plane Motion

Example 1:

The solid homogeneous cylinder is released from rest on the ramp. If $\theta = 40°$, $\mu_s = 0.30$, and $\mu_k = 0.20$, determine the acceleration of the mass center G and the friction force exerted by the ramp on the cylinder.



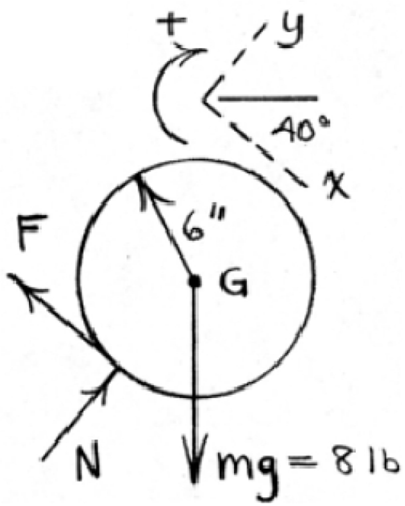
Solution: $mg = 8 \text{ lb}$, $\bar{I} = \frac{1}{2}mr^2$, $\mu_s = 0.3$, $\mu_k = 0.20$, $\theta = 40°$

$\Sigma F_x = m\bar{a}_x$: $-F + 8\sin 40° = \frac{8}{32.2}a$ (1)

$\Sigma F_y = 0$: $N - 8\cos 40° = 0$ (2)

$\Sigma M_G = \bar{I}\alpha$: $F\left(\frac{6}{12}\right) = \frac{1}{2}\frac{8}{32.2}\left(\frac{6}{12}\right)^2\alpha$ (3)

Assume rolling with no slip : $a = \frac{6}{12}\alpha$ (4)



Solution of (1) - (4) : $F = 1.714 \text{ lb}$ $a = 13.80 \frac{\text{ft}}{\text{sec}^2}$

$N = 6.13 \text{ lb}$ $\alpha = 27.6 \frac{\text{rad}}{\text{sec}^2}$

$F_{max} = \mu_s N = 0.3\,(6.13) = 1.839 \text{ lb} > F$

Assumption valid.

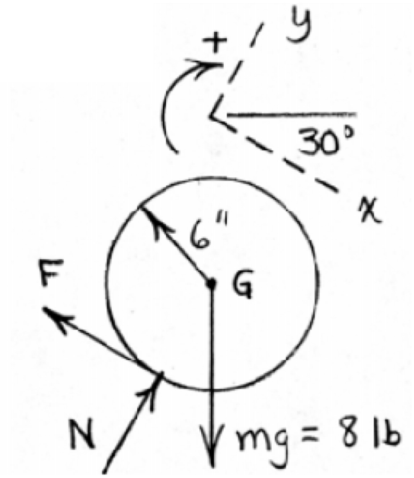
6.5 General Plane Motion

Example 2:

Repeate Example 1, except let $\theta = 30°$, $\mu_s = 0.15$, and $\mu_k = 0.10$.

Solution:

$$\begin{cases} mg = 8 \text{ lb}, \quad \bar{I} = \frac{1}{2} m r^2 \\ \mu_s = 0.15, \quad \mu_k = 0.10 \\ \theta = 30° \end{cases}$$



$$\Sigma F_x = m\bar{a}_x : \quad -F + 8 \sin 30° = \frac{8}{32.2} a \qquad (1)$$

$$\Sigma F_y = 0 : \quad N - 8 \cos 30° = 0 \qquad (2)$$

$$\Sigma M_G = \bar{I}\alpha : \quad F\left(\frac{6}{12}\right) = \frac{1}{2} \frac{8}{32.2} \left(\frac{6}{12}\right)^2 \alpha \qquad (3)$$

Assume rolling with no slip: $a = \frac{6}{12}\alpha$ (4)

Solution of (1) - (4): $F = 1.333$ lb, $a = 10.73 \frac{ft}{sec^2}$

$N = 6.93$ lb, $\alpha = 21.5 \frac{rad}{sec^2}$

$F_{max} = \mu_s N = 0.15(6.93) = 1.039 \text{ lb} < F \Rightarrow$ slips

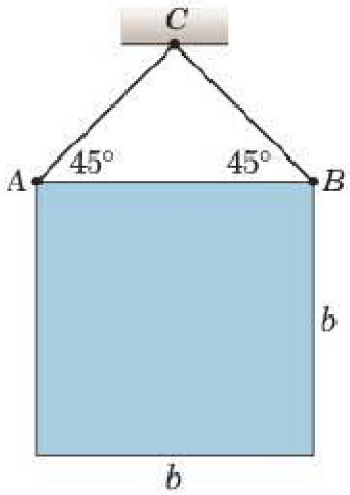
$F = \mu_k N = 0.10(6.93) = \underline{0.693 \text{ lb}}$

From Eqs. (1) & (3): $\underline{a = 13.31 \text{ ft/sec}^2}$, $\alpha = 11.15 \frac{rad}{sec^2}$

6.5 General Plane Motion

Example 3:

The uniform 12-kg square panel is suspended from point C by the two wires at A and B. If the wire at B suddenly breaks, calculate the tension T in the wire at A an instant after the break occurs.



Solution:

$$\Sigma M_A = \bar{I}\alpha + m\bar{a}d$$

$$mg\frac{b}{2} = \frac{1}{6}mb^2\alpha + m\frac{b}{\sqrt{2}}\alpha\frac{b}{\sqrt{2}}$$

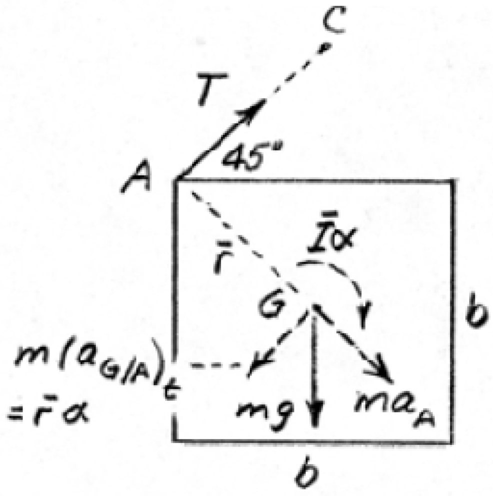
$$\alpha = \frac{3g}{4b}$$

$$\Sigma M_G = \bar{I}\alpha$$

$$T\frac{b}{\sqrt{2}} = \frac{1}{6}mb^2\left(\frac{3g}{4b}\right)$$

$$T = \frac{\sqrt{2}}{8}mg = \frac{\sqrt{2}}{8}(12)(9.81)$$

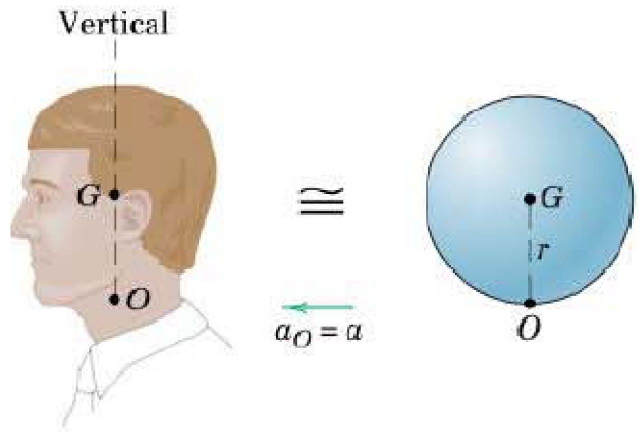
$$= \underline{20.8\ N}$$



6.5 General Plane Motion

Example 4:

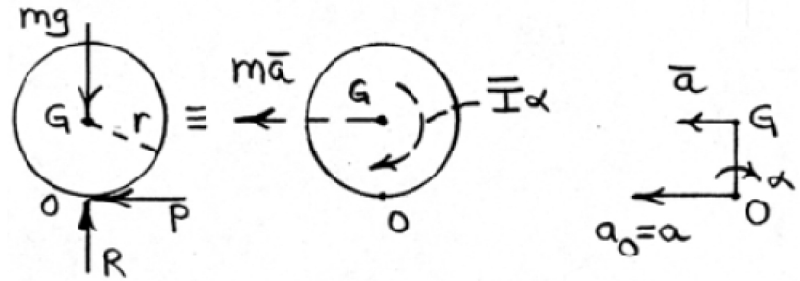
In an investigation of whiplash resulting from rear-end collisions, sudden rotation of the head is modeled by using a homogeneous solid sphere of mass m and radius r pivoted about a tangent axis (at the neck) to represent the head. If the axis at O is given a constant acceleration a with the head initially at rest, determine expressions for the initial angular acceleration α of the head and its angular velocity ω as a function of the angle θ of rotation. Assume that the neck is relaxed so that no moment is applied to the head at O.



6.5 General Plane Motion

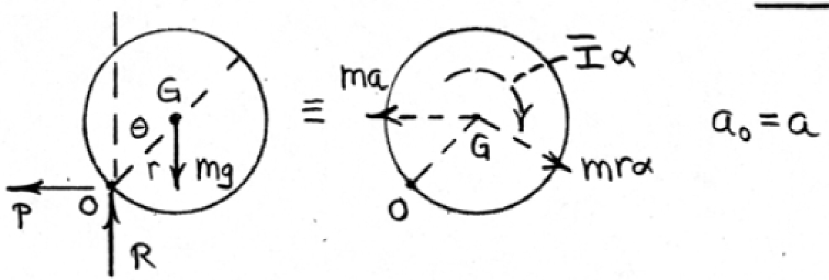
Solution:

$$\bar I = \frac{2}{5}mr^2$$



$$\Sigma M_O = \bar I\alpha - m\bar a r: \quad 0 = \frac{2}{5}mr^2\alpha - m\bar a r, \quad \bar a = \frac{2}{5}r\alpha$$

$$a_G = a_O + a_{G/O}: \quad \bar a = a - r\alpha = \frac{2}{5}r\alpha \Rightarrow \alpha = \frac{5}{7}\frac{a}{r}$$



$$\Sigma M_O = \bar I\alpha + \Sigma m\bar a d: \quad mgr\sin\theta = \frac{2}{5}mr^2\alpha + mr^2\alpha - mar\cos\theta$$

$$\alpha = \frac{5}{7r}\left(g\sin\theta + a\cos\theta\right)$$

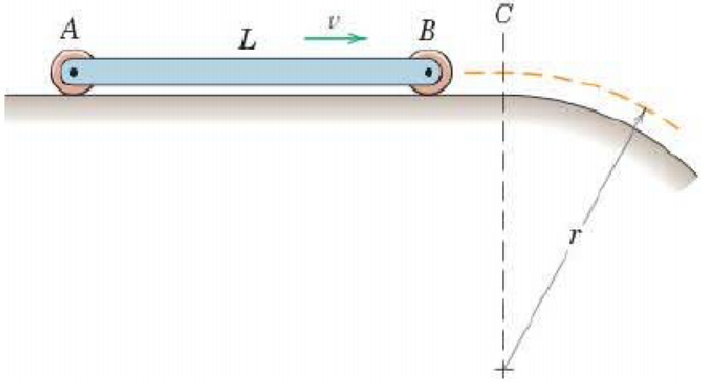
$$\omega\, d\omega = \alpha\, d\theta: \quad \int_0^\omega \omega\, d\omega = \frac{5}{7r}\int_0^\theta \left(g\sin\theta + a\cos\theta\right)d\theta$$

$$\omega = \sqrt{\frac{10}{7r}}\sqrt{g(1-\cos\theta) + a\sin\theta}$$

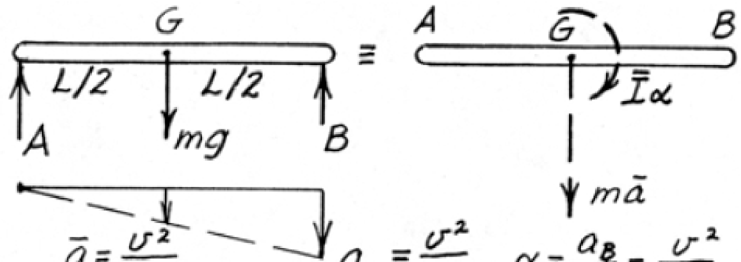
6.5 General Plane Motion

Example 4:

The uniform bar of mass m and length L is moving horizontally with a velocity v on its light end rollers. Determine the force under roller B an instant after it passes point C and prior to mechanical interference with the path. At what velocity v will the force under roller B reach zero?



Solution:



$a_A = 0$ $\qquad \bar{a} = \dfrac{v^2}{2r}$ $\qquad a_B = \dfrac{v^2}{r}$ $\qquad \alpha = \dfrac{a_B}{L} = \dfrac{v^2}{Lr}$

$\omega_{AB} = 0$

$$\Sigma M_A = m\bar{a}\frac{L}{2} + \bar{I}\alpha : \quad mg\frac{L}{2} - BL = m\frac{v^2}{2r}\frac{L}{2} + \frac{1}{12}mL^2\frac{v^2}{Lr}$$

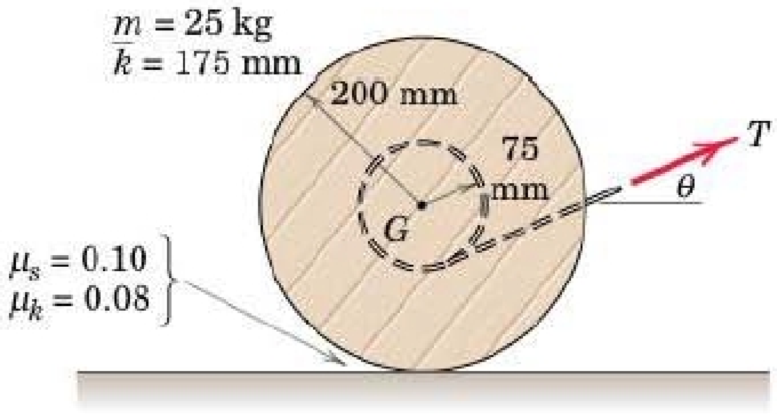
$$B = m\left(\frac{g}{2} - \frac{v^2}{3r}\right)$$

$$B = 0 \text{ if } \frac{g}{2} - \frac{v^2}{3r} = 0, \quad v = \sqrt{3gr/2}$$

6.5 General Plane Motion

H.W:

6/88 The circular disk of 200-mm radius has a mass of 25 kg with centroidal radius of gyration $\bar{k}$ = 175 mm and has a concentric circular groove of 75-mm radius cut into it. A steady force T is applied at an angle θ to a cord wrapped around the groove as shown. If T = 30 N, θ = 0, μ_s = 0.10, and μ_k = 0.08, determine the angular acceleration α of the disk, the acceleration a of its mass center G, and the friction force F which the surface exerts on the disk.



Ans. α = -2.12 rad/s^2, a = 0.425 m/s^2, F = 19.38 N

6/89 Repeat Prob. 6/88, except let T = 50 N and θ = 30°.

Ans. α = 0.295 rad/s^2, a = 1.027 m/s^2, F = 17.62 N

6/90 Repeat Prob. 6/88, except let T = 30 N and θ = 70°.

Ans. α = 0.1121 rad/s^2, a = - 0.0224 m/s^2, F = 10.82 N