LECTURE 1:
ELASTIC BEHAVIOR & PROPERTIES OF MATERIALS

Learning Outcomes:

You should be able to:

- Describe how a material responds to an applied load
- Define elastic modulus
- List examples of applications in which elastic modulus is a design criterion.
- Calculate from a given selection the best material for elastic-limited design

Brief Introduction

- Materials are subjected to forces or loads.

  e.g. Al alloy in airplane wing; steel in automobile axle

- Mechanical behavior of materials reflects the relationship between its response or deformation to an applied load or force

- Factor to consider in design $\rightarrow$ nature of applied load, duration, environmental conditions

- Stress states $\rightarrow$ tensile, compressive, bending or shear $\rightarrow$ constant or fluctuate continuously with time

Mechanical Properties: Definition

- **Ductility** – ability to deform without fracture
- **Malleability** – ability to withstand deformation under compression without rupture
- **Toughness** – resistance to crack propagation
- **Hardness** – resistance to indentation
- **Stiffness** – resistance of bonds to deform
- **Strength** – resistance to permanent deformation

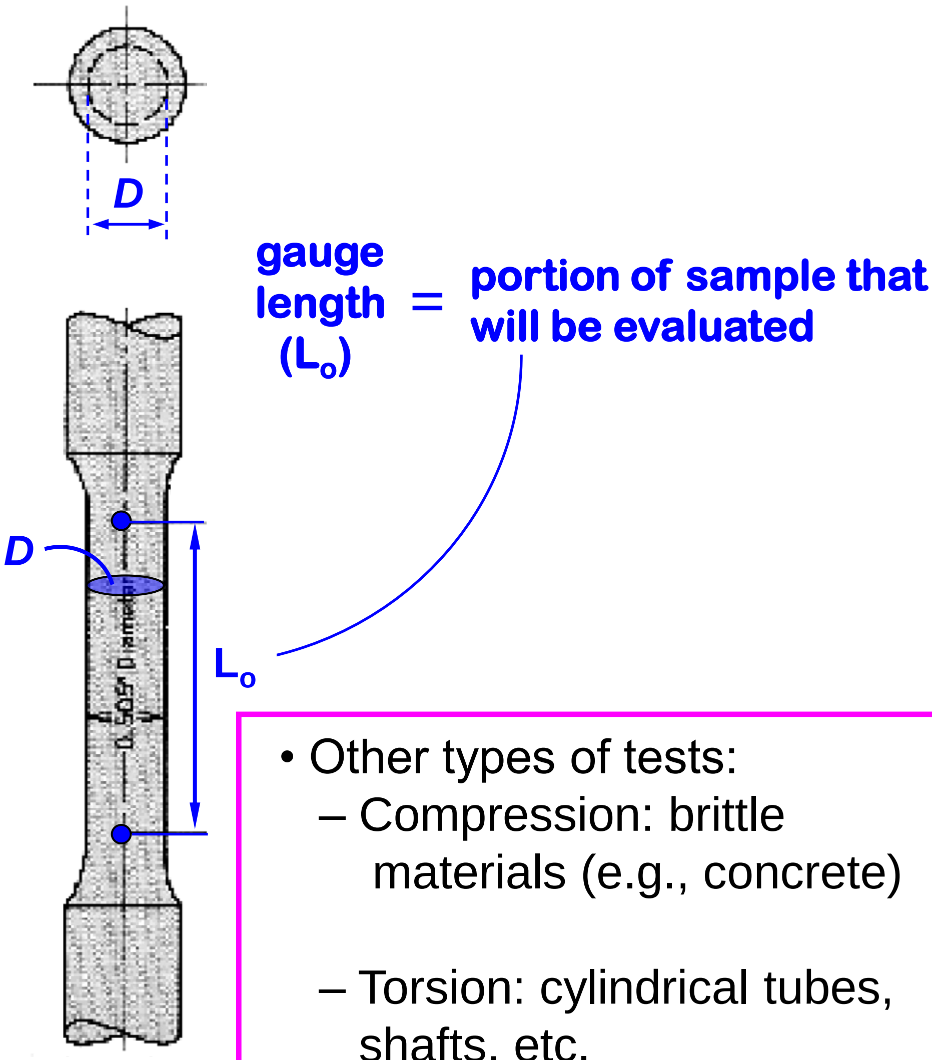
Definition of Stress and Strain

(a) Tensile stress produces an elongation and +ve linear strain

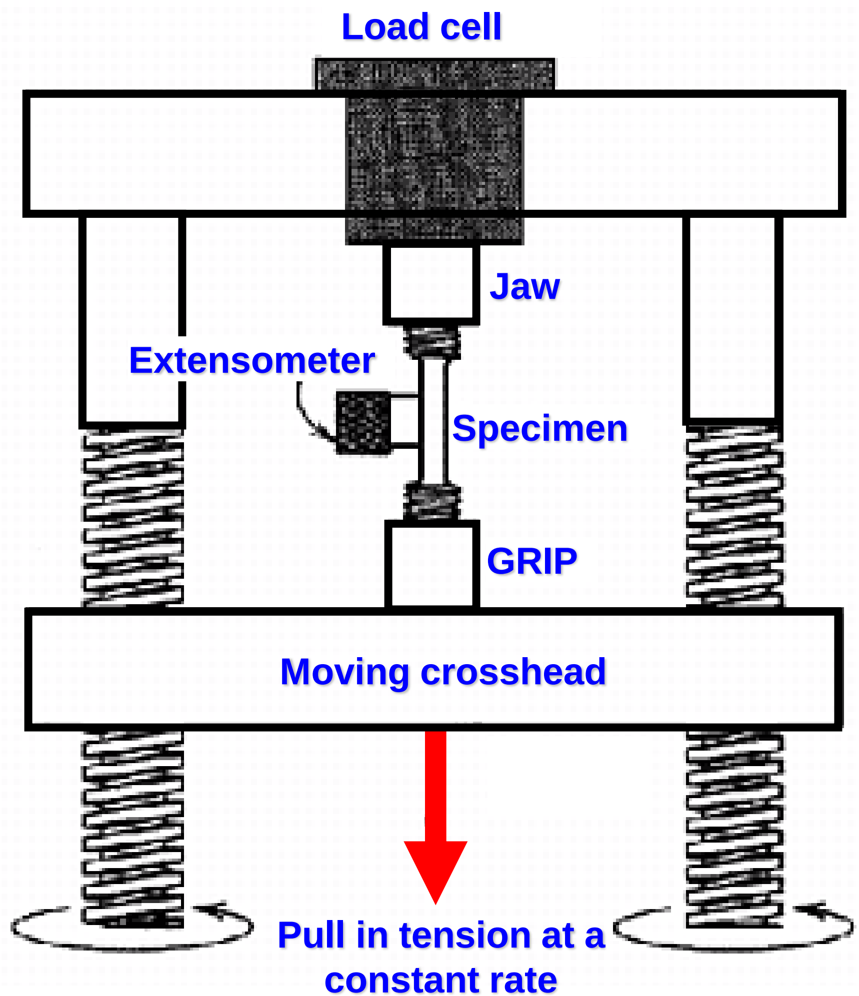
(b) Compressive strain produces contraction and –ve linear strain

Stress-Strain Testing

• Typical tensile specimen

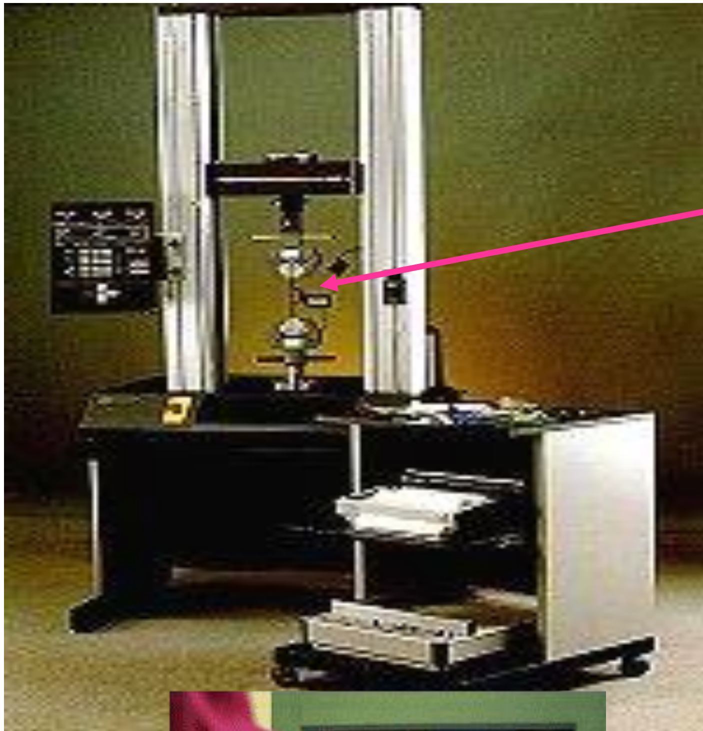


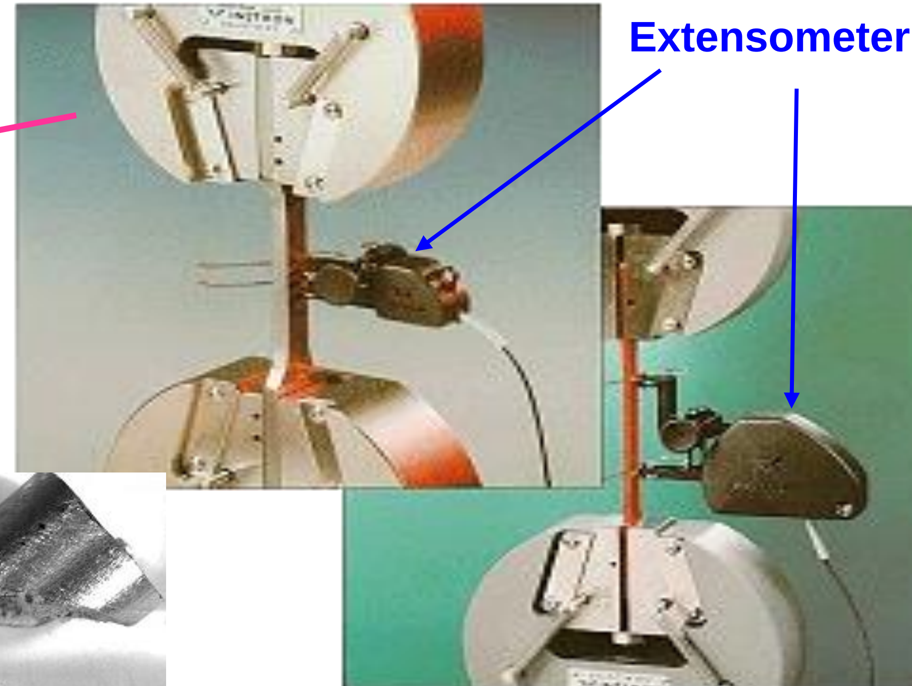
• Geometry of a typical tensile test machine

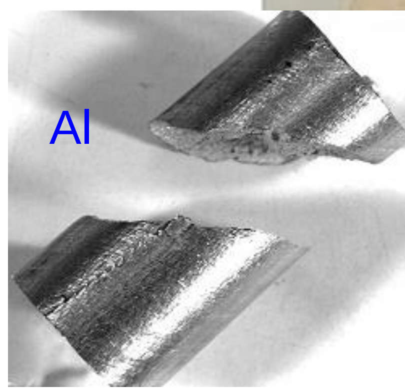


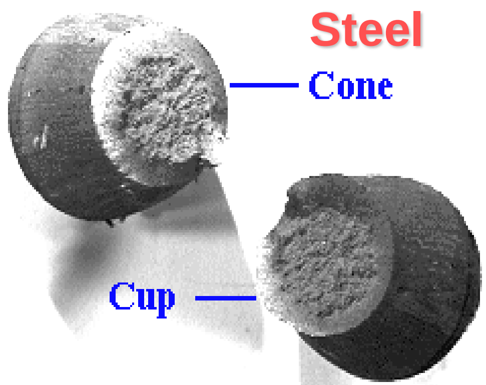
• Other types of tests:
  – Compression: brittle materials (e.g., concrete)
  – Torsion: cylindrical tubes, shafts, etc.

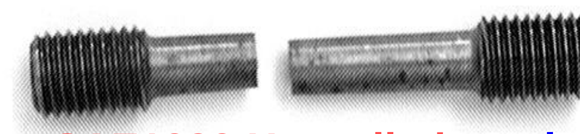
Universal Tensile Machine (UTM)









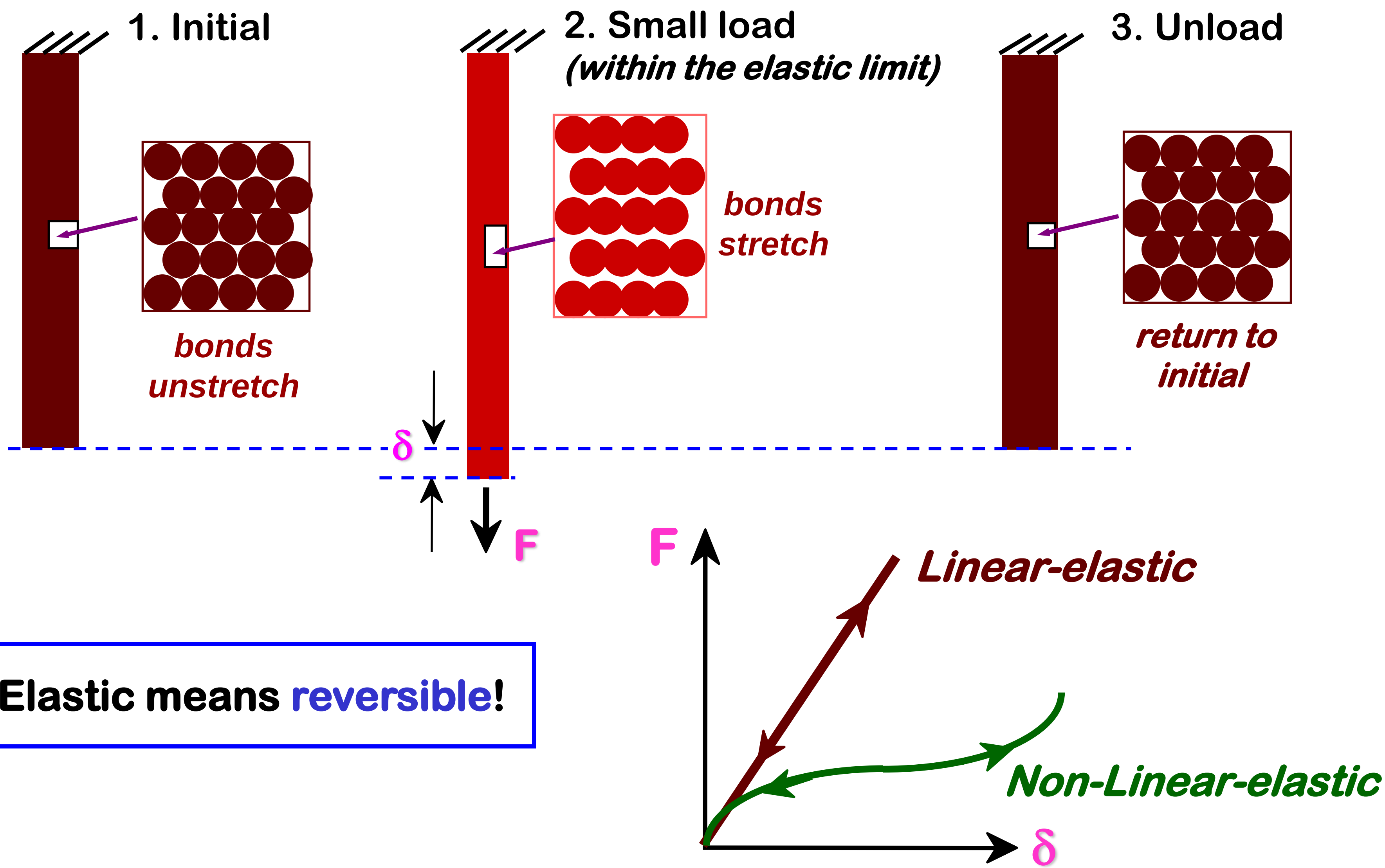


SAE1090 Hot rolled **steel**

SAE1095 Spheroidized **steel**

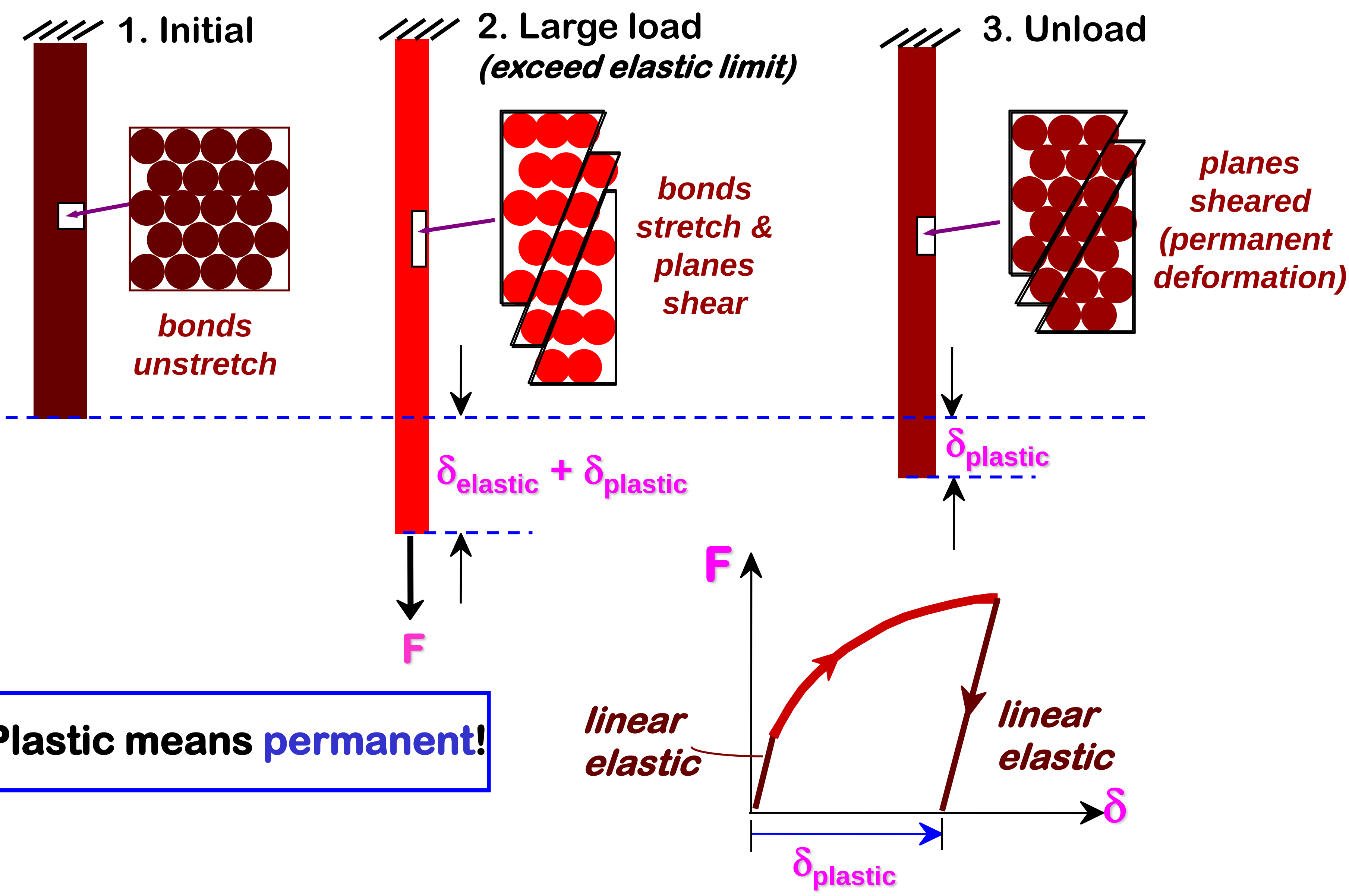
SAE1045 Hot rolled **steel**

Elastic Deformation



Elastic means reversible!

Plastic Deformation

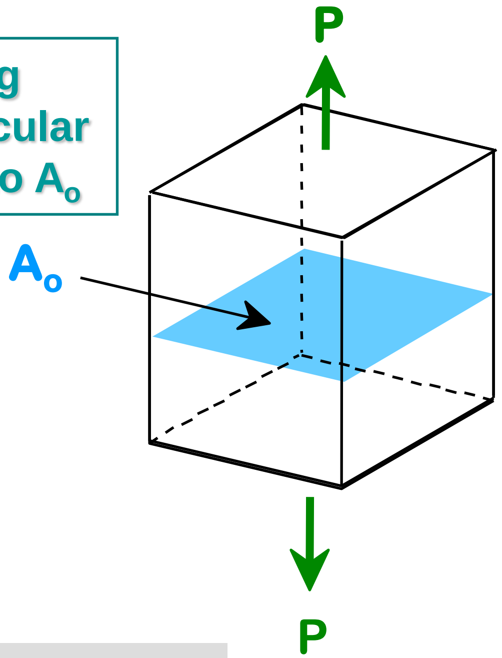


Plastic means permanent!

Nominal (or Engineering) Stress

- **Tensile** stress, σ:

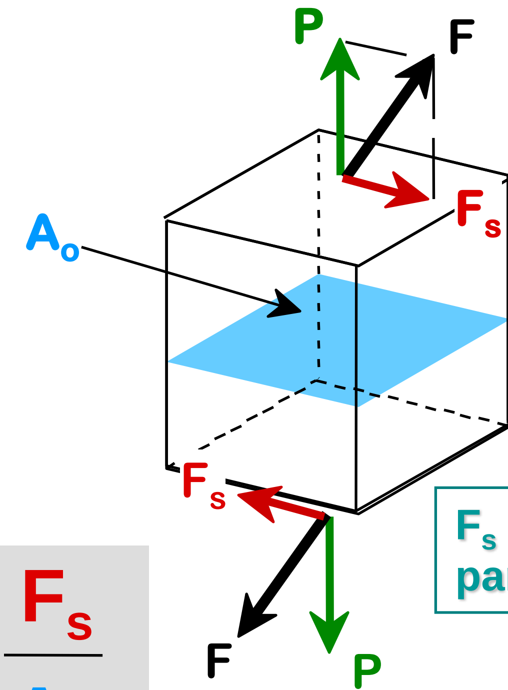
P is acting perpendicular (i.e. 90º) to A_o



$$= \frac{P}{A_o}$$

original area before loading

- **Shear** stress, τ:



F_s is acting parallel to A_o

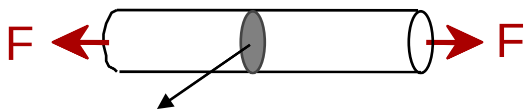
$$\tau = \frac{F_s}{A_o}$$

Stress has units of N/m² or Pa
Note:
1 N/m² = 1 Pascal (Pa)
1 N/mm² = 1 MPa

Stress and strain (rather than force and extension) are generally used when describing the deformation of solids because it is necessary to take into account the geometry and size of the sample.

Common States of Stress

- **Simple tension: Cable**

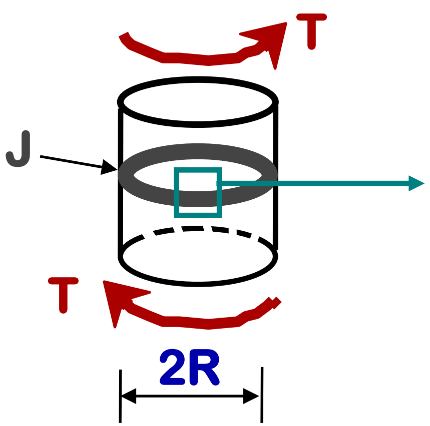


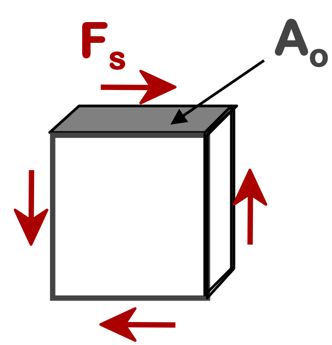
A_o = original CSA
(when unloaded)

$$\sigma = \frac{F}{A_o}$$

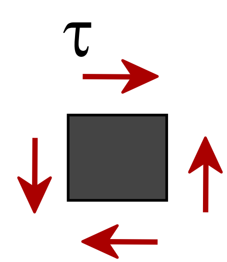
Tensile stress = +ve

- **Simple shear: Drive shaft**





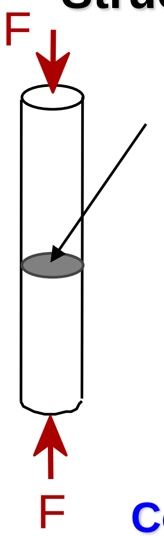
$$\tau = \frac{F_s}{A_o}$$



Note: $\tau = T.R/J$
where T = torque
R = radius of shaft
J = polar second moment of area

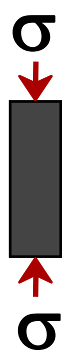
Other Common States of Stress

- **Simple compression: Structural member**

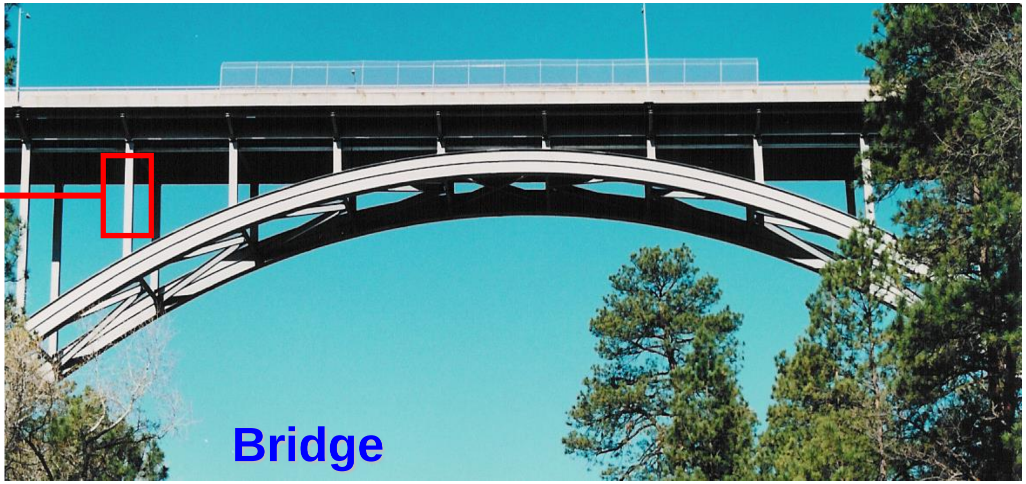


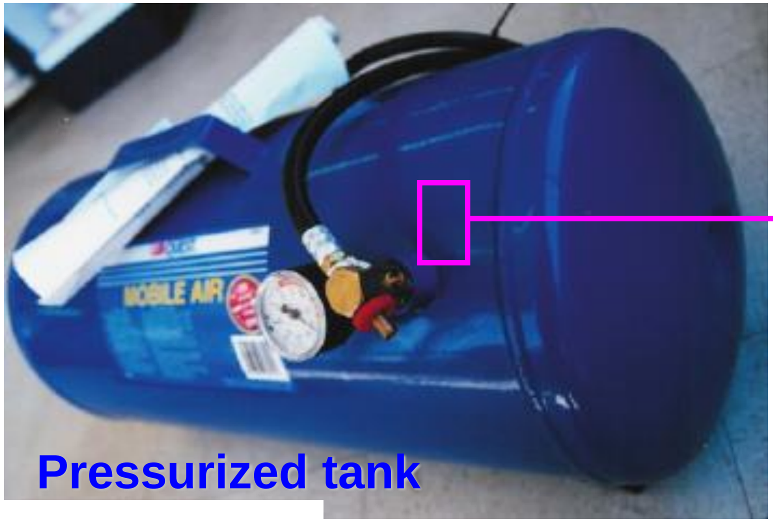
A_o = original CSA (when unloaded)

$$\sigma = \frac{F}{A_o}$$

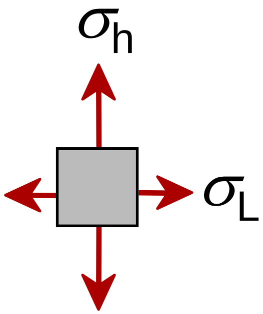


Compressive stress = – ve





- **Bi-axial tension: Pressure vessel**



where σ_h = hoop or circumferential stress

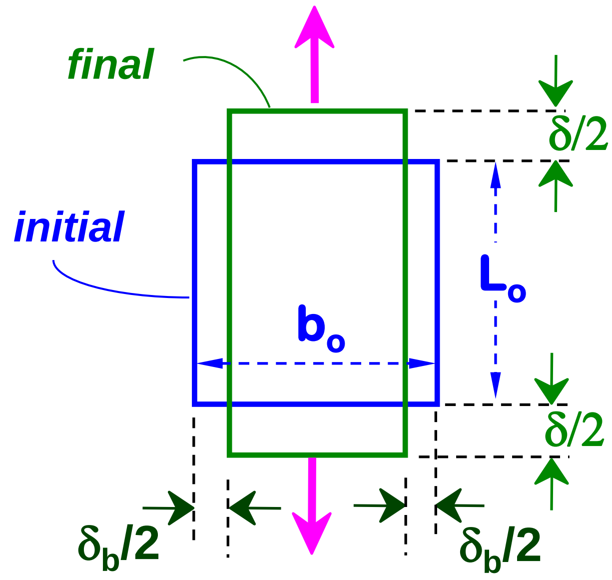
σ_L = longitudinal stress

Nominal (or Engineering) Strain

- **Tensile** strain or **Linear** strain:

$$\varepsilon = \frac{\delta}{L_o}$$

+ve strain coz' tensile nature

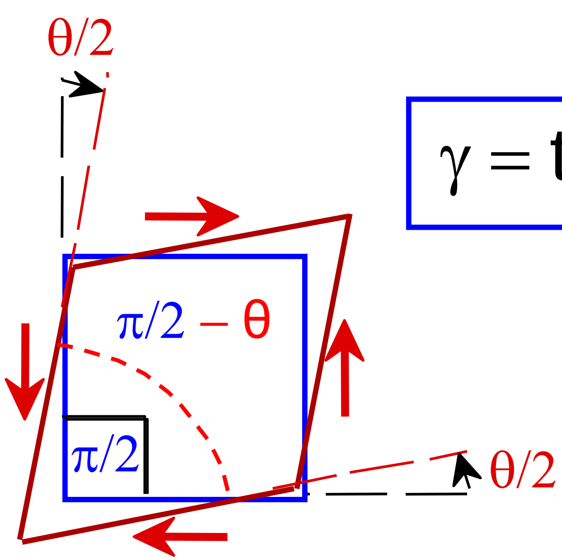


- **Lateral** strain:

$$\varepsilon_b = \frac{-\delta_b}{b_o}$$

–ve strain coz' compressive nature

- **Shear** strain:



$$\gamma = \tan\theta$$

Where θ = angle of shear in **radians**

Strain is always DIMENSIONLESS & often expressed in terms of percentage

e.g. $\varepsilon = 0.03$ or 3%

Elastic Properties

- Hooke's Law: When the material is **stressed within the elastic limit**; the stress varies in proportional to the applied strain and can be related through:

$$\sigma = E\,\varepsilon$$

Where:
E is the matrix stiffness known as the modulus of elasticity or Young's modulus ; ε is the linear strain

$$\text{since } \sigma = \frac{F}{A_o} \quad \& \quad \varepsilon = \frac{\delta}{L_o} \quad ; \qquad \therefore \frac{F}{A_o} = E \cdot \frac{\delta}{L_o} \quad \rightarrow \quad \delta = \frac{FL_o}{A_oE}$$

- Elastic Shear Modulus, G:

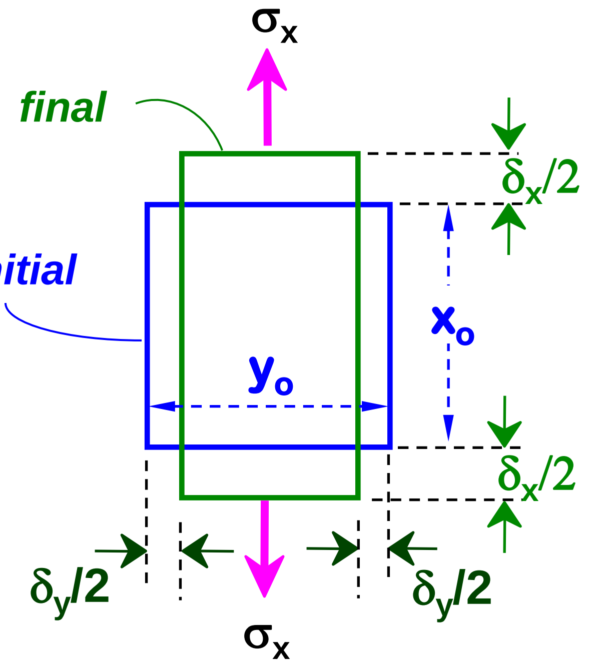
$$\tau = G\,\gamma$$

Units:
E = N/m^2 or GPa
G = N/m^2 or GPa

Elastic Properties: Poisson's Ratio

- **Consider a material loaded within the elastic limit, say in the x-direction as shown:**

- **When we stretch the material in the x-direction, what happen in the radial (y- and z-) directions?**
  elongates in the x-dir. but
  gets thinner (compressed) in the y- and z-directions!
- **So we have axial or linear strain in the x-dir. which is taken as positive strain (i.e. tensile nature) and we have lateral strain in the lateral direction which is taken as negative strain because compression!**



- **Thus, the ratio between the two strains give Poisson's Ratio (μ):**

$$\mu = -\frac{\text{lateral strain}}{\text{linear strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

where $\varepsilon_x = \dfrac{\delta_x}{x_o}$ & $\varepsilon_y = \dfrac{-\delta_y}{y_o}$

μ = dimension less, positive & is less than 1
e.g. metals: μ ~ 0.33
ceramics: μ ~ 0.25
polymers: μ ~ 0.40

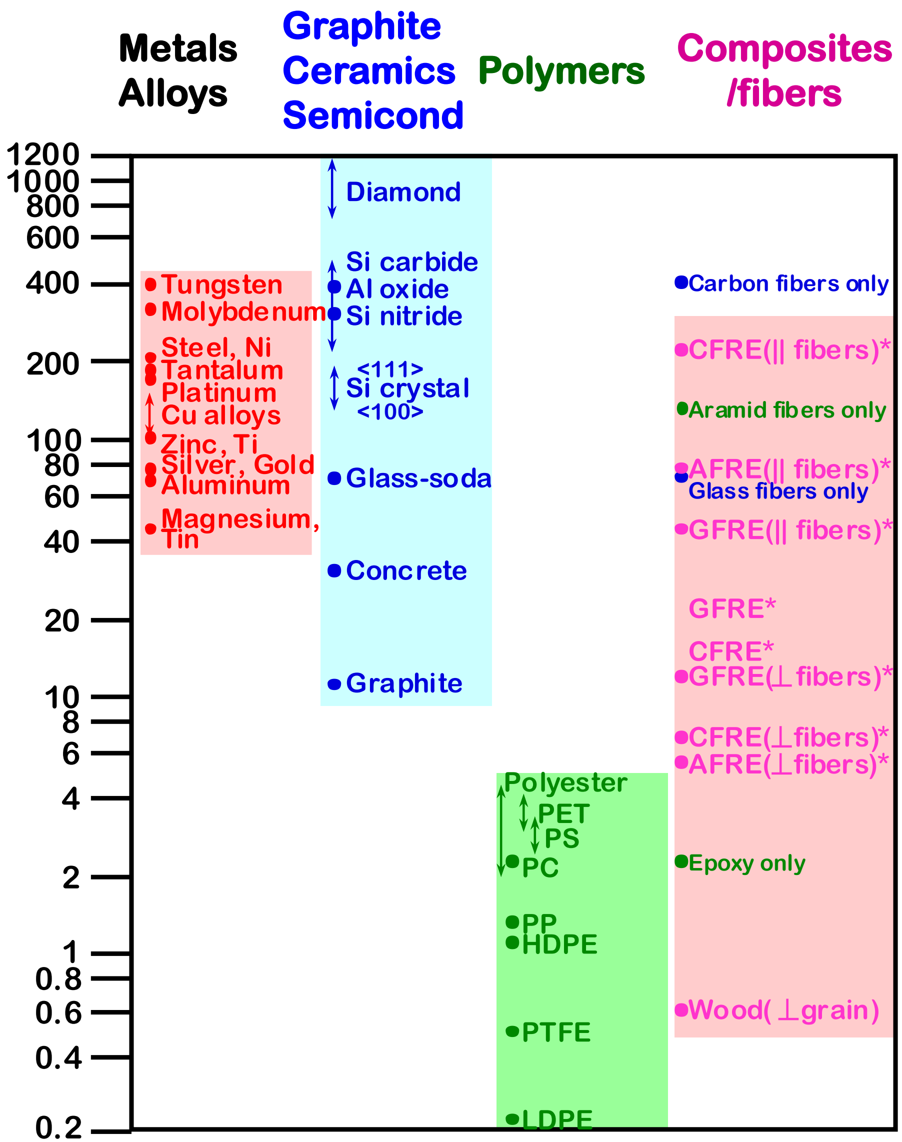
- **For Isotropic material: E = 2G(1 + μ);**
  For most metals G ~ 0.4E

- **However, many materials, especially crystals, are NOT Isotropic; so their properties depend on crystals orientation**

Young's Moduli: Comparison

E(GPa)

1 GPa = 10^9 Pa



$E_{ceramics}$ > E_{metals} >> $E_{polymers}$

Based on data in Table B2, *Callister 6e*.

Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Stiffness and Modulus

- ***Stiffness*** is a measure of an ***object's*** resistance to elastic deformation or strain
  - – depends on material type
  - – depends on thickness and shape of object

$$E \propto \left(\frac{dF}{dr}\right)_{r_o}$$

- ***Modulus*** is a measure of a ***material's*** resistance to elastic deformation or strain
- or modulus is a measure of resistance to separation of adjacent atoms, i.e., inter-atomic bonding forces
  - – independent on shape

$\uparrow$ E $\propto$ Stiffness $\uparrow$ $\propto$ Elastic strain $\downarrow$

Definition of True Stress and True Strain

- True stress and True strain are calculated based on **incremental change** in sample geometry.

- Consider the case of axial loading:
  - True Stress:
  - True Strain:

$$\sigma_t = \frac{F}{A_t}$$

Where
F = applied force
A_t = instantaneous CSA or true area
A_o = initial CSA
L_f = final length
L_o = initial length

$$d\varepsilon = \frac{d\delta}{L}$$ **…..for limits between L_o and L_f:**

$$\varepsilon_t = \int_{L_o}^{L_f} \frac{d\delta}{L} = log_e\left[\frac{L_f}{L_o}\right] = ln\left[\frac{L_f}{L_o}\right] = ln\left[\frac{A_o}{A_f}\right]$$

Since there is no lost of material during deformation, ∴vol. = constant !:

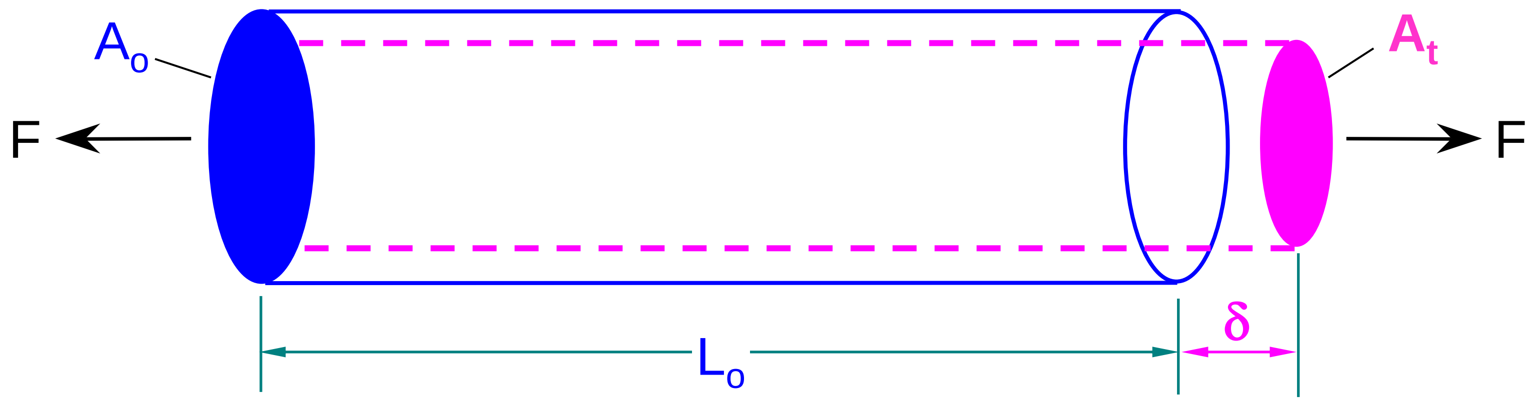
$V_o = V_f$
$A_oL_o = A_fL_f$

$$\therefore \frac{L_f}{L_o} = \frac{A_o}{A_f}$$

- For **small values of strain** **(within the elastic limit)**, engineering and true strains **are equivalent** – **but they rapidly diverge for LARGE Strains (i.e. when loaded above elastic limit)**

- True stresses and strains must be considered during manufacturing processes that involve **plastic deformation (PD).**

Nominal (or Engineering) Stress Vs. True Stress



- Consider the homogeneous bar shown above, subjected to an axial tensile force (F).

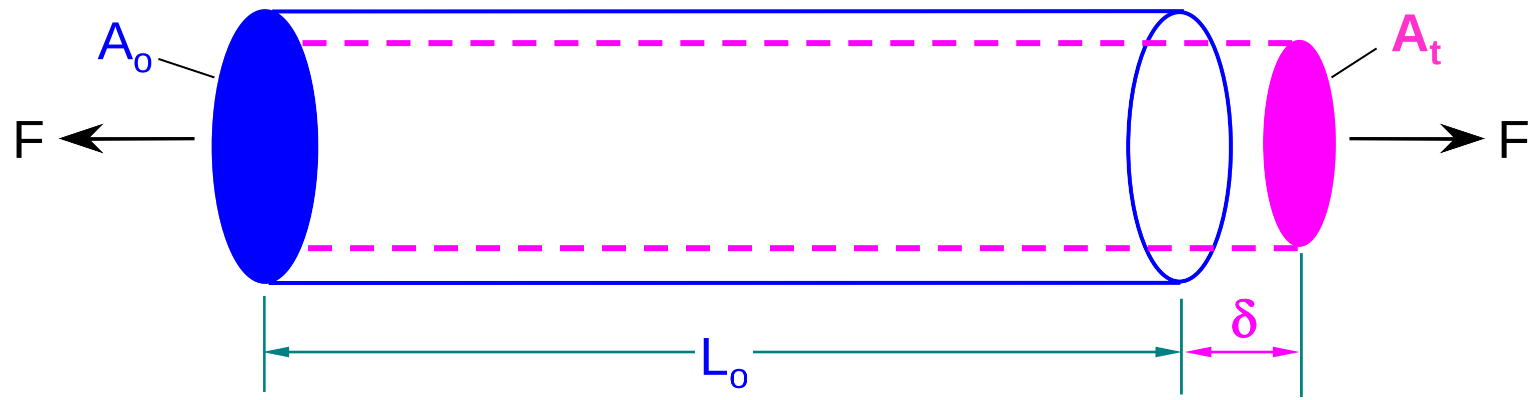
- There are two ways to calculate stress:

  – Based on the original area, ***nominal or eng. stress***: $$\sigma_n = \frac{F}{A_o}$$

  – Based on the **instantaneous area**, ***true stress***: $$\sigma_t = \frac{F}{A_t}$$

Where
A_o = initial cross sectional area (CSA); L_o = initial length;
A_t = instantaneous CSA; δ = change in length;

Nominal (or Engineering) Strain Vs. True Strain



- Consider the homogeneous bar shown above, subjected to an axial tensile force (F).

- There are two ways to calculate strain:
  - Based on the **original length**, ***nominal strain***:

$$\varepsilon_n = \frac{\delta}{L_o} = \frac{L_f - L_o}{L_o}$$

  - Based on the **instantaneous length**, ***true strain***:

$$\varepsilon_t = ln\left(\frac{L_f}{L_o}\right) = ln\left(\frac{A_o}{A_f}\right)$$

Where
L_o = initial length; δ = change in length; L_f = instantaneous or final length

Relationship Between Nominal (or Eng.) & True Values

From previous slide:

$$\varepsilon_t = \ln\left(\frac{L_f}{L_o}\right) --- (1) \qquad \varepsilon_n = \frac{L_f - L_o}{L_o} \quad \textit{or} \quad L_f = L_o + \varepsilon_n L_o --- (2)$$

Put (2) into (1): $\varepsilon_t = \ln\left(\frac{L_o + \varepsilon_n L_o}{L_o}\right)$; $\boxed{\therefore \ \varepsilon_t = \ln(1+\varepsilon_n) --- (3)}$

During deformation, note that: Initial vol. of sample = instantaneous vol. of sample (*since mass & density remain the same*)

$$\text{i.e. } V_o = V_t \ ; \ A_o L_o = A_t L_f \quad (\text{substitute for } L_f \text{ from Eq. 2})$$

$$A_o L_o = A_t(L_o + \varepsilon_n L_o)$$

$$\therefore \ A_o = A_t(1+\varepsilon_n) --- (4)$$

Also, at any instance during the loading:

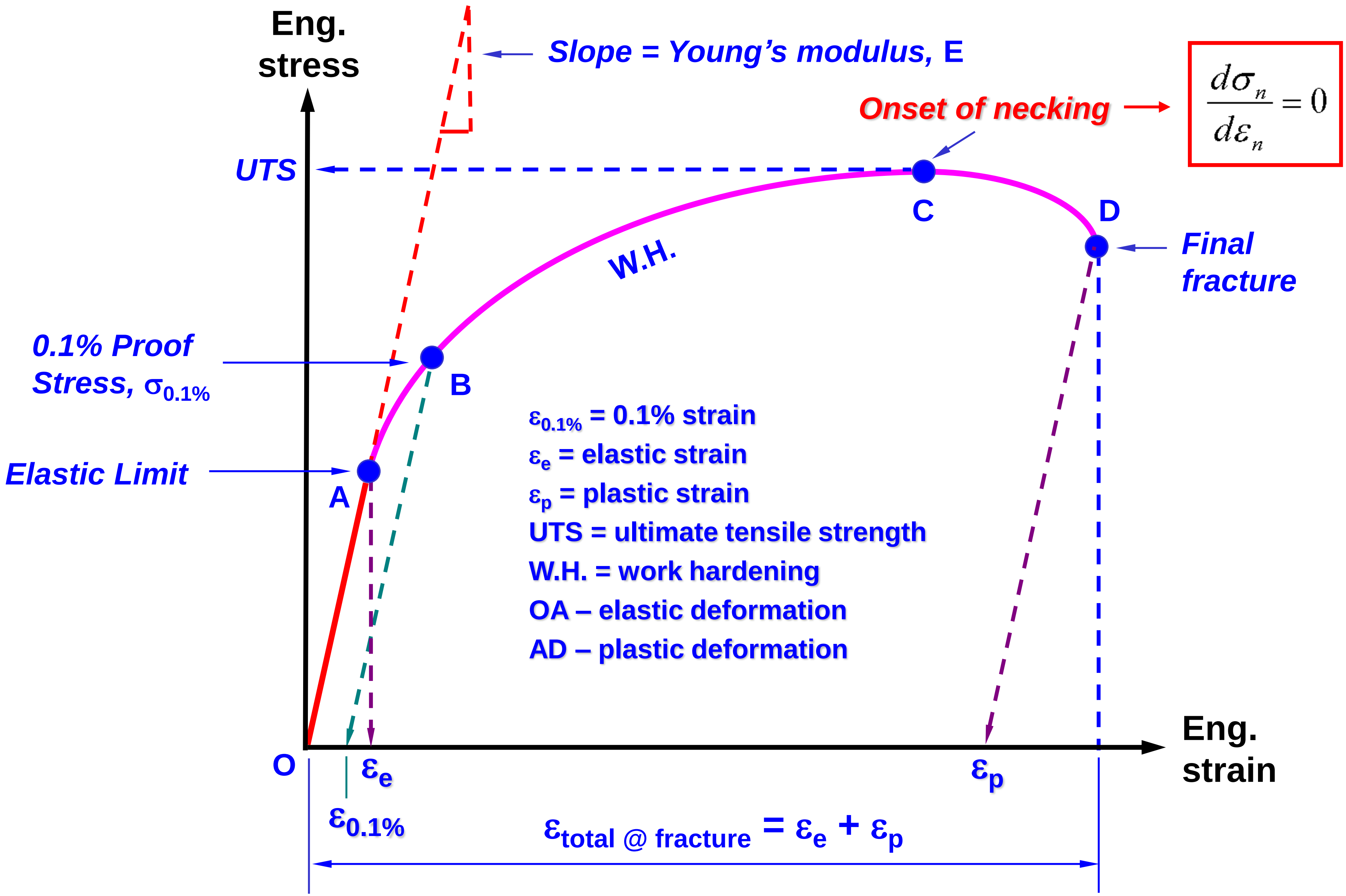
$$F_n = F_t$$

$$\sigma_n A_o = \sigma_t A_t \quad (\text{substitute for } A_o \text{ from Eq. 4})$$

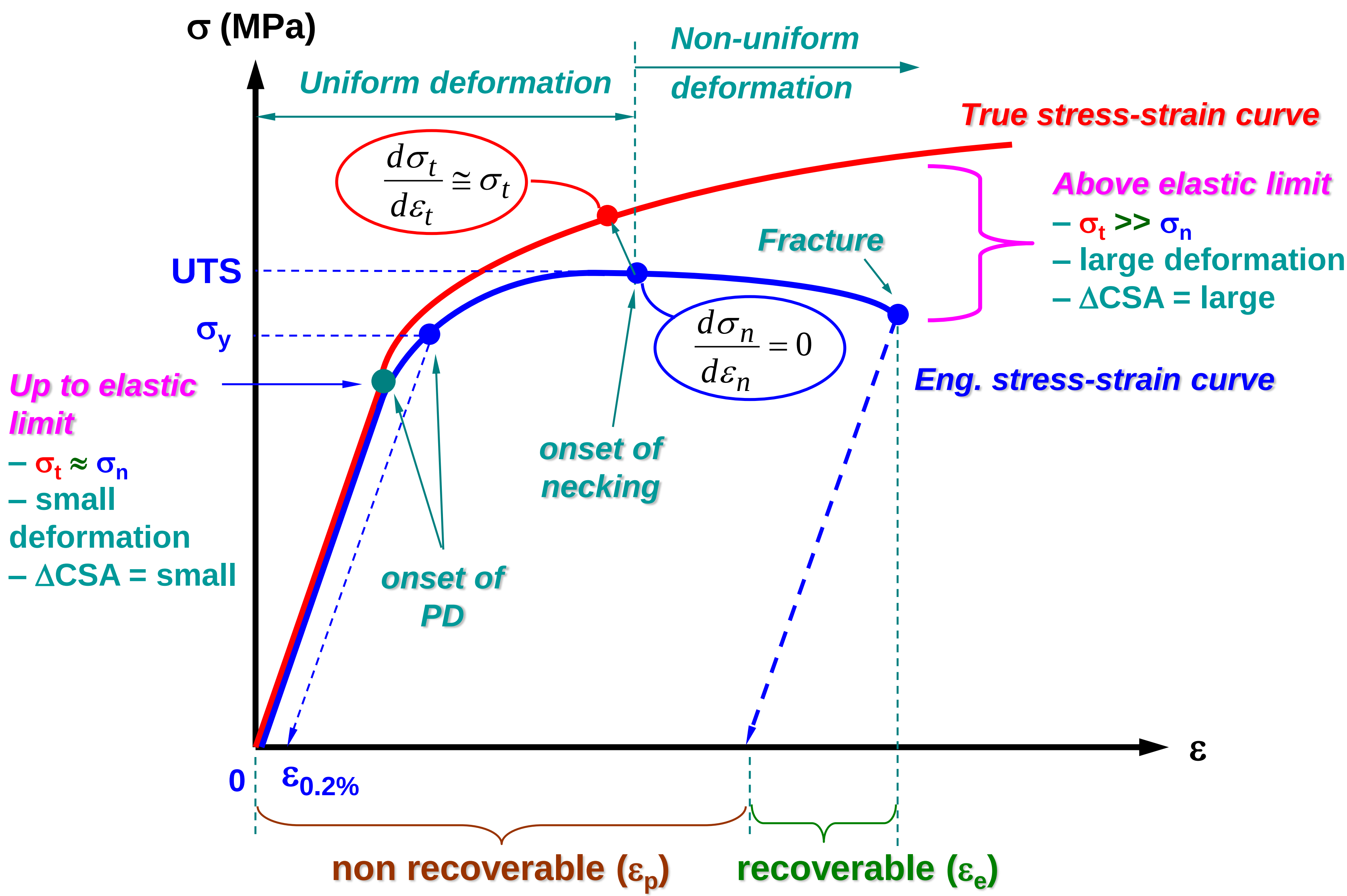
$$\sigma_n A_t(1+\varepsilon_n) = \sigma_t A_t \ ; \quad \boxed{\therefore \ \sigma_t = \sigma_n(1+\varepsilon_n) --- (5)}$$

$\sigma_t \cong \sigma_n$ for high E and small ε but not true for materials undergo large ε, e.g., rubber where CSA may change significant

Engineering (or Nominal) Stress–Strain Curve



True Stress–Strain Curve: Comparison



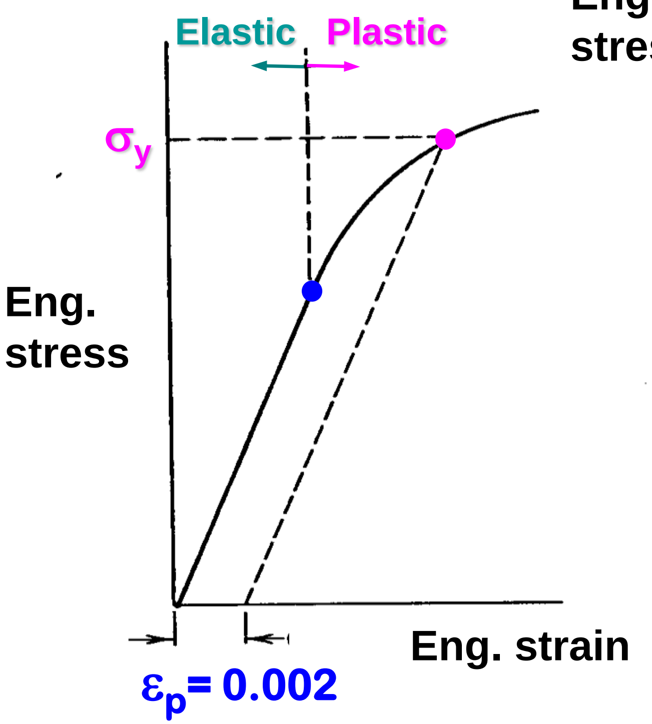
Plastic (Permanent) Deformation (PD)

- 1-PD is permanent and non-recoverable
- 2-PD occurs when the applied load exceed the elastic limit
- 3-From this point onwards, the material DO NOT obeys the Hooke's law and stress is no longer proportional to the strain
- 4 -PD normally occurs at lower temperatures, i.e. $T < T_{melt}/3$
- 5-PD corresponds to breaking of bonds with original atom neighbors and reforming bonds with new neighbors; $\therefore$atoms or molecules move relative to one another
- 6-Transition from elastic to plastic is gradual for most metals
- 7-For crystalline solids, deformation process is by means of slip which involves dislocation motion.
- 8-Yielding = stress level at which PD begins
- $\therefore$9- The yield stress, $\sigma_y \rightarrow$ is a measure of material's resistance to PD

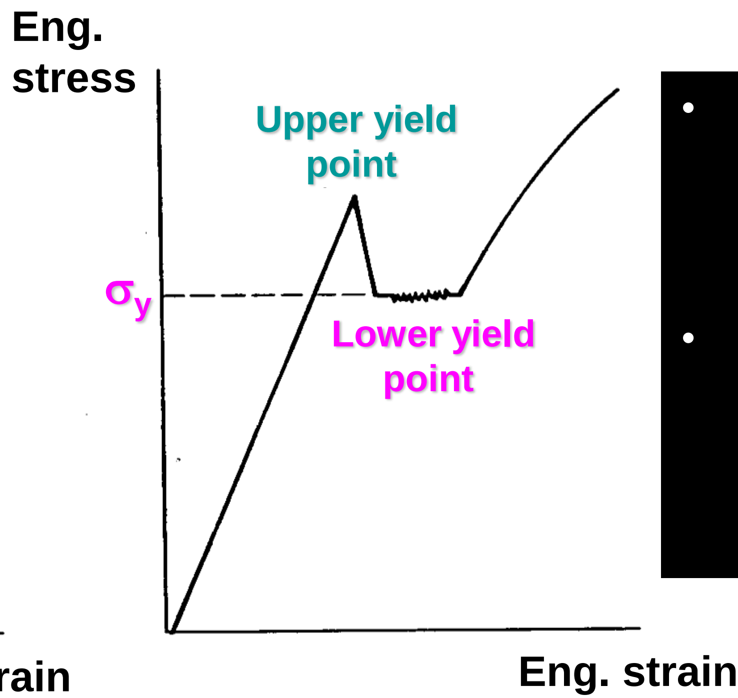
Yield Strength or Stress (σ_y)

- σ_y is the stress at which *noticeable* plastic deformation has occurred; For most metals yielding occurs when the plastic strain is, **ε_p = 0.002 or 0.2%**

e.g. Grey cast iron



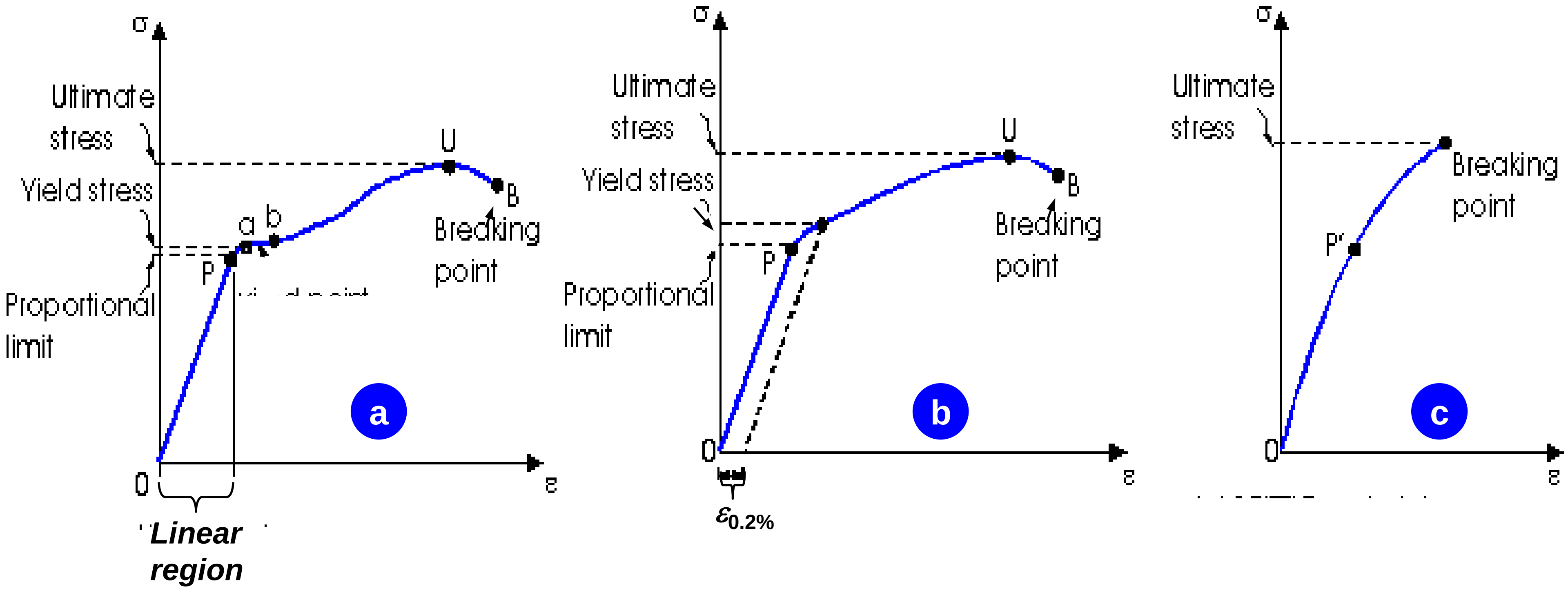
e.g. Low carbon steels



- **Point of yielding** = initial departure from linearity of Eng. Stress Vs. Eng. Strain curve
- In design, eng. components must not deform plastically. Thus, **σ_y is taken as design criteria !!**

- Sometimes, it is difficult to determine exactly the position of the yield point (as shown above for grey cast iron); It is common to take the yield stress as the 0.2% proof stress, i.e. the stress corresponds to **0.002 strain offset** as shown above.

Yield Strength of Materials



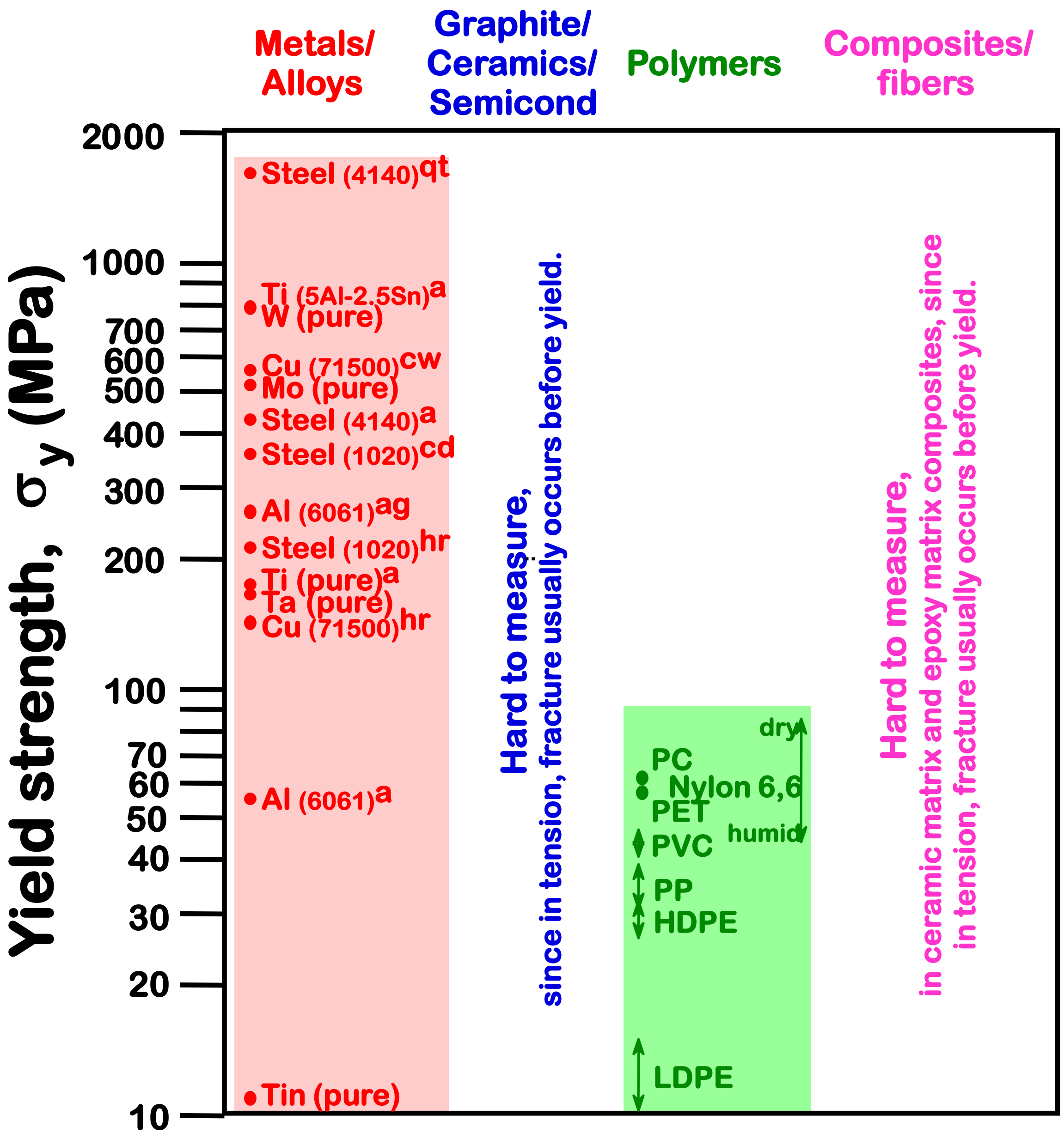
The stress-strain curves for three different types of material:

(a) Low-carbon steel, a ductile material with a distinct yield point.

(b) A ductile material, such as Al alloy → no define yield point → take σ_y @ 0.2% strain ($\varepsilon_{0.2\%}$)

(c) A brittle material, e.g. glass, ceramics, cast iron, in compression → take $\sigma_y = \sigma_{fracture}$

Yield Strength: Comparison



$\sigma_{y(ceramics)}$

$> \sigma_{y(metals)}$

$>> \sigma_{y(polymers)}$

Room T values

Based on data in Table B4, *Callister 6e.*

a = annealed

hr = hot rolled

ag = aged

cd = cold drawn

cw = cold worked

qt = quenched & tempered

Yield Strength: Design Criteria

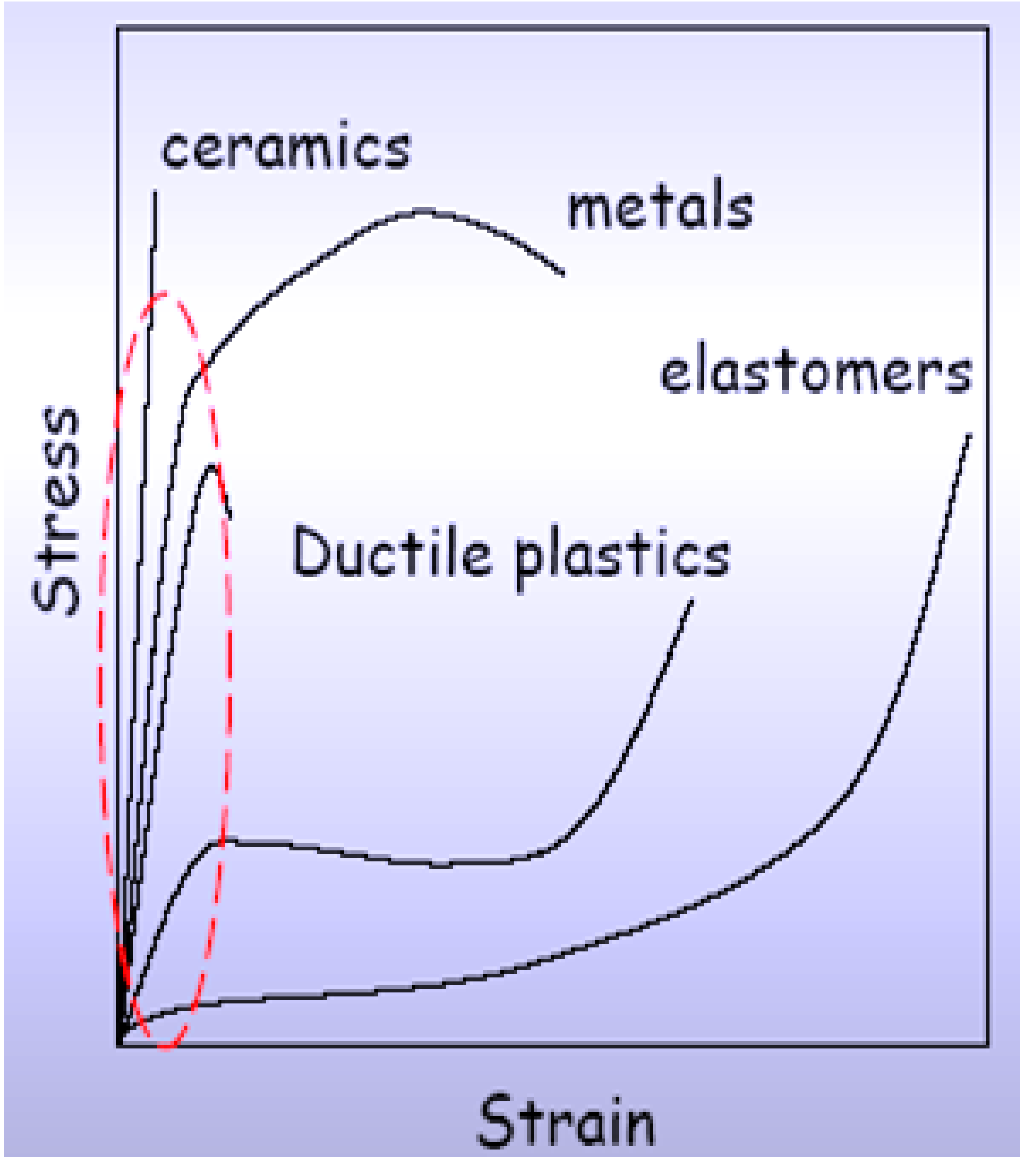
- Schematic comparison of tensile behavior of common engineering materials.
- Design criteria ***vary with material classes***.
- A general rule of thumb:

– For metals: $\sigma_y = \sigma_{0.2\%}$

– For ceramics & brittle plastics: $\sigma_y = \sigma_{fracture}$

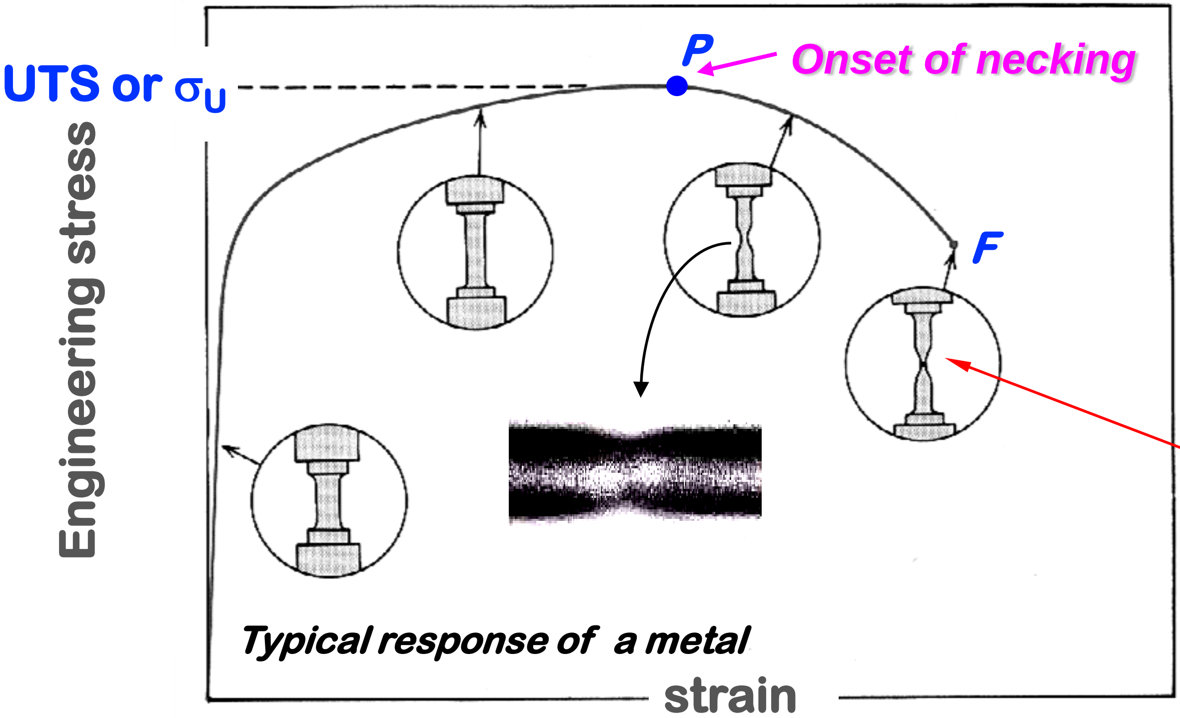
– For ductile plastics: $\sigma_y = \sigma_{1\%}$

– For composites: $\sigma_y = \sigma_{0.5\%}$

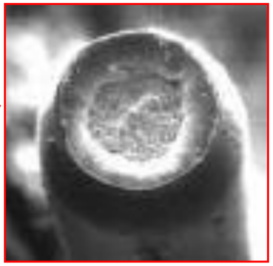


Tensile Strength (Ultimate Tensile Strength, UTS)

- UTS or σ_U is the **maximum possible** engineering stress in tension.



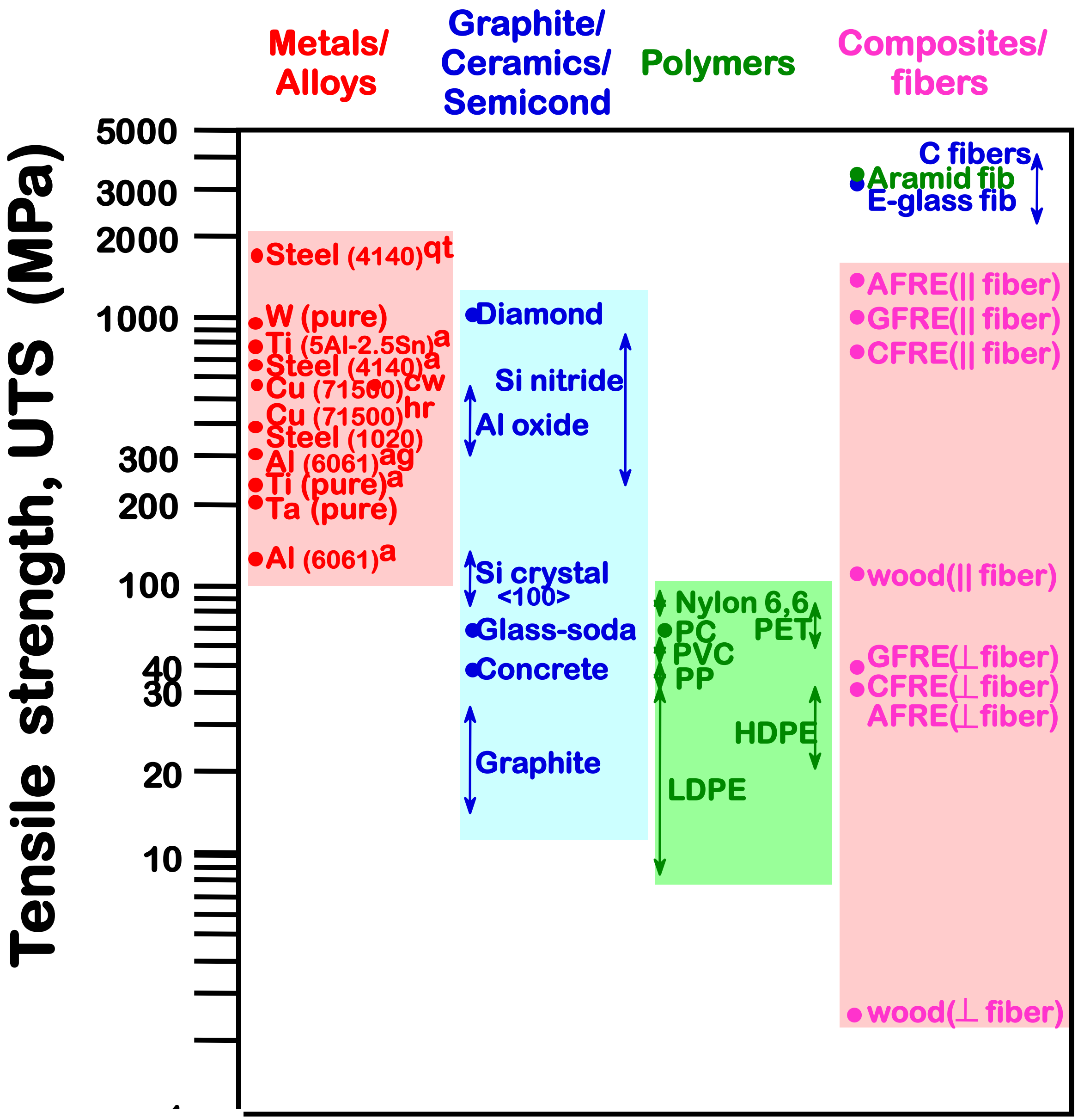
- Uniform deformation up to *P*
- Beyond point *P*, necking resulted; i.e., deformation confined at this constriction.



Fracture strength = Stress at fracture

- Metals: occurs when noticeable necking starts.
- Ceramics: occurs when crack propagation starts.
- Polymers: occurs when polymer backbones are aligned and about to break.
- For design, use σ_y and not UTS (due to large PD had occurred at UTS). *UTS is only use for deep drawing or other metal forming process.*

Tensile Strength: Comparison



$UTS_{(ceramics)}$
$\sim UTS_{(metals)}$
$\sim UTS_{(composites)}$
$\gg UTS_{(polymers)}$

Room T values

Based on data in Table B4, *Callister 6e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

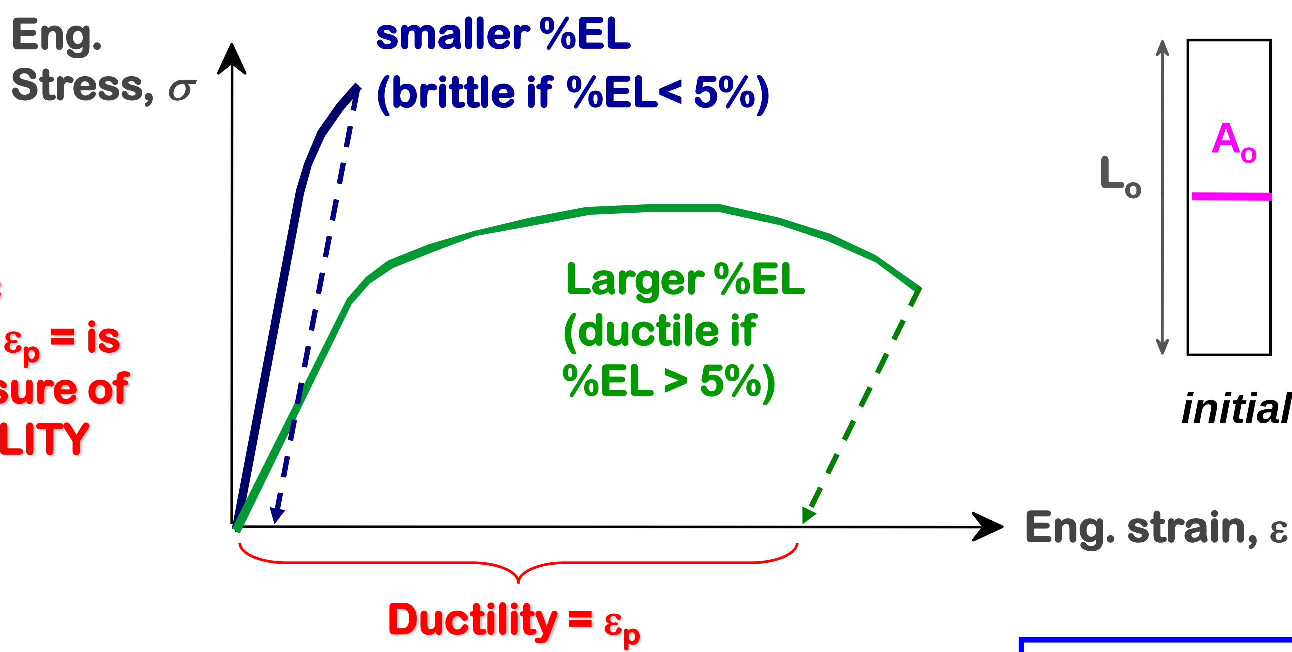
AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

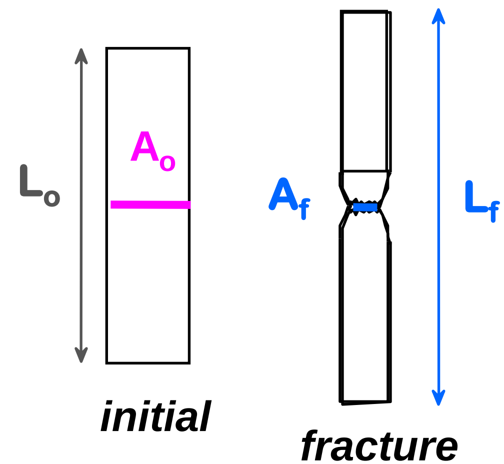
Ductility

- Is define as **plastic tensile strain at fracture** and can be expressed in terms of %elongation (EL) or % reduction in area (AR):

$$\%EL = \frac{(L_f - L_o)}{L_o} \times 100$$

Plastic strain, ε_p = is a measure of DUCTILITY





- % reduction in area (AR):

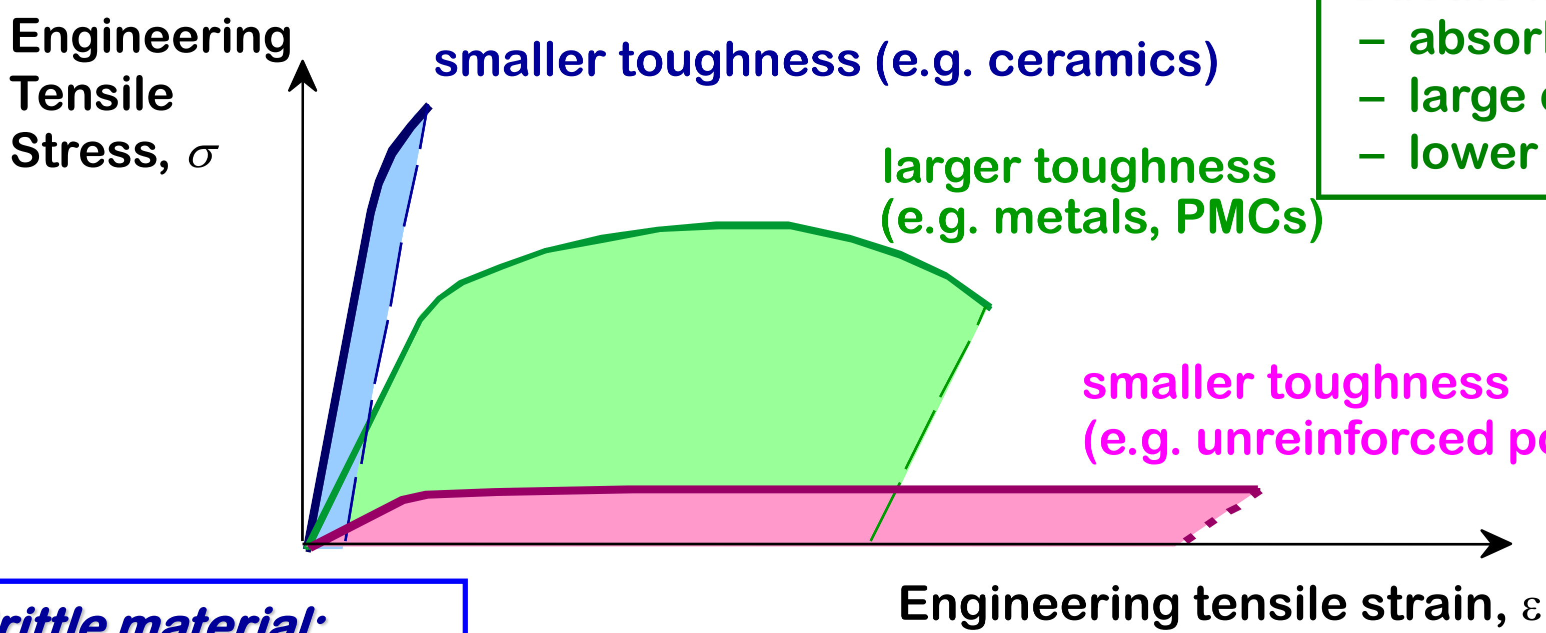
$$\%AR = \frac{(A_o - A_f)}{A_o} \times 100$$

- Note that %AR and %EL are often comparable.
  – Reason: crystal slip does not change material volume.

  – However it is possible that %AR > %EL if internal voids form in neck region.

Toughness

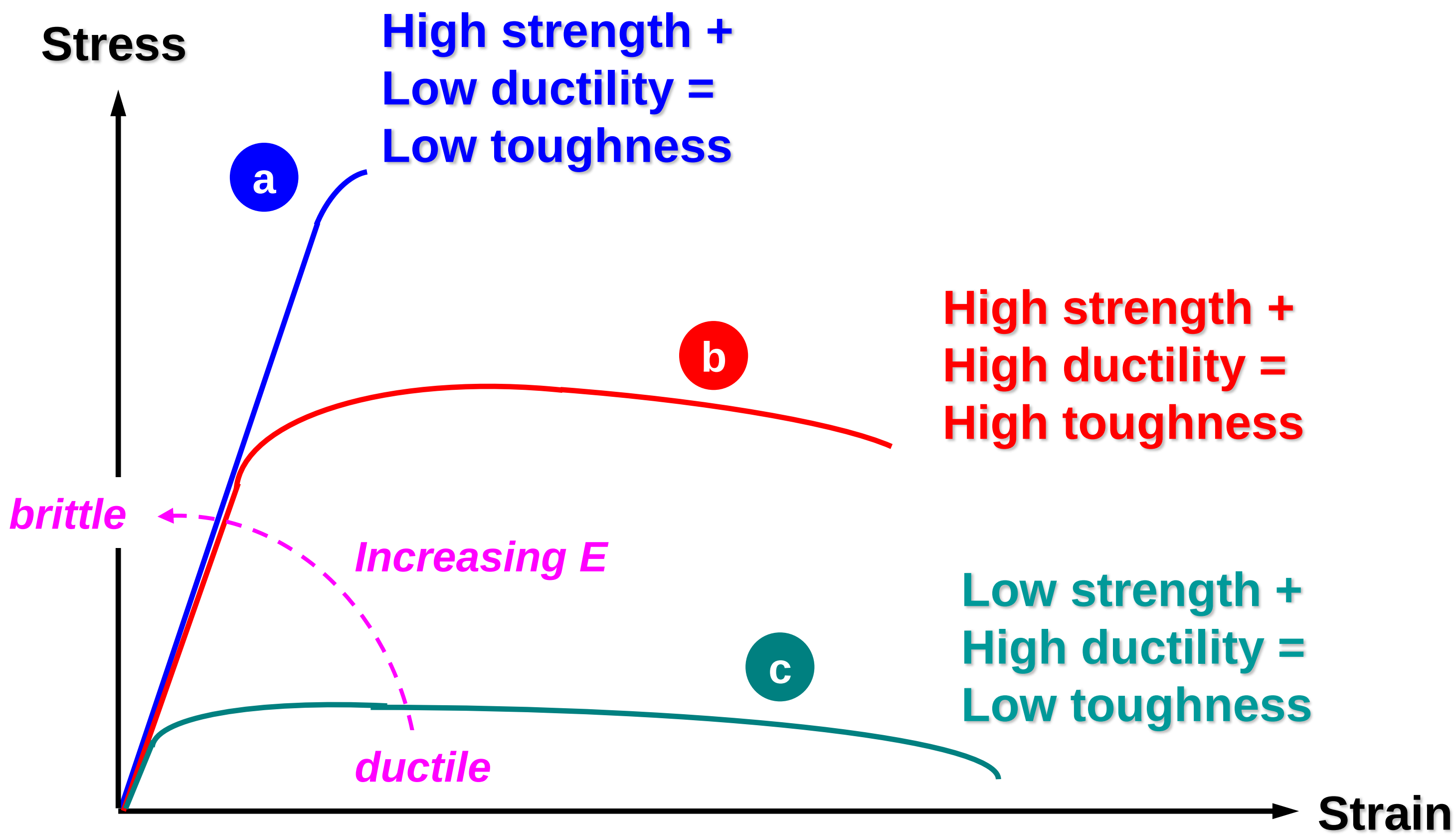
- A measure of the energy needed to break a unit volume of material.
- Can be approximated by the **area under the engineering stress-strain curve**.



Ductile material:
- absorb energy
- large elongation
- lower strength

Brittle material:
- absorb less energy
- small elongation
- high strength

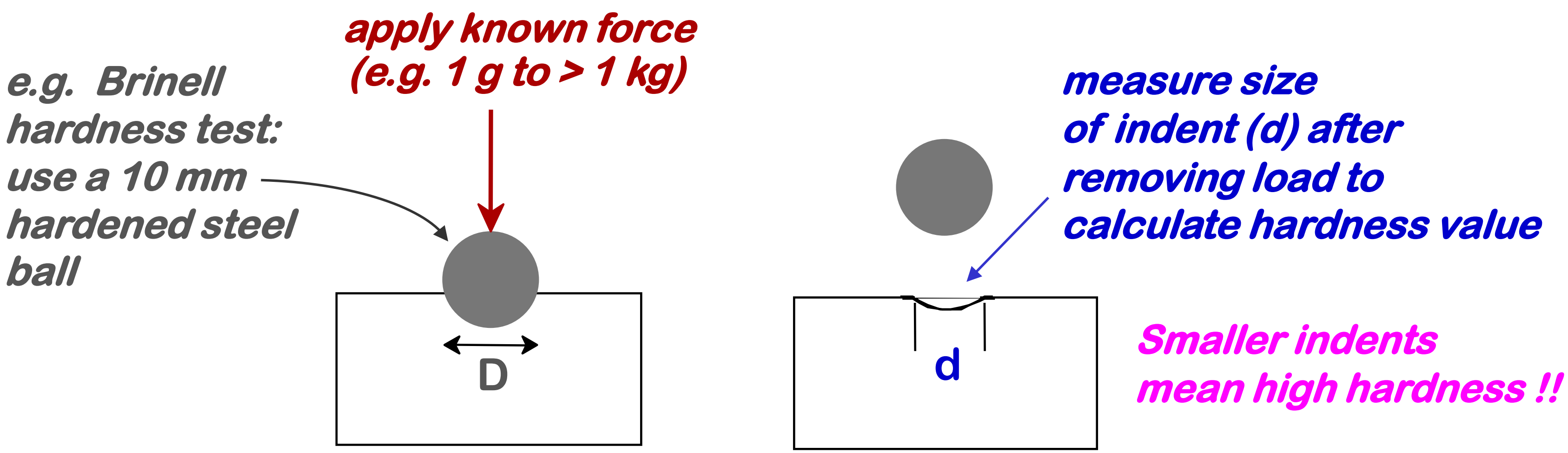
Strength, Toughness & Ductility

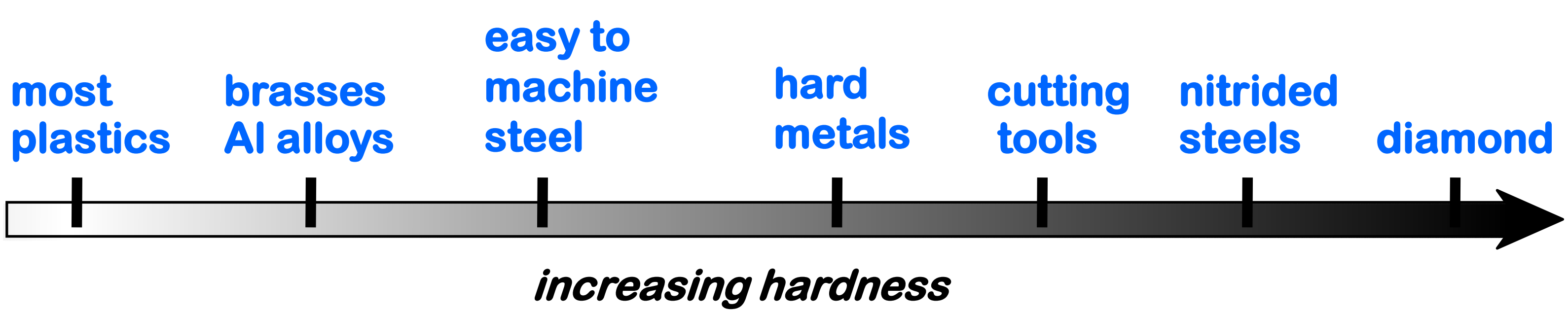


- High toughness depends on the proper combination of strength and ductility
- Which material, based on their stress-strain curve shown above, is most suitable for cold working process such as rolling process?

Hardness

- Resistance to surface scratching or indentation by another body.
- Large hardness means:
  - resistance to plastic deformation or cracking in compression
  - better wear properties (i.e. high wear resistance)





Safety Factor (SF)

- It is impossible to perfectly analyze stresses and material properties in any design problem.

- Safety Factor (SF) is thus introduced to account for design uncertainties; the idea is to lower the working design stress limit.

- The yield stress (σ_y) is normally taken as the **maximum design limit** and **NOT** the UTS since at the UTS severe plastic deformation had taken place.

- Engineers will not design components to work at the design limit but at a safe limit which is lower than the yield stress.

- Thus the **safe working design stress** ($\sigma_{working}$) can be found as follow:

$$\sigma_{working} = \frac{\sigma_y}{SF}$$

– *SF may vary depending on the design requirements, from 1 to 5*

– *(SF)$_{ave.}$* may be taken as 2

Safety Factor (SF)

$$\sigma_{\text{working}} = \frac{\sigma_y}{SF}$$

Selection of SF is based upon:

- Previous experience
- Accuracy for which mechanical properties & material properties can be determined
- Consequences of failure in terms of loss of life, and/or property damage
- Economics & cost considerations

Designing using low SF:

- Safety may not be an issue based on past experiences or the design itself
- Select materials with small variability of properties (may incur high cost)
- Increase routine inspections to detect incipient failures

Designing using high SF:

- When safety is of ultimate concern
- High cost
- Inspection are not routine

Example

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm. Determine the magnitude of the load required to produce a 2.5 x 10^{-3} mm change in diameter if deformation is entirely elastic.
Take for brass: μ = 0.34 and E = 97 GPa

Solution:

Given: d_o = 10 mm ; Δd = – 2.5 x 10^{-3} mm ; μ = 0.34 ; E = 97 GPa or 97 x 10^3 MPa

→ strain in *y*-dir. or *lateral strain*: $\varepsilon_y = \Delta d / d_O = (-2.5 \times 10^{-3})/(10) = -2.5 \times 10^{-4}$

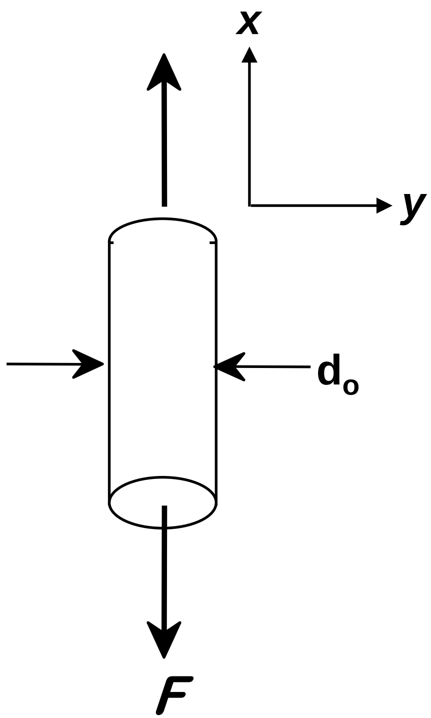
→ strain in *x*-dir. or *axial strain* (ε_x) can be found from:

$$\mu = -\frac{\varepsilon_y}{\varepsilon_x} \rightarrow 0.34 = -\frac{-2.5\times10^{-4}}{\varepsilon_x} \quad ; \quad \therefore \underline{\varepsilon_x = 7.35\times10^{-4}}$$

→ To find applied force (F):

$$E = \frac{\sigma_x}{\varepsilon_x} = \frac{F}{A_o \varepsilon_x} \rightarrow 97\times10^3 \text{ MPa} = \frac{F}{\pi\left(\frac{10\text{ mm}}{2}\right)^2 (7.35\times10^{-4})}$$

$$\therefore F = \underline{5600\text{ N or } 5.6\text{ kN}} \quad \text{(ans.)}$$



Example

For a bronze alloy (*E* = 115 GPa), the stress at which PD begins is 275 MPa.

(a) What is the maximum load that may be applied to a specimen with a CSA of 325 mm^2 without experiencing plastic deformation?

(b) If the original specimen length is 115 mm, what is the maximum length to which it may be stretched without causing plastic deformation?

Solution:

Given: σ_y = 275 MPa ; A_o = 325 mm^2 ; L_o = 115 mm ; E = 115 GPa or 115 x 10^3 MPa

(a) Since SF is not specified, take σ_y as the working limit.

To find F:

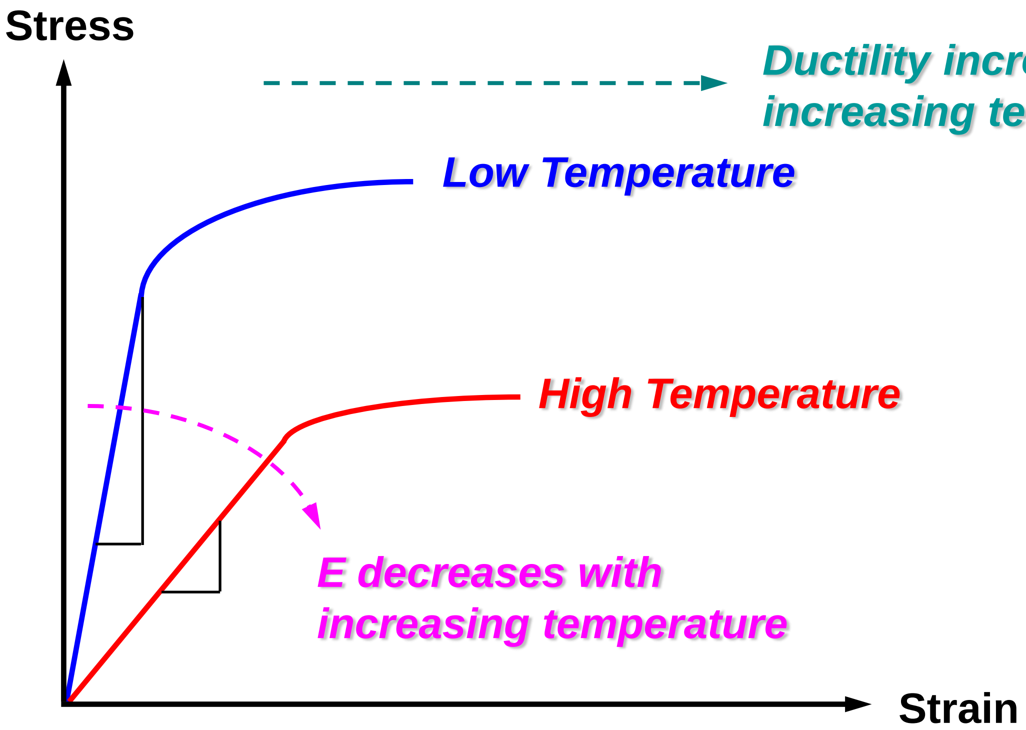
$$\sigma_y = \frac{F}{A_o} \quad \rightarrow \quad 275\ \text{MPa} = \frac{F}{325\ \text{mm}^2} \quad ; \quad \therefore\ \underline{F = 89{,}375\ \text{N or } 89\ \text{kN}} \quad \text{(ans.)}$$

(b) To find L_f before PD occurs i.e. still within the elastic limit:

$$\textit{Elastic ext.,}\quad \delta = \frac{FL_o}{A_o E} = \frac{(89{,}375\ \text{N})(115\ \text{mm})}{(325\ \text{mm}^2)(115\times 10^3\ \text{MPa})} = 0.275\ \text{mm}$$

$$\therefore\ \text{L}_\text{f} = \text{L}_\text{o} + \delta = 115 + 0.275 = \underline{115.275\ \text{mm}} \quad \text{(ans.)}$$

Temperature Effect on Young's Modulus & Ductility



- WH also decreases with increasing temperature !

- Mechanical properties such as σ_y, UTS and *E* are temperature dependent.
- These properties decreased with increasing temperature !
  ➔ Reason: distance of atom or ion separation ↑ with temperature, resulting in a decrease in inter-atomic forces between adjacent atoms or ions.
- Ductility increases with temperature.

Summary

- Stress and strain: These are **size-independent** measures of load and displacement, respectively.
- Elastic behavior: This reversible behavior often shows a linear relation between stress and strain.
- To minimize deformation, select a material with a large elastic modulus (E or G).
- Plastic behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σ_y.
- Toughness: The energy needed to break a unit volume of material.
- Ductility: The plastic strain at failure.

Resources

- Callister
  Chapter 6

THANK YOU FOR YOUR ATTENTION ☺