College of Engineering
Bio. Eng. Dept.
Subject: Numerical Analysis
Third Stage

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Lecture 3

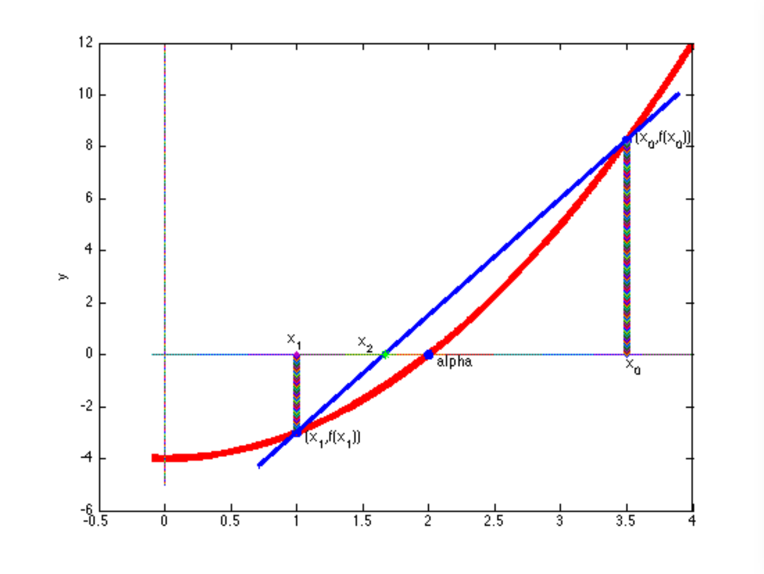
3. The Secant Method

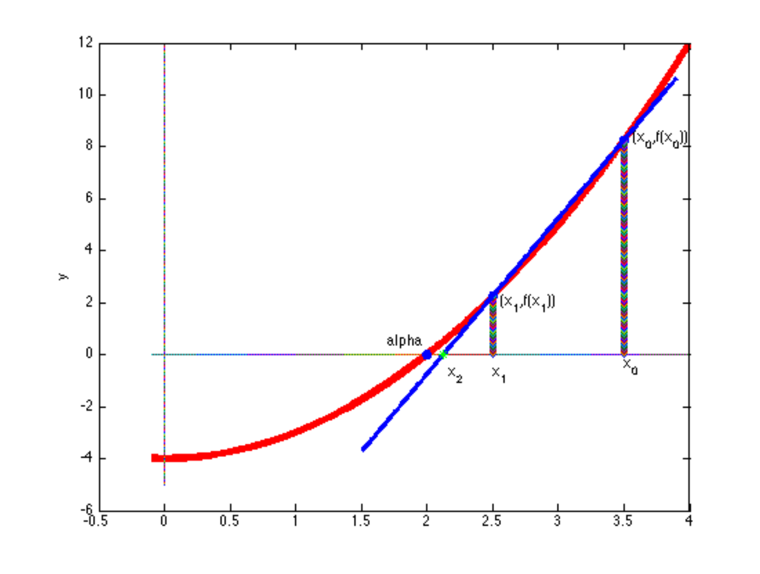
Introduction

The Newton method is based on approximating the graph of $y = f(x)$ with a tangent line and on then using a root of this straight line as an approximation to the root α of $f(x)$.
From this perspective,
other straight-line approximation to $y = f(x)$ would also lead to methods of approximating a root of $f(x)$.
One such straight-line approximation leads to the secant method . Assume that two initial guesses to α are known and denote them by x_0 and x_1.

✓ **They may occur**

- on opposite sides of α ,or
- on the same side of α





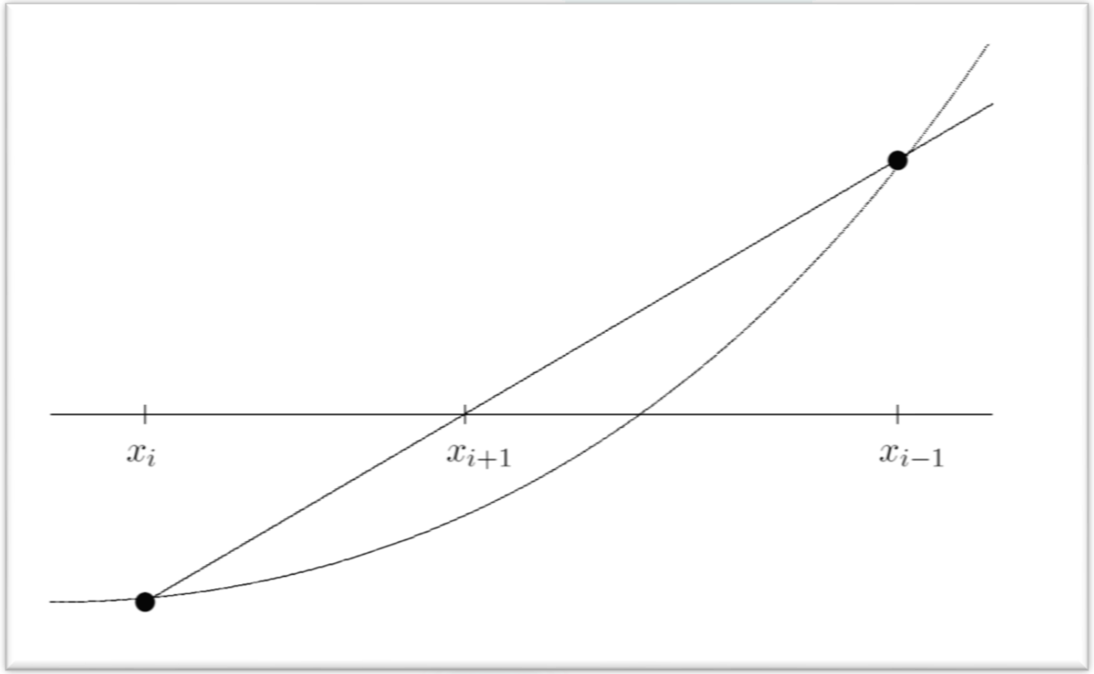
The secant method requires two initial approximations x_0 and x_1, preferably both reasonably close to the solution x^*. From x_0 and x_1 we can determine that the points (x_0 , y_0=$f(x_0)$) and (x_1 , y_1=$f(x_1)$) both lie on the graph of f. Connecting these points gives the (secant) line

$$y - y_1 = \frac{y_1 - y_0}{x_1 - x_0}(x - x_1)$$

Since we want $f(x) = 0$, we set y= 0, solve for x, and use that as our next approximation. Repeating this process gives us the iteration

$$x_{i+1} = x_i - \frac{x_i - x_{i-1}}{y_i - y_{i-1}} y_i$$

with , y_i=$f(x_i)$. See Figure for an illustration.

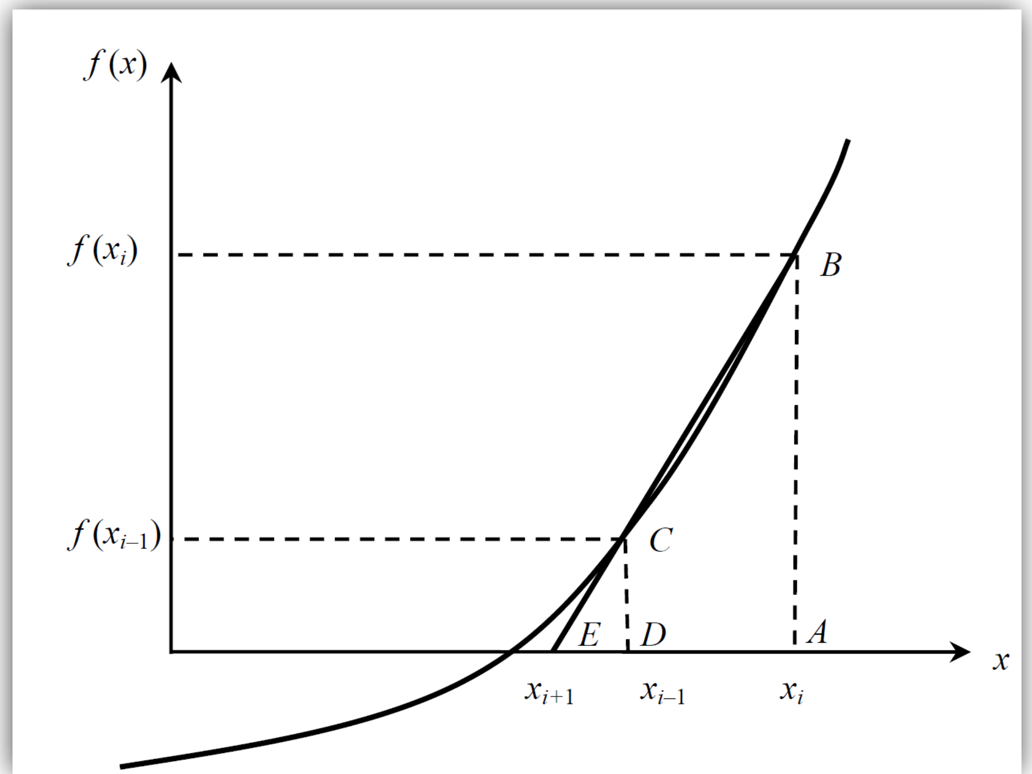


✓ The secant method can also be derived from geometry, as shown in Figure .
Taking two initial guesses, x_{i-1} and x_i , one draws a straight line between $f(x_i)$ and $f(x_{i-1})$ passing through the x -axis at x_{i+1}. ABE and DCE are similar triangles.

Hence

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$



On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Example:

Use the Secant method to estimate the root of $f(x) = e^{-x} - x$.
Start with initial estimates $x_{i-1} = 0$ and $x_i = 1$?

Solution:

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

- ✓ For i=0

$x_{-1} = 0$ ≫ $f(x_{-1}) = e^{-x_{-1}} - x_{-1} = e^0 - 0 = 1$

$x_0 = 1$ ≫ $f(x_0) = e^{-x_0} - x_0 = e^{-1} - 1 = -0.63212$

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})} = 1 - \frac{-0.63212(1-0)}{-0.63212 - 1} = 0.61270$$

✓ For i=1

$$x_0 = 1 \gg f(x_0) = e^{-x_0} - x_0 = e^{-1} - 1 = -0.63212$$

$$x_1 = 0.61270 \gg f(x_1) = e^{-x_1} - x_1 = e^{-0.61270} - 0.61270 = -0.07081$$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 0.56384$$

✓ For i=2

$$x_1 = 0.61270 \gg f(x_1) = e^{-x_1} - x_1 = e^{-0.61270} - 0.61270 = -0.07081$$

$$x_2 = 0.56384 \gg f(x_2) = e^{-x_2} - x_2 = e^{-0.56384} - 0.56384 = 0.00518$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} = 0.56717$$

✓ For i=3

$x_2 = 0.56384 \gg f(x_2) = e^{-x_2} - x_2 = e^{-0.56384} - 0.56384 = 0.00518$

$x_3 = 0.56717 \gg f(x_3) = e^{-x_3} - x_3 = e^{-0.56717} - 0.56717 = -0.00004$

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)} = 0.56714$$

i	x_{i-1}	$f(x_{i-1})$	x_i	$f(x_i)$	x_{i+1}	$\lvert\varepsilon_a\rvert = \left\lvert\frac{x_m^{new} - x_m^{old}}{x_m^{new}}\right\rvert * 100\%$
0	0	1	1	-0.63212	0.61270	100 %
1	1	-0.63212	0.61270	-0.07081	0.56384	77.3553 %
2	0.61270	-0.07081	0.56384	0.00518	0.56717	8.0276 %
3	0.56384	0.00518	0.56717	-0.00004	0.56714	0.5819 %
4	0.56717	-0.00004	0.56714	0.00001	0.56715	0.0035 %
5	0.56714	0.00001	0.56715	-0.00001	0.56715	0.0018 %

Homework:

Find the roots of the following equation $x^6 - x - 1 = 0$

Use the secant method ($\varepsilon_a = 0.00001$) ?