Chapter One :- Concepts and Definitions

-<u>Thermodynamic System</u> :- is defined as a definite area or space where some thermodynamic process is taking place, and it is a region where our attention is focused for studying a thermodynamic process.

<u>- Boundary</u> :- is defined as the envelope which contains the system. The boundary may be fixed like that of a tank enclosing a certain mass of compressed gas or movable like boundary of certain volume of liquid in a pipe.

-<u>Control Volume</u> :- A control volume is a fixed region in space chosen for the thermodynamic study of mass and energy balances for flowing system .the boundary of the control volume may be a real or imaginary envelope.

Classification of Thermodynamic System :-

<u>1-Closed System</u> :- The system whose boundary does not permit the transfer of mass between the system and its surrounding i.e. its mass is constant and only energy being allowed to transfer across the boundary.



2- <u>Open System</u> :- is defined as a region is which the mass is not necessarily constants besides , the mass as well as energy transfer cross its boundary .



3-Isolated System :- is defined as a system of fixed mass and neither mass nor energy cross its boundary or its is a system which is completely non-influenced by the surrounding .

- <u>Pure Substance</u>:- is a single substance or mixture of substance which has the same consistent composition throughout, or it is a homogeneous substance and its molecular structure does not vary .ex: steam, water, air and mixture of water and steam.

- Thermodynamic Properties

Properties can be classified as

1-Extensive Property :- The properties of a system whose value for the entire system is equal to the sum of their value for the individual parts of the system , these properties are related to the mass of the system. Like (volume, mass, all kinds of energy).

2- <u>Intensive Property</u> :- the properties of the system whose value for the entire system is not equal to the sum of their values for the individual parts of the system .These properties does not depend on the mass of the system.(like pressure ,Temperature ,Specific volume and density).

-<u>Steady State</u> :- is that circumstance in which there is no accumulation of mass or energy within the control volume , and the properties at any point within the system are independent of time.

-<u>Equilibrium</u>:- When a system is in equilibrium with regard to all possible changes in state, the system is in thermodynamic equilibrium, the temperate will be the same through hot the entire system if the gas that comprises a system is in thermal equilibrium.

a-isothermal in equilibrium (uniform temp.)

b-mechanical equilibrium (uniform pressure)

c-chemical equilibrium (uniform composition)

- <u>**Phase</u></u> :- is the homogeneous part of the substance having the same intensive properties it can be either solid , liquid or gas (vapor)</u>**

- <u>Homogeneous and Heterogeneous</u> :- A substance exist at a single phase called a homogeneous , while one consists of two or more phase is called ⁽⁽Heterogeneous⁾⁾.

-**Process** :- is a change in the state of a system by any change in a system properties.



The path where the succession of state passes in called a ((process))

-<u>Cycle</u> :- is a sequence of processes which return the system to its original state.



-<u>Units</u> :- is an arbitrary amount of quantity be measured with assigned numerical value of unit.

<u>Fundamental units</u>		(S.I)
Length	L	m
Mass	Μ	kg
Time	t	S
Temp.	Т	°C

Thermodynamics	Mech.Eng.Dept. 2 nd stag		
- <u>Secondary units</u>			
1-Area	L^2	m^2	
2-Volume	L^3	m^2	
3-Velocity	Lt^{-1}	m/s	
4-Acceleration	Lt ⁻²	m/s^2	
5-Force	MLt ⁻²	Kg ms ⁻²	(Newton)
6-Density	ML^{-3}	Kg/m ³	
7-Perssure($\frac{F}{A}$)	$ML^{-1}t^{-2}$	N/m ²	$(\text{Kgm}^{-1} \text{ S}^{-2})$
8-Work	ML^2t^{-2}	N.m	(Joule)
9-Power	ML^2t^{-3}	J/S	(Watt)

-**Volume** :- Is the space required by the system and is measured in (m^3) .

-<u>Specific Volume</u> :-(V) is the total volume of that substance divided by the total mass of that substance, it is measured in (m^3/Kg)

$$V = \frac{V}{M} \frac{m^2}{kg}$$

-**Density** (ρ) :- is the total mass of that substance divided by the total volume . it is measured in (Kg/m^3) .

$$\rho = \frac{M}{V} \quad \text{Kg/m}^3$$

Zero Law of thermodynamic :-

If cold body (A) in contact with hot body (B) then the cold body become hoter and the hot body become colder, This process continue till this exchange of Heat attains a stat of thermal equilibrium state, the two bodies will be at the same temp.

(i.e. TA=TB)



If a third body (C) has same temp as TA & Then TC=TB i.e.⁽⁽⁾ when a two bodies are in thermal equilibrium with a third body. They are also in thermal equilibrium with each other⁽⁾⁾.

-<u>Temperature</u> :- is a measure of the molecular activity of a subst. the greater the movement of molecules , the higher the temp. it is a measure of how ((hot)) or ((cold)) a substance is and can be used to predict the direction of heat transfer .

Two scales are commonly used for measuring temperature.

<u>Scale</u>		water freezing point	water boiling point
1-Centigrate	°C	0 °C	100 °C
2-Faherhheit	F	32 F	212 F

The absolute scale relate to the Celsius or centigrade is Kelvin (K)

 $T_{(K)} = T_{(°C)} + 273.15$

The a absolute scale relate to the Fahrenheit is the RanKine (R)

 $T_{(R)} = T_{(F)} + 459.57$, $\Delta T(R) = \Delta T(^{\circ}F)$

 $T_{(F)}=1.8 T(^{\circ}C) + 32, \Delta T(^{\circ}C) = \Delta T(K)$

<u>Ex(1)</u>

What is the Kelvin equivalent of 80 °F?

°C
$$=\frac{1}{1.8}$$
(°F-32) $=\frac{1}{1.8}$ (80-32)
= 26.7 °C

 $T_{(k)} = T(^{\circ}C) + 273$

-<u>Pressure</u> :- is a measure of the force exerted per unit area on the boundaries of a substance (or system)

$$P = \frac{F}{A} \left(\frac{N}{m^2}\right)$$
(Pascal)

When pressure is measured relative to a perfect

vaccume, it is called absolute pressure (Pisa), When measured relative to atmospheric pressure (14.7 Pisa) it is called gauge pressure (Psig). The latter pressure scale was developed because almost all pressure gauges register Zero when open to the atmosphere. Therefore, pressure gauges measure the difference between the pressure of the fluid to which they are connected and that of the surrounding air.

$$P_{abs.} = P_{atm} + P_{guge}$$
.

 $P_{abs} = P_{atm} - P_{vacc}$.

P_{atm} : atmospheric (barometric) pressure.

When pressure is measured relative to a perfect vaccume, it is called absolute pressure (Pisa), When measured relative to atmospheric pressure (14.7 Pisa) it is called gauge pressure (Psig). The latter pressure scale was developed because almost all pressure gauges register Zero when open to the atmosphere. Therefore, pressure gauges measure the difference between the pressure of the fluid to which they are connected and that of the surrounding air. Pabs.= Patm +Pguge .

Pabs.=Patm – Pvacc.

Pabs.=Patm – Pvacc.

Patm= atomospheric (barometric) pressure.

 $1 \text{ bar} = 10^5 \text{ Pa}$ = 100 KPa

= 0.1 MPa

1 atm= 101325 Pa

= 1.01325 bar

1 Psi = 6.894 KPa

Pressure read in mm Hg usually for the mano meter

$$\Delta P = P_1 - P_2 = \rho g L$$

 ρ = density of L liquid (Kg/m³)

g= gravitational acceleration (m/sec^2)

L=height (m)

Ex: Find the Gauge pressure and height of the liquid?





$$L = \frac{P - P_{atm}}{gg}$$

= $\frac{(104.0 - 101.33) kP_{a}}{(13.59 g/cm^{3})(9.81 m/s^{2})} \frac{10^{3} N/m^{2}}{1 kP_{a}} \frac{10^{3} g}{1 kg} \frac{1 m^{3}}{10^{6} cm^{3}} \frac{1 kg \cdot m/s^{2}}{1 N}$
= $0.02 m = 2 cm$
 $P_{gage} = P - P_{atm} = 104.0 - 101.33 = 2.67 kP_{a}$
 $P_{gage} = P - P_{atm} = 104.0 - 101.33 = 2.67 kP_{a}$

Ex(2) :- The air flow inside the apparatus is determined by measuring the pressure manometer a cross an orifice with a height different of zoo mm what is the pressure drop ?

T=5 °C ,
$$\rho_{\rm Hg}$$
= (139595- 2.5 T_C°_C) $\frac{kg}{m^3}$

 $\Delta P = P_1 - P_2 = \rho \text{ gL}$ $\rho = 13595 - 2.5(5) = 13582.5 \text{ Kg/m}^3$ $\Delta P = 13582.5 \times 9.8 \times \frac{200}{1000} = 25.8 \text{ KPa}$



-<u>Energy</u> :- is defined as the capacity of a system to perform work or produce heat. There are different types of energy:

1-<u>Potential energy (P.E.)</u> :- is defined as the energy of position, it is defined as :-

```
P.E=mgz
```

m= mass (Kg) , g=gravitational acc. (m/sec²)

z= height of body (m) P.E. in Joule (J)

2-<u>Kinetic Energy (K.E.)</u>:- is defined as the energy of motion, it is defined as:-

$$K.E. = \frac{1}{2} mv^2 \qquad (J)$$

V=speed of body (m/sec.)

3-Internal Energy (u) :- is the energy of mass composition fund a mentally, it is due to the rotation, vibration, translate interactions among the molecules of the substance.

4-Specific Enthalpy (H):- is defined as ((H=u+pV)), where u is the specific internal energy (J) of the system being studied ,p is the pressure of the system (Pa) and (V) is the specific volume of the system (m^3/kg) .it is usually used in connection with ((open)) system problem in thermodynamic , it is a property like pressure , Temp. and volume but it cannot be measured directly .

Ex:- An automobile of 1200 Kg accelerates from 50 Km/hr. to 100 Km/hr. Determine the initial and the change in the kinetic energy.

Sol.:

 $K.E_1 = 1/2 mV_1^2$

Ex:- A river is flowing at an average velocity of (3 m/sec.) and flow rate with 500 m³/sec. with an elevation of (90 m) of the free surface of the river .Determine the total mechanical energy of the river water per unit mass , and the power geneneration potential of the entire river ?



Sol:

E_{mesh}.=P.E+KE= gh+
$$\frac{v^2}{2}$$

=((9.81)(90)+ $\frac{(3)^2}{2}$)($\frac{1 kJ/kg}{1000^{m^2}/s^2}$)

=0.887 KJ/Kg

 $\dot{m}=P\dot{V}=(1000)(500)=500000 \text{ kg/sec.}$ w= $\dot{m} \text{ E}=(500000)(0.887)=444000 \text{ KW}=444 \text{ MW}$

-Path function and state function :-

<u>State Function</u> :- The properties which does not depend on the past history of the substance nor on the path it followed in reaching a given state , like U,H,T,S,P.

<u>Path Function</u> :- the properties which are opposite to the state functions, i.e the properties which depend on the path like heat and work.

Properties Of Pure Substance

The phase change of substances in a system is very important to thermodynamics. It is possible to design systems to take advantage of the phase changes between solid and liquid or between liquid and vapor to enhance the performance of the system.

Consider as a system 1 kg of water contained in the piston/cylinder arrangement shown in Fig. 1*a*. Suppose that the piston and weight maintain a pressure of 0.1 MPa in the cylinder and that the initial temperature is 20°C. As heat is transferred to the water, the temperature increases appreciably, the specific volume increases slightly, and the pressure remains constant. When the temperature reaches 99.6°C, additional heat transfer results in a change of phase, as indicated in Fig. 1*b*. That is, some of the liquid becomes vapor, and during this process both the temperature and pressure remain constant, but the specific volume increases considerably. When the last drop of liquid has

vaporized, further transfer of heat results in an increase in both the temperature and specific volume of the vapor, as shown in Fig.1.c.



Figure (1) constant pressure change from a liquid to a vapor.

-The Phase Diagram :-

As illustrated by Fig.(2), when the surface is projected in this way (Pressure-Temperature Plane), The two phase regions represented by lines. A point on any of these lines represents all two phase mixtures that particular temperature and pressure.





The term saturation temperature designates the temperature at which phase change takes place at a given pressure , and this pressure is called the saturation pressure for the given temperature.

The line representing the two phase solid liquid region on the phase diagram, slopes to the left for water that expand on freezing and to the right for those that contract.

P-V Diagram :-

The appearance of constant temperature lines (isotherms) ,By inspection of Fig(3) ,it can be seen that for any specified temperature lines less than the critical temperature ,pressure remains constant as the two phase liquid –vapor region is traversed ,but in the single phase liquid and vapor regions the pressure decreases at fixed temp. is specific volume increases .for temperature greater than or equal to the critical temperature ,pressure decreases continuously at fixed temperature at specific volume increases .There is no puss age across the two phase liquid –vapor region.

T-V diagram:-

The T-V diagram is often convenient for problem ,The appearance of constant pressure lines (is 0 bars) .For pressure less than the critical pressure ,such as the 10 MPa is 0 bars on Fig.(2), the pressure remains constant with temperature as the two phase region is traversed .In the single phase liquid and vapor regions ,The temp. increases at fixed pressure as the specific volume increases .For pressure greater than or equal to the critical pressure , such as the one marked 30 Mpa on Fig(4).



-**Quality** :- the fraction of the total mass of a mixture that is in vapor phase.

-<u>Saturated liquid</u> :- pure liquid which may be equilibrium with vapor.

-<u>Saturated vapor</u> :- pure vapor which may be equilibrium with liquid.

-<u>Steam tables</u> :-

Steam is widely used in power stations, steam tables listed according to the in dependent variables Temp.(T) & Pressure (P) ,for each state at a

set T&P the table contains values for four variables: specific volume (v), internal energy (u), specific enthalpy (h) and specific entropy.

Suffix (f) represent to saturated liquid (v_f , u_f , h_f , s_f)

Suffix (g) represent to saturated vapor (V_g , u_g , h_g , s_g)

-<u>Properties of wet vapor</u> :- in a wet vapor region pressure and Temp. are dependent variable and dryness fraction (quality) are need.

dryness factor(x) = $\frac{mg}{mg+mf}$

Where :-

mg=mass of vapor

m_f= mass of liquid

-for specific volume of wet vapor

$$V_{m=} \frac{v}{m} = \frac{Vf+Vg}{mf+mg}$$

$$= -\frac{m_f \nabla_f + m_g \nabla_g}{m_f + m_g} \qquad -\frac{\nabla_f m_f}{m_f + m_g} + \frac{m_g \nabla_g}{m_f + m_g}$$
$$= -\frac{m_f + m_g - m_g}{m_f + m_g} \nabla_f + x \nabla_{fg}$$
$$= (1 - x) \nabla_f + x \nabla_g$$

14

$$V_{m} = (1-x) V_{f} + x V_{g}$$
$$V_{m} = V_{g} - (1-x) V_{fg} = V_{f} + x V_{fg}$$
$$V_{fg} = V_{g} - V_{f}$$

Similarly for other properties

$$-$$
 um= (1- x) u_f + x ug

Um=ug-(1-x) u_{fg}= u_f+xu_{fg}

- hm=(1-x) h_f+x_{hg}

$$=h_g - (1-x)h_{fg} = h_f + x x h_{fg}$$

-Entropy (S):

Sm=(1-x) S_f+xs_g

$$=$$
sg-(1- x) sfg=sf+ x Sfg

Ex:- A vessel having a volume of 0.285 m^3 contains 1.36 kg of liquid water and water vapor in mixture at a pressure of 7 bar ,calculate

1-the volume and mass of liquid.

2-the volume & mass of vapor.

Sol.:-

 $V=0.285 \text{ m}^3$, $P_{sat.}=7 \text{ bar} (700 \text{ kpa})$

m=1.36 kg
$$V_g$$
=272.68×10⁻³ m³/kg , V_L =1.108×10⁻³ m³/kg

 $v=m_g V_g+m_L V_L$

 $0.285 = (272.68 \times 10^{-3}) m_g + (1.108 \times 10^{-3}) mL$ 1

Also $m_g + m_L = 1.36$ 2

from equ. 1 and 2 $M_g=1.045 \text{ kg}, m_L=0.314 \text{ Kg}$

 $V_{g=}m_gV_g = 1.04 \ (0.27268) = 0.2849 \ m^3$

 $V_L = m_L V_L = 0.314 (1.108 \times 10^{-3}) = 3.47 \times 10^{-4} m^3$

Ex:- Calculate the specific internal energy of steam having a pressure of 0.6 Mpa and a quality of 0.952 ?

Sol:- the state is saturation

 $u = x u_g + (1 - x) u_L$

from steam table at p=0.6 Mpa u_g =2566.2 KJ/kg

u_L=669.762 KJ/kg

u=0.95 (2566.2)+ (0.05) (669.762)

= 2471.3 KJ/Kg

Ex:- A tank contains (0.5) kg of liquid and vapor water mixture at equilibrium at 7 bar .If each of liquid and vapor occupies half volume of the tank ,find the volume of the tank and the enthalpy of the tank contains.

Sol:- at 0.7 Mpa $V_f=1.108 \times 10^{-3} \frac{m^3}{kg}$, $V_g=0.2725 \text{ m}^3/kg$ $h_f=697 \text{ KJ/kg}$, $h_g=2762 \text{ KJ/kg}$ $m_g+m_L=0.5 \dots 1$ $m_Lv_L=m_g V_g=\frac{1}{2}v$ $1.108 \times 10^{-3} m_L = 0.2725 m_g \dots 2.01$

From 1 & 2 $m_L=0.497 \text{ K}_g, m_g=2.01\times 10^{-3} \text{ k}_g$ $x = \frac{mg}{mg+mL} = 4.02\times 10^{-3}$ $V_L = m_L V_L = 0.497(1.108\times 10^{-3}) = 5.506 \times 10^{-4} \text{ m}^3$ $V_g = m_g V_g = 2.01 \times 10^{-3} (0.2725) = 5.477 \times 10^{-4} \text{ m}^3$

H=
$$xH_g$$
+ (1- x) H_L
=(4.02 ×10⁻³) (2762)+(1-4.02 ×10⁻³)(697)
= 705.3 KJ/kg

Equation of state :-

The results of certain experiments with gases at relatively low pressure led Rober Boyle to formulate a well known law :-

(⁽The pressure of gas expanding at constant temperature varies inversely to the volume ⁾⁾

$$P\alpha \xrightarrow{1}{V} PV = Const.$$

 $P_1V_1 = P_2V_2 = P_3V_3 = constant$

Charles Law :-

During a change of state of any gas in which mass and pressure remain constant, the volume varies in proportion with a absolute temperature.

$$V\alpha T$$

$$\frac{V}{T} = C$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \longrightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{Or } \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$1 \longrightarrow A = \text{Boyles law}$$

$$P_1 V_1 = P_A V_A$$

$$P_A = P_2$$

$$V_A = \frac{P_1 V_1}{P_2} \quad \dots \quad 1$$



$$A - 2$$
 Charles law

...

$$\frac{V_A}{T_A} = \frac{V_2}{T_2} \qquad T = \text{Const.}$$

$$\frac{V_A}{T_1} = \frac{V_2}{T_2} \qquad \therefore V_A = V_2 \frac{T_1}{T_2} \qquad \dots \dots 2$$

Subst. equal 2 into 1

$$V_2 \, \frac{T_1}{T_2} = \frac{P_1 V_1}{P_2}$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{const.}$$

$\frac{PV}{T}$ = Const.	
1	

Ideal Gas Law :-

By combination the results of Charle's and Boyle's laws, the relationship $(\frac{PV}{T} = \text{const.})$ is obtained.

The constant in the above equation is called $^{(\prime}$ Ideal gas const." designed by R , thus the ideal gas equation becomes :-

 $PV = \overline{R} T$

For (m) kg of gas ,multiple both sides by (m)

 $P(mv) = m\overline{R}T \quad \overline{R} = \frac{J}{Kg.k} , R = \frac{J}{mole.k}$

divided by (M.wt) molecular weight

Pv=mRT

PV=nRT

where $n = \frac{m}{M.wt}$ no.of moles

this is equation of state or ideal gas law .for low densities gases or vapors.

Real gas :-

The compressibility factor (Z) is the measure of deviation from the ideal gas behavior .

To calculate the compressibility factor ,we reduce the properties with respect to the values of the critical point

Reduced pressure :	$P_r = \frac{P}{PC}$	Pc: Critical pressure
Reduced Temperature :	$T_r = \frac{T}{TC}$	Tc: Critical Temp.

Ex:- A tank has a volume of 0.5 m³ and contains 10 kg of an ideal gas having a molecular weight of 24. The temperature is 24 °C .what is the pressure?

Sol:-

PV=nRT

$$PV = \frac{m}{M.wt} RT \longrightarrow P = \frac{MRT}{V.m.wt}$$
$$= \frac{10 \times 8.314 + (298)}{0.5 \times 24} = 2066 \text{ KPa}$$

Ex:- Calculate the specific volume of propane of 7 MPa and a temp. of 150 $^{\circ}$ C , and compare this with the specific volume given by the ideal gas equation of state ?.

For propane :-

$$T_{c}= 369.8 \text{ K}$$
 $P_{c}=4.25 \text{ MPa}$
 $R= 0.18855 \text{ KJ/kgk}$
 $T_{r}= = \frac{423}{369.8} = 1.144$
 $P_{r}=\frac{7}{4.25} = 1.647$
From the compressibility chart Z=0.523

$$V = \frac{ZRT}{P} = \frac{0.523 \times 0.18855 \times 423.0}{7000} = 0.00596 \text{ m}^3/\text{kg}$$

For ideal gas

$$V = \frac{RT}{P} = \frac{0.18855 \times 423}{7000} = 0.0114 \text{ m}^3/\text{kg}$$

EX:-

for water at 100 KPa with a quality 10 % find the volume fraction of vapor ?

This is a two phase state at a given pressure :-

Table: B.1.2 $V_f = 0.001043$ m³/kg , $V_g = 1.6940$ m³/kg fraction = $\frac{Vg}{v} = \frac{XVg}{XVg(1-x)V_f}$

0.1×1.694 0.1×1.694+0.9×0.00143

=0.9945

EX:- Determine the phase of the substance at the given state.

a-water at 100 °C , 500 KPa

P_{sat}(100 °C) =101.3 KPa

500 KPa > Psat then it is compressed liquid.

or T_{sat} (500 KPa) = 152 °C

100 °C <T_{sat}.it is compressed liquid

b-Ammonia NH_3 : at – 10 °C , 150 KPa

Table :- B.2.1 $P < P_{sat} (-10 \ ^{\circ}C) = 291 \text{ KPa}$

Super heated vapor

c- R- 12 $T=0 \ ^{\circ}C$, P= 350 KPa

Table B.3.1 $P > P_{sat} \cdot (0 \circ C) = 309 \text{ KPa}$

Compressed liquid

<u>Work</u> :- work is a form of energy , but it is energy in transit , work is not a property of a system , work is a process done by or on a system , but a system contains no work , work is defined for mechanical system as the

action of a force on an object through a distance . it equals the product of the force times the displacement (dL).

W = FdL

For a piston, the work

F=PA

∴ δ W=PAdL

AdL= dv change in volume

δW=PdV

P: pressure of the gas

A: area of the pistion

Work is a path function

P_e: external pressure acting on the system by surround.

P:internal pressure acting by the gas.

Pe= P when the process is assumed reversible frictionless (quasi- equilibrium)

 $\int_{1,A}^{2} P dv \neq \int_{2,B}^{1} P dv$ $\int_{1}^{2} \delta W = \int_{1}^{2} P dv$

Unit for work is watt $\left(\frac{J}{sec}\right)$

1 hp = 0.746 KW

Work is a path dependence relation between pressure and volume

P=f(v)



Positive value for work done by the system (+w) negative value for work done on the system.(- W).

$$W_{12} = \int_{1}^{2} dW = \int_{1}^{2} p dv$$

1-graphical solution

2-Analytical solution

$$a-PV^{n}=constant = P_1V_1^{n}=P_2V_2^{n}$$

 $\int_{1}^{2} P dV = \int_{1}^{2} \frac{const}{V^{n}} dV$ $= const \left[\frac{1}{1-n} (V^{1-n} - 1) \right]_{1}^{2}$ $= \frac{const.}{1-n} \left[V2^{1-n} - V1^{1-n} \right]$ $= \frac{P2V2 - P1V1}{1-n}$



b- $PV = const. = P_1V_1 = P_2V_2$

$$= \int_{1}^{2} P dv = P_{1} V_{1} \int_{1}^{2} \frac{dv}{v} = P_{1} V_{1} \ln \frac{V_{2}}{V_{1}}$$
$$= P_{2} V_{2} \ln \frac{V_{2}}{V_{1}}$$

Ex:- consider as a system the gas in the cylinder which is fitted with a piston on which a number of small weights are placed . the initial pressure is 200 KPa , and the initial volume of the gas is 0.04 m^3 , calculate the work for the following conditions.

a- Let a Burner be placed under the cylinder , and the volume of the gas increase to 0.1 m^3 , while pressure remains constant.

$$W_{12} = \int_{1}^{2} P dV = \int_{1}^{2} dV = P(V_2 - V_1)$$

=200(0.1-0.04)=12 KJ

b- consider the same initial conditions, but at the same time the Burner is under the cylinder and the piston is rising where the temperature remains constant .(assume ideal gas).

PV= mRT

$$W_{12} = \int_{1}^{2} P dV = P_{1} V_{1} \ln \frac{V_{2}}{V_{1}}$$
$$= 200 * 0.04 \ln \frac{0.1}{0.04} = -7.33 \text{ KJ}$$

C-the same as (b) but for $PV^{1.3} = const$ and the final volume (0.1) m³.

$$P_{1}V_{1}^{1.3} = P_{2}V_{2}^{1.3} \longrightarrow P_{2} = 200 \left(\frac{0.04}{0.1}\right)^{1.3}$$
$$= 60.77 \text{ KPa}$$
$$W_{12} = \int_{1}^{2} P dV = \frac{P_{2}V_{2} - P_{1}V_{2}}{1 - 1.3} = \frac{60.77 \times 0.1 - 200 \times 0.04}{1 - 1.3}$$
$$= 6.41 \text{ KJ}$$

<u>Heat</u> :-

Heat, like work is energy in transit .the transfer of energy as heat, however, occurs at the molecular level as a result of a temperature difference .the symbol Q is used to denote heat . A positive value for heat indicates that heat is added to the system by its surrounding, Negative value indicates that heat is rejected from the system.

$$q = \frac{Q}{m}$$

$$q = heat \text{ transferred per unit mass (J/Kg)}$$

$$Q = heat \text{ transferred (J)}$$

$$m = mass (kg)$$

$$Cp = \frac{Q}{mDT} = \frac{q}{4T}$$

...

C_P=specific heat at constant pressure (J/kg.k)

 ΔT =temperature change (k)

as
$$Q = mCp \quad \Delta T$$

For flow rate $\dot{Q} = \dot{m} \text{ Cp}\Delta T$

For heat capacity at constant volume

$$Cv = \frac{Q}{mdT} \longrightarrow Q = mCv \Delta T$$

 $\dot{Q} = mCv \Delta T$

Cv= specific heat at constant volume (J/kg.k).

Chapter Five "First Law Of Thermodynamic"

The energy can neither created or destroyed ,though it can be transferred from one form to another . This law will be state for

1-For a system under goes a cycle .

2-For a change in state of a system.

3-For a control volume.

1-For a system undergoes a cycle :-

Figure below shows a system (gas) goes through a cycle comprise W0 processes , work done on the system by rotating a paddle , and heat transfer from a system until the system returns to its initial state.

Then 1st law states that cyclic integral of the heat is proportional to the cyclic integral of the work.

 $J\int dQ = \int \delta W$

J is a proportionality factor depends on the units of heat & work in I unit both work and heat have the same unit:-

 $\int dQ = \int dW$



Fig.(5.1) The first law applied to a cycle.

Ex:- A closed system consisting of (2 kg) ammonia undergoes a cycle composed of three processes.

Process (1-2) : constant volume from P= 10 bar , X_1 = 0.6 to Saturated vapor.

Process (2-3) constant temperature, $Q_{23} = 228$ KJ

Process (3-1) constant pressure, Determine the cycle net work and the heat transfer for each process.

Sol:-State1:- P₁= 10 bar, X₁= 0.6 $u_f= 296.1 \frac{kJ}{kg}, u_g= 1334.66 \frac{kJ}{kg}$ $V_f=0.016841 \frac{m^2}{kg}, V_g= 0.1075 \frac{m^2}{kg}$

$$u_1 = \chi u_g + (1 - \chi) u_f = (0.6) (1334.66) + (0.4)(296.1)$$

 $V_1 = \chi V_g + (1 - \chi)V_f = (0.6)(0.016841) + (0.4)(0.1075)$

$$= 0.07776 \text{ m}^3/\text{kg}$$

State 2:-

Sat.vapor. $V_2 = V_2$ by interpolation

 $u_2 = 1341.26 \text{ kJ/kg}$, $T_2 = 42.52 \text{ °C}$

State 3 $P_3=P_1 = 10$ bar $T_1=T_2=42.52$ °C

at P= 10 bar T_{sat} = 24.89 °C

T (42.52°C)>T_{sat} \therefore superheated

by interpolation

$$u_3 = 1374.95 \text{ kJ/kg}$$
 and $V_3 = 0.14027 \frac{\text{m}^3}{\text{kg}}$

process 1—2: $\Delta U_{12} = Q_{12} - W_{12}$ W=0 Const. volume $Q_{12} = m(U_2 - U_1) = 2(1341.26 - 919.24) = 844$ KJ. Process 2—3: $\Delta U_{23} = Q_{23} - W_{23}$ $W_{23} = Q_{23} - m(U_3 - U_2) = 228 - 2(1374.95 - 1341.26) = 160.6$ KJ Process 3—1: $\Delta U_{31} = Q_{31} - W_{31}$ $W_{31} = mP(V_1 - V_3) = 2(10*100) (0.07776 - 0.14027) = -125$ KJ $Q_{31} = 2 (919.24 - 1374.95) - (-125) = -1036.4$ KJ $W_{cycle} = W_{12} + W_{23} + W_{31} = 0 + 160.6 + (-125) = 35.6$ KJ 2- <u>For a change in a state of system</u> :- Consider a system undergoes a cycle , in which it change from state 1 to state 2 by process A , and returns from state 2 to state 1 by process B

now consider another cycle in which the control mass change from state 1 to state 2 by process C and returns to state 1 by process B.

$$\int \delta Q = \int \delta W$$

$$\int_{1}^{2} dQ_{A} + \int_{2}^{1} dQ_{B} = \int_{1}^{2} \delta W_{A} + \int_{2}^{1} \delta W_{B} \qquad \dots \qquad (1)$$

$$\int_{1}^{2} dQ_{C} + \int_{2}^{1} dQ_{B} = \int_{1}^{2} dW_{C} + \int_{2}^{1} dW_{B} \qquad \dots \qquad (2)$$
Subtracting equation (2) from eqn.(1)
$$\int_{1}^{2} dQ_{A} + \int_{1}^{2} dQ_{C} = \int_{1}^{2} dW_{A} + \int_{1}^{2} dW_{C}$$

$$\int_{1}^{2} (dQ - dW)_{A} = \int_{1}^{2} (dQ - dW)_{C}$$

(dQ-dW) depend only on the initial and final state , and not on the path followed between the two state. Therefore it is a point function and it is a differential of a property of the system of the system this properly is the energy of the system.

dQ-dW=dE

 $dQ_{12} = dE_{12} + dW_{12}$

E= Internal energy + kinetic energy + potential energy

= u +K.E +P.E

dE=du+d(K.E)+d(P.E)

$$\int_{1}^{2} dQ = \int_{1}^{2} du + \int_{1}^{2} \left(\frac{mv^{2}}{2}\right) + \int_{1}^{2} d(mgz) + \int_{1}^{2} dw$$

$$Q_{12} = (u_2 - u_1) + \frac{1}{2}m(v_2^2 - v_1^2) + mg(z_2 - z_1) + W_{12}$$

-For K.E = 0, P.E=0

 $Q=\Delta u+W$

-for K.E= 0 , P.E=0 and W= $\int_{1}^{2} p dv$

$$\int_{1}^{2} dQ = \int_{1}^{2} du + \int_{1}^{2} p dv$$
$$Q_{12} = (u_{2} - u_{1}) + p(v_{2} - v_{1})$$

$$Q_{12} = (u_2 + pv_2) - (u_1 + pv_1)$$

H=u+pv

$$Q_{12} = H_2 - H_1 \rightarrow Q_{12} = \Delta H$$

For Ideal gas:

$$\mathbf{C}_{\mathbf{P}} = \frac{1}{m} \left(\frac{dQ}{dT} \right)_{\mathbf{P}} = \frac{1}{m} \left(\frac{dH}{dT} \right)_{\mathbf{P}}$$

 $Q_{12} = \Delta H = mCp\Delta T$

a- for P.E=0 , K.E= 0 , constant volume (isometric) (W=0)

 $\Delta u = Q$

$$\operatorname{Cv}=\frac{1}{m}(\frac{\partial Q}{\partial T})_{V}=\frac{1}{m}(\frac{du}{\partial T})_{V}$$

 $\Delta u = mCv\Delta T$

-For ideal gas Pv= RT

H=u+Pv

H=u+RT

$$\left(\frac{\partial H}{\partial T}\right)_P = \left(\frac{\partial u}{\partial T}\right)_V + R$$

Cp=Cv+R ,

(constant pressure)

b- <u>Constant Pressure Process (isobaric)</u>

$$\Delta u = Q - W$$

$$W_{12} = P \Delta V_{12}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\Delta u_{12} = Q_{12} - P \Delta V_{12}$$

$$\Delta H = Q$$
For ideal gas $\Delta H_{12} = \int_1^2 Cp dT$

c- <u>Constant Temp. Process (isothermal)</u>

$$\Delta u = Q-W$$

$$W = \int_{1}^{2} P dv$$

$$P_{1}V_{1} = P_{2}V_{2}$$

$$\Delta u_{12} = Q_{12} - \int_{1}^{2} P dv$$

For ideal gas $\Delta u_{12} = 0$ sine u = f(T) only

 $W_{12} = Q_{12} = \int_{1}^{2} \frac{nRT}{V} dv = nRT \ln \frac{V_2}{V_1} = nRT \ln \frac{P_1}{P_2}$



$$P_1V_1 = P_2V_2 = const.$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

d- Adiabatic Process (isentropic)

 $\Delta u_{12} = Q_{12} - W_{12}$ $P_{1}V_{1}^{v} = P_{2}V_{2}^{v}$ $Q_{12} = 0$ $\frac{T_{2}}{T_{1}} = \left(\frac{V_{2}}{V_{1}}\right)^{1-v}$ For ideal gas $\frac{T_{2}}{T_{1}} = \left(\frac{P_{2}}{P_{1}}\right)^{v-1/v}$ $\Delta u_{12} = C_{v}(T_{2} - T_{1})$ $\frac{V_{2}}{V_{1}} = \left(\frac{P_{2}}{P_{1}}\right)^{-1/v}$ $W_{12} = \frac{nRT_{1}}{v-1} \left[1 - \left(\frac{P_{2}}{P_{1}}\right)^{v-1/v}\right]$ Cp = Cv + R $q = \frac{P_{1}V_{1}}{V_{r-1}} \left[1 - \left(\frac{P_{2}}{P_{1}}\right)^{v-1/v}\right]$ 31



E- Polytropic Process :-

** *

The process which is not isothermal nor adiabatic

$$Q = \Delta u_{12} + W_{12}$$

$$W_{12} = \frac{P2V2 - P1V1}{1 - n}$$
For perfect gas
$$W_{12} = \frac{R(T2 - T1)}{1 - n}$$

$$\Delta u = \int Cv dT$$

$$Q = W_{12}(\frac{\gamma - n}{\gamma - 1})$$
otes:-
1-for diatomic gas (O₂, N₂, H₂)
$$Cp = \frac{7}{2} R$$



Not

1-for diatomic gas
$$(O_2, N_2, H_2)$$
 $Cp = \frac{7}{2} R$
 $Cv = \frac{5}{2} R$
2-for monatomic gas (A_r, He) $Cp = \frac{5}{2} R$
 $Cv = \frac{3}{2} R$

Ex:- when a system is taken from state (a) to a state (b) in Fig. below along path <u>abc</u>, 100 J of heat flows into the system and the system does 40 J work How much heat into the system along path aeb if the work done by the system is 20 J? the system returns from (b) to (a) along the path bda if the work done on the system is 30 J, does the system absorb or librate heat? How much?.

Sol:- nonflow system

Path I $\Delta u = Q - W$

= 100-40 = 60 J

Path II : Δu is constant is all.

Since Δu is state function

 $\Delta u = -60 J$



-60 = Q-W $-60 = Q-(-30) \rightarrow Q = -90 J[$ Heat out of the system] Path III :- $\Delta u = 60 J$, W=20 J $\Delta u = Q-W$ $60 = Q-20 \rightarrow \qquad Q=80 J$

 Q_T = 100-90+80= 90 J [system absorbs a heat]

Ex:

A closed well insulated tank contains 10 kg of air at 300 K, an electrical motor drive a propeller inside the tank. Until the temp. of air rise to 400 k. How much work had done by the motor?.

From Table	T=300) K	u ₁ = 214.364 KJ/kg
	T= 400 K	$\mathbf{u}_2 =$	286.487 KJ/kg

Sol:- it is a closed system (non flow process)

Zero $\Delta u = \sqrt{Q} - W$ $\Delta u = -W$ [Q=0 tank is insulated , no heat in or out] $\Delta u = 286.487 - 214.364 = 72.123$ KJ/kg

 \therefore W= - Δ u = -72.123 KJ/Kg

Ex:- water at 93 °C is pumped from a storage tank at a rate 50 gal/min the pump supplied work at the rate 2 hp. The water passes a heat exchanger where it gives up heat at a rate of 400,000 $\frac{Btu}{min}$ and is delivered to a second storage tank at an elevation (50ft) of soft a above the first tank, what is the temp. of water delivered to the second tank?.

Sol:- flow system

Zero

$$\Delta H + \frac{\Delta t'}{2} + g\Delta z = Q-W$$
 1m=3.28 ft
 $g\Delta z = 9.8 \text{ m/s}^{2*}50/3.28 = 149.4 \frac{m^{2}}{s^{2}}$
 $= 149.4 \text{ J/kg}$
 $= 0.1494 \text{ KJ/kg}$
 $Q = 40000 \frac{Btu}{min}$
1 Bute = 1055 J
1 Bute = 1055 J

Q=40000 $\frac{B(u)}{\min} \left(\frac{1.055 \text{ KJ}}{B+u}\right) \left(\frac{1 \min}{605}\right) = 703 \text{ KJ/S}$

Volumetric flow rate = $50 \frac{\text{gal}}{\text{min}} \times \frac{1 \text{m}^3}{264 \text{ gallon}} \times \frac{1 \text{min}}{60 \text{ sec.}} = 3.144 \times 10^{-3} \text{ m}^3/\text{ sec.}$

 $1 \text{ m}^3 = 265 \text{ gall}$

From steam table at 93°C $~v_f\!\!=1.03{\times}10^{\text{-3}}~m^3\!/kg$, H= 390 KJ/kg

$$\rho = 1/V_f = 970 \text{ kg/m}^3$$

Mass flow rate = 970 kg/m³* 3.144×10^{-3} m³/sec.=3.052 kg/sec.

$$Q = \frac{703}{3.052} = -230 \text{ KJ/kg}$$

الاشارة السالبة تعني ان الحرارة خارجية من النظام.

$$W = 2hp \times \frac{745 \text{ kw}}{hp} = 1.496 \frac{\text{kJ}}{\text{sec.}}$$

$$W = \frac{1.496}{3.052} = -0.488 \frac{kJ}{kg}$$

 $\Delta H = Q - W - g \Delta z = 230 - (-0.488) - 0.144$

ΔH=- 230.6 KJ/kg

 $\Delta H = H_2 - H_1 \rightarrow H_2 = \Delta H + H_1 = -230.6 + 390 = 159.3 \frac{KJ}{kg}$

From steam tables at H= 159.3 $\frac{\text{KJ}}{\text{kg}} \rightarrow \text{T}=38 \text{ °C}$

Ex:- A vessel having a volume of 5 m^3 containing 0.05 m^3 of sat. liquid & 4.95 m^3 of sat .water vapor at 0.1 MPa. Heat transfered until the vessel is filled with sat. vapor Determine the heat transfer to the process.

Sol:- initial state : Two phase

From steam table at 0.1 MPa

 $\text{V}_{g}\text{=}1693.7{\times}10^{\text{-3}}\text{ m}^{3}\text{/kg}\text{, V}_{f}\text{=}1.043{\times}10^{\text{-3}}\text{ m}^{3}\text{ /kg}$

 $u_g\!=\!2506.1~\text{KJ/kg}$, $u_f\!\!=417.7~\text{KJ/kg}$

$$m_{g} = \frac{V_{g}}{V_{g}} = \frac{4.95}{1.6937} = 2.922 \text{ kg}, m_{f} = \frac{V_{L}}{V_{L}} = \frac{0.05}{1.043 \times 10^{-3}} = 47.93 \text{ kg}$$
$$X = \frac{m_{g}}{m_{g} + m_{L}} = \frac{2.922}{2.922 + 47.93} = 0.05746 \quad , \quad u_{1} = xu_{g} + (1 - x) u_{f}$$

State 2 Sat. vapor

$$V = \frac{V}{m} = \frac{5}{2.922 + 47.93} = 0.0983 \text{ m}^3/\text{kg}$$

From steam table at $V = 0.0983 \text{ m}^3/\text{kg} \rightarrow u_2=2597 \text{ KJ/kg}$

 $\Delta u = Q - W$ zero (no change in volume)

 $Q = \Delta u = -537.4 + 2597 = 2059.5 \text{ KJ/kg}$

T=300 k u_1 = 1.41 kJ/kg T=400 k u_2 =72.93 Kj/kg

 $\Delta u = Q - W$

Q=0 closed (isolated system)

 $\Delta u = -W$

 $\Delta u = u_2 - u_1 = 72.93 - 1.41 = 71.52 \text{ KJ/kg}$

 $W = -\Delta u = -71.52 \text{ KJ/kg}$

Ex:- A vessel having a volume of 100 ft³ contains 1 ft³ of Saturated liquid water and 99 ft³ of saturated vapor at 14.7 1bf/ in² Heat is transferred until the vessel is filled with saturated vapor .Determine the heat transfer for this process?

$$Q_{12} = u_2 - u_1$$

at 14.7 lbf/in ² $V_f = 0.01672 \text{ ft}^3 / 1 \text{ bm}$, $V_g = 26.8 \text{ lft}^3 / 1 \text{ bm}$ $u_f = 180.1 \text{ Btu} / 1 \text{ bm}$, $u_g = 1077.6 \text{ Btu} / 1 \text{ bm}$ $m_f = \frac{V_f}{V_f} = \frac{1}{0.01672} = 59.81 \text{ lbm}$ $m_g = \frac{V_g}{V_g} = \frac{99}{26.80} = 369 \text{ lbm}$

36

 $u_1 = m_f u_f + m_g u_g = 59.81(180.1) + 3.69(10776)$

at state 2 $m=m_f+m_g = 59.81+3.69= 63.5$ 1bm

$$V = \frac{V}{m} = \frac{100}{63.5} = 1.575 \ ft^3 / \ 1bm$$

by interpolation , $P=294 \ 1bf/in^2$
$u_2 = 1117 \text{ Btu}/1\text{bm}$ $u_2 = mu_2 = 63.5 (1117) = 70930 \text{ Btu}$ $Q = u_2 - u_1 = 70930 - 14748$ = 56182 Btu

Ex: A cylinder fitted with a piston has an initial volume of 0.1 m^3 and contains a nitrogen at 150 KPa, 25 °C , the piston is moved , compressing the nitrogen until the pressure is 1 MPa and the temperature is 150 °C .During this compression process heat is transferred from the nitrogen ,and the work done on the nitergon is 20 KJ. Determine the amount of this heat transfer.

Q=
$$\Delta u$$
+W
Q=mCv(T₂-T₁) +w
 $m = \frac{PV}{RT} = \frac{150 \times 0.1}{(0.2968)(298.15)} = 0.1695 \text{ kg}$

Cv at $\frac{T_2 + T_1}{2} = \frac{150 + 25}{2} = \frac{175}{2} = 87.5$ °C

by interpolation with table

Ex:- one Kilogram of air heated reversibility at constant pressure from an initial state of 300 k and 1bar until its volume triples calculate W,Q, Δu and ΔH for the process. Assume that air obeys the relation Pv/T =83.14 bar cm³ mole⁻¹ k⁻¹ and that Cp= 29 J/mole.k.

Sol:

m=1 kg , T₁= 300 K , P₁= 1 bar n= $\frac{1}{29}$ (1000) =34 mole

$$Pv = nRT \rightarrow V_{1} = \frac{nRT}{p} = \frac{(34)(83.14)(300)}{1} \times 10^{-6}$$
$$= 0.848 \text{ m}^{3}$$
$$\frac{V_{1}}{V_{2}} = \frac{T_{1}}{T_{2}} = 3 + \frac{V_{1}}{3V_{1}} = \frac{300}{T_{2}} = 3 + T_{2} = 900 \text{ K}$$
$$Q = \Delta H = \int nCp dT = 34(29)(900 - 300) = 591600 \text{ J}$$
$$\Delta u = Q - W \quad , W = \int p dv = 100(2.5 = 0.848)$$
$$= 165.2 \text{ KJ}$$
$$\Delta u = Q - W = 591.6 - 165.2 = 426.4 \text{ kJ}$$

 $\Delta H = \Delta u + P \Delta V$

Chapter Six: First Law Analysis for a control volume :

-Conservation of mass :-

assume the following for a control volume :-

1-some fluid inter the (C.V.) through the inlet pipe at section $1 \dots 1$ (δmi)

2-some fluid leaves the control volume (C.V.) through the outlet pipe at section 2....2 δme

3- the mass contained in the (C.V.) change from m_t to $m_t+\delta_t$





$$\begin{split} \delta_{mi} - \delta m_e &= m_{t+\delta t} - m_t \\ (m_{t+\delta t} - m_t) + (\delta m_e - \delta m_i) &= 0 \\ (\frac{m_t + \delta_t - m_t}{\delta t}) + \frac{\delta m_t}{\delta t} - \frac{\delta m_i}{\delta t} &= 0 \end{split}$$

take the unit with $\delta t \rightarrow 0$

$$\frac{dm_{c.v.}}{dt} + \varepsilon me^{\cdot} - \varepsilon m^{\cdot} i = 0$$
$$m^{\cdot} = \rho A V_{av.}, \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

 $Q = A V_{av} is the volumetric flow rate \left(\frac{m^{3}}{sec.}\right)$ $V_{av} = average flow velocity (m/sec.)$ for steady state $(dm_{c,v}/dt) = 0$ i.e. $\dot{m}_{1} = \dot{m}_{e}$ $E_{2}-E_{1}=Q_{12}-W_{12} \text{ for fixed mass (closed system)}$ $\frac{dE_{c,v}}{dt} = \dot{Q} - \dot{W}$ $\dot{W}_{flow} = pdv = p\dot{m} (V)$ $\frac{dE_{c,v}}{dt} = \dot{Q}_{c,v} - \dot{W}_{c,v} + m_{i}e_{i} - \dot{m}_{e}e_{e} + \dot{W}_{flowin}$ $= \dot{Q}_{c,v} - \dot{W}_{c,v} + \dot{m}_{i}(e_{i} + p_{i} V_{i}) - \dot{m}_{e}(e_{e} + p_{e} V_{e})$ $e + pV = u + pV + gz + \frac{1}{2}v^{2} = h + \frac{1}{2}v^{2} + gz$ $\frac{dE_{c,v}}{dt} = \dot{Q}_{c,v} - \dot{W}_{c,v} + \dot{m}_{i}(hi + \frac{1}{2}Vi^{2} + gzi) - \dot{m}_{e} (he + \frac{1}{2}Ve^{2} + gze)$

this form of energy equation which work term is the sum of all shaft work terms and boundary work terms.

- For several entering or leaving mass flow rates :

$$\frac{dE_{C,V}}{dt} = \dot{Q}_{C,V} - \dot{W}_{C,V} + \varepsilon mi (hi + \frac{1}{2}Vi^2 + gzi) - (he + \frac{1}{2}Ve^2 + gze)$$
$$E_{C,V}^J = \rho e^{J/kg} v = me = m_A e_A + m_B e_{B+\cdots}.$$

-For steady state :- $\dot{m}_i = \dot{m}_e = \dot{m}$

$$\frac{dE_{C.V}}{dt} = 0$$
 and assume K.E and P.E= 0

 $\dot{m}(\Delta H) = \dot{Q} - \dot{W}$

If we neglect $K_E \& P_E$

 $dQ+dm_i(Hi) = dw+dm_0[H_0] + d [mu]$

 $d(mu) + dm_0(H_0) - dm_i(H_i) = dQ - dw$

a- For closed system

 $dm_0 = 0$

dmi=0

d(mu) = dQ - dW

 $\Delta u = Q-W \quad 1^{st}$ law for closed system.

b- For open system (steady state)

 $d(mu) + dm_0(H_0) - dmi(Hi) = dQ - dw$

d(mu)=0

 $m_0H_0-m_iH_i=Q-W$

 $\Delta H= Q-W$ first law for open system.

Ex:- steam at a pressure of 1.4 MPa and 300 °C is flowing in a pipe connected to this pipe through a value is an evacuated tank, the value is open and the tank is fill with steam until the pressure is 1.4 MPa and the value is closed .The process take place A adiabatically. Neglect P_E and K_E .Determine the final temp. of the steam.

Sol:-

A adiabatic process dQ=0

No change in volume dW=0

No exit system $\rightarrow dm_0 = 0$

Neglect kinetic and potential energy

$$\rightarrow \frac{v^2}{2} = 0 , gz = 0
d[mu + \frac{mv^2}{2} + mgz] + dm_0 [H_0 + \frac{v^0}{2} + gz_0] - dm_i [H_i + \frac{vi^2}{2} + gz_i] = dQ - dw
d[mu] - dmiHi = 0
m_2u_2 - m_1u_1(m_iH_i)
m_1 = 0 (tank evacuated)
m_2u_2 = m_iH_i m_i = m_2
u_2 = H_i from steam table at 300 °C & & 1.4 MPa$$

 $u_2 = H_i = 3041.6 \text{ KJ/kg}$

 \rightarrow at H= 3.41.6 & 14 MPa \rightarrow T₂= 458 °C

Ex:- the volume of a certain vessel is 2 m^3 contains steam at a pressure of 20 bar and a temp. at $127^{\circ}C$, the vessel is heated at constant pressure until its temp. reached $227^{\circ}C$. Notes that the vessel is provided with relief value to keep the pressure constant at 20 bar. calculate the heat required in kJ units for this process.

At 2 MPa,127 °C	At 2 MPa,227 °C
Superheated	superheated steam
$V = 151.13 \times 10^{-3} \frac{m^2}{kg}$	V =175.55 m ³ /kg
u=2945.4 KJ/kg	u=3116.2 kJ/kg
H=3248.7 KJ/kg	H= 3467.3 KJ/kg
Sol	

Sol:-

 $d(mu)+dm_0(H_0)-dm_i(H_i)=dQ-dw$

 $d(mu)+dm_0(H_0)=dQ$

 m_2u_2 - m_1u_1 + m_0H_0 =Q

 $m_1 - m_2 = m_0$

<u>initial</u> superheated $T = 127^{\circ}C$ P = 2 MPa

m =
$$=\frac{V}{V}\frac{2}{151.13 \times 10^{-3}}$$
 = 13.23 kg , u₁ = 2945.5 kJ/kg

<u>final state</u> superheated (T=227°C, P=2 MPa)

$$m_2 = \frac{V}{V} = \frac{2}{175.55 \times 10^{-3}} = 11.392 \text{ kg}$$

 $u_2 = 3116.2 \text{ kJ/kg}$

 $m_0 = m_1 - m_2 = 13.23 - 11.392 = 1.68 \text{ kg}$

 $Q = m_2 u_2 - m_1 u_1 + m_0 H_0$

= 11.392(3116.2) - 13.23(2945.5) + 1.838(3467.3)

= 2905 KJ

Example of steady state Flow processes:-

-<u>Heat Exchanger :-</u>

A steady state flow heat exchanger is a simple fluid flow through a pipe or a system of pipes, where heat is transferred to or from the fluid the fluid may be heated or cooled, and may or may not boil, liquid to vapor, or condense, vapor to liquid.

Ex:- consider a water –cooled condenser in a large refrigeration system in which R-134a is the refrigerant fluid, the refrigerant enters the condenser at 1.0 MPa , 60 °C , at the rate of 0.2 kg/sec. and exist as a liquid at 0.95 MPa, 35°C . Cooling water enters the consider at 10 °C and exist at 20 °C . Determine the rate at which cooling water flows through the condenser .

Sol:-

εṁihi = εmėhe

 $\dot{m}i(hi)_r + \dot{m}w(hi)_w = \dot{m}r(he)_r + \dot{m}w(he)_w$



P=0.95 MPa = 9.5 bar, T= 35 °C (he)_r= 98.78 $\frac{kJ}{kg}$

$$\dot{mw} = \frac{\dot{m}_r(hi)_r - (he)_r}{(he - hi)_W} = (0.2) \frac{(291.36 - 98 - 78)}{(83.95 - 42.01)} = (0.2) \frac{192.58}{41.94}$$

= 0.918 kg/sec.

$$\dot{Q}_{c,v} = \dot{m}_r (he - hi)_r = (0.2)(98.78-291.36) = -38.5 \text{ KJ/sec.}$$

 $\dot{Q}_{c,v} = \dot{m}_w (he - hi)_w = 0.98 (83.95-42.01) = -38.5 \text{ KJ/sec.}$

Turbine (Expander) :-

A turbine is a rotary machine whose purpose is the production of shaft work at the expense of the working fluid. The Kinetic energy that converts into shaft work when the stream impinges on blades attached to a rotating shaft .Thus a turbine consists of alternative sets of nozzles and rotating blades, when a steam is the working fluid, the device called a turbine, when a high pressure gas used called an expander.

Ex:- the mass rate of flow into a steam turbine is 1.5 kg/sec., and the heat transfer from the turbine is 8.5 KW. The following data are known for the steam entering and leaving the turbine. Determine the output power from the turbine.

	<u>In let</u>	out let
Р	2.0 MPa 0	.1 MPa
Т	350 °C	
Х		100 %
٧	50 m/sec.	100 m/sec.
Ζ	6 m	3 m
	$**\frac{kJ}{kg} = 1000 \frac{m}{se}$	1 ² 2C.

Sol:-

$$\dot{Q} + \dot{m}(hi + \frac{Vi^2}{2} + gzi) = \dot{m}(he + \frac{Ve^2}{2} + gze) + \dot{W}_{c.v.}$$

$$\frac{Vi^2}{2} = \frac{(50)^2}{2 \times 1000} = 1.25 \text{ KJ/kg} , gz_i = \frac{6*9.8}{1000} = 0.059 \text{ KJ/kg}$$

$$\frac{Ve^2}{2} = \frac{(100)^2}{2 \times 1000} = 5.01 \text{ KJ/kg} , gz_e = \frac{3 \times 9.8}{1000} = 0.029 \text{ KJ/kg}$$
From steam table:- h_i= 3137 kJ/kg , h_e = 2675.5 kJ/kg
-8.5+1.5(3137+1.25+0.059) = 1.5(26755+5.01+0.029) + \dot{W}_{c.v}
$$\dot{W}_{c.v.} = 678.2 \text{ KJ/s}(\text{KW})$$

-<u>Nozzle :-</u>

A nozzle is a steady state flow device whose purpose is to create a high velocity fluid stream at the expense of its pressure. It is contoured in an appropriate manner to expand a flowing fluid smoothly to a lower pressure, there by increasing its velocity.

Ex:- steam at 0.6 MPa , 200 °C enters an inuslated nozzle with a velocity of 50 m/sec. It leaves at a pressure of 0.15 MPa and a velocity of 600 m/sec. Determine the final temperature if the steam is superheated in the final state, and the quality if it is saturated.

Slo:-



= 2671.4 kJ/kg

And if it is saturated. P=0.15 MPa by interpolation

$$h_f = 221.615 \text{ kJ/kg}$$
, $hg = 2597.2 \text{ kJ/kg}$

$$he = \boldsymbol{\chi} hg + (1 - \boldsymbol{\chi})h_f$$

 $2671.4 = \chi$ (hg) + (1-x)h_f

- Diffuser :-

Ex:- for a steady state flow diffuser with a data provided below, Determine the ratio of exit flow area to the inlet flow area and the exit temperature?

$$\begin{split} h_2 &= h_1 + \frac{V1^2}{2} - \frac{V2^2}{2} \\ h_2 &= 300.19 + \left(\frac{250^2 - (140)^2}{2}\right) \frac{1 \ kJ}{1000 \ N.m} \\ &= 321.64 \ KJ/kg \\ then by interpolation , h_2 &= 321.64 \ KJ/kg \\ T_2 &= 321.3 \ K \end{split}$$

$$\rho_{2}A_{2}V_{2} = \rho_{1}A_{1}V_{1}$$

$$\left(\frac{P_{2} M.W}{RT_{2}}\right)A_{2}V_{2} = \left(\frac{P_{1} M.W}{RT_{1}}\right)A_{1}V_{1}$$

$$\frac{A_{2}}{A_{1}} = \frac{P_{1}}{P_{2}}\frac{T_{2}}{T_{1}}\frac{V_{1}}{V_{2}}$$

$$= \left(\frac{1}{1.13}\right) \left(\frac{321.2}{3001}\right) \left(\frac{250}{140}\right)$$

$$\frac{A_{2}}{A_{1}} = 10692$$

Compressor / Pump

The purpose of a compressor or pump (liquid) is the same, to increase the pressure of a fluid in a shaft work (power), the internal processes are essentially the opposite of the two processes occurring in side a turbine.

Ex:-The centrifugal air compressor of a gas turbine receiver air from the ambient atmosphere where the pressure is 1 bar and Temp. is 300 k, At the discharge of the compressor P=4 bar, T=483 k, V=100 m/sec. the mass flow rate into the compressor is 15 kg/s. Determine the power required to drive the compressor.

Sol:-

$$\dot{mh}_{i} = \dot{mh}_{e} + \frac{Ve^{2}}{2} + \dot{W}_{C.V}$$
$$-\dot{W}_{C.V} = \dot{m} [(h_{e} - h_{i}) + \frac{Ve^{2}}{2}]$$
$$= \dot{m} [Cp(T_{e} - T_{i}) + \frac{Ve^{2}}{2}]$$
$$= 15 [1.0035 (480 - 300) + \frac{(100)^{2}}{2 \times 1000}]$$
$$= 2784 \text{ KW}$$

Throttle:-

A throttling process occurs when a fluid flowing in a line suddenly encounters a restriction in the flow passage .This may be a plate with a small role in it or it may be a partially valve producing into the flow passage , or it may be change to a much smaller diameter tube (capillary tube).

Assuming a throttle to be a constant –enthalpy process leads us to define a property called the Joule- Thomson Coefficient m μ_i



Throttling devices. (a) Orifice plate. (b) Globe value

Ex:- steam at 800 KPa ,300 °C is throttled to 200 KPa .change in Kinetic energy are negligible for this process .Determine the final temperature of the steam and the average Joule Thomson coefficient.

Sol:-

 $\dot{m}\Delta H = \dot{Q} + \dot{W}$ hi=he at P= 800 KPa , 300 °C by interpolation since :- he= hi= 3056.5 kJ/kg he= 3056.5 kJ/kg , P= 200 KPa by interpolation $T_e= 292.4$ °C $\mu_J = (\frac{\Delta T}{\Delta P})_h = \frac{-7.6}{-600} = 0.0127$ K/KPa

Ex:- A tank of 50 ft³ volume contains saturated a ammonia at a pressure of 200 1 bf/in with 50 % vapor by volume .vapor is with drawn from the top of the tank until the pressure is 100 1 bf/ in .Assuming that only vapor (no liquid) leaves, adiabatic process.Determine the mass of ammonia that is withdrawn.

Sol:-

$$Q_{c.v.} = 0 , W_{c.v.} = 0 , m_i = 0 , K.E\&P.E=0$$

$$m_e h_e + m_2 u_2 - m_1 u_1 = 01$$

$$(m_2 - m_1)_{c.v.} + m_e = 02$$

$$(m_2 - m_1)_{be+} m_2 u_2 - m_1 u_1 = 0$$

$$m_2(h_e - u_2) = m_1(h_e - u_1)3$$
at 200 1 bf /in² , 100 1 bf/in
$$V_{f1} = 0.02732 \frac{ft^3}{1b} V_{f2} = 0.02564 \text{ ft}^3/1b$$

$$V_{g1} = 1.501 \frac{ft^2}{1b} V_{g2} = 2.9497 \text{ ft}^3/1b$$

Second Law Of Thermodynamics :-

Two alternative statements of the second law are frequently used in engineering thermodynamic ,They are the Claudius and Kelvin- planck statements.

The Claudius statements of the second law asserts that :It is impossible for any system to operate in such a way the sole result would be an energy transfer by heat from a cooler to a hotter body.

The Kelvin- Planck statement, It is impossible for any system to operate in a thermodynamic cycle and deliver a net amount of energy by work to its surrounding while receiving energy by heat transfer from a single thermal reservoir.



Irreversible and reversible processes :-

A process called irreversible if the system and all parts of its surrounding cannot be exactly restored to their respective initial states after the process has occurred .A process is reversible if both the system and surrounding can be returned to their initial states.

Irreversibility processes normally include one or more of the following Irreversibilities :-

- Heat transfer through a finite temperature difference.

-unrestrained expansion of a gas or liquid to a lower pressure .

-Spontaneous chemical reaction.

-Friction –sliding friction as well as friction in the flow of fluids.

-Electrical current flow through are assistance.

Heat Engine :-

Is defined simply as a system operating in a cycle and producing useful work by abstracting heat from a suitable heat source η .



 η (Heat engine thermal efficiency)= $\frac{\text{net work output}}{\text{Total heat input}}$

$$=\frac{Q_H Q_R}{Q_H}=1-\frac{Q_R}{Q_H}$$

Heat Pump and Refrigerator :-

It is a device that required network in order to transfer heat from low temperature to a high temperature reservoir while operating in a thermodynamic cycle.



Coefficient of performance $= \frac{Desired \ output}{Required \ input}$

For refrigeration Coefficient Performance (C.O.P) $_R$

$$(C.O.P)_{R} = \frac{Cooling \ demand \ (Refrigeration \ effect)}{work \ input} = \frac{Q_{R}}{W} = \frac{Q_{R}}{Q_{H} - Q_{R}}$$

For a heat pump coefficient of performance :-

$$(C.O.P)_{hp} = \frac{heating \ demand}{work \ input} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_R}$$

$$(C.O.P)_{hp} = \frac{Q_H}{Q_H - Q_R}$$

The claim cannot be valid ,since the thermal efficiency of the cycle exceeds the maximum theoretical cycle value.

Ex:- By steadily circulating a refrigerant at low temperature through passage in the walls of the freezer compartment .the rate of heat transfer from the freezer to the refrigerant is 8000 kJ/h and the power input required to operate the refrigerant is 3200 kJ/h.Determine the coefficient of performance of the refrigerant and compare with the coefficient of performance of reversible refrigerant cycle operating between the reservoirs at the same temperatures.

$$(C.OP.) = \frac{Q_R}{W_{Cycle}} = \frac{8000}{3200} = 2.5$$
$$(C.OP.)_{max} = \frac{T_R}{T_H - T_R} = \frac{268}{295 - 268} = 9.9$$

-The Carnot Cycle:-

Carnot recognized that conditions for converting heat into work with maximum efficiency were as follows:-



a- The cycle must consist of a series of processes each of which is fully reversible.

b- All heat reception occurs at the maximum cycle temperature and all heat rejection occurs at the minimum cycle temperature.

The carnot cycle of four reversible processes ,Two isothermal and two adiabatic .

1-2: isothermal expansion at $T = T_{max}$.

2-3: Reversible adiabatic expansion :

$$\frac{P_2}{P_3} = (\frac{T_2}{T_3})^{v/v-1} = (\frac{T_{max}}{T_{min}})^{v/v-1} \dots (2)$$
$$\frac{V_2}{V_3} = (\frac{T_2}{T_3})^{-1/v-1} = (\frac{T_{max}}{T_{min}})^{-1/v-1} \dots (3)$$

3-4: isothermal compression at T=Time

$$Q_{R} = \mathrm{RT}_{\min} \ln \frac{\mathrm{V}_{2}}{\mathrm{V}_{4}} \dots \dots \dots (4)$$

$$\frac{P_{1}}{P_{4}} = (\frac{T_{1}}{T_{4}})^{\sqrt[n]{y-1}} = (\frac{T_{max}}{T_{min}})^{\sqrt[n]{y-1}} \dots \dots (5)$$

$$\frac{\mathrm{V}_{1}}{\mathrm{V}_{2}} = (\frac{T_{1}}{T_{2}})^{-1/\sqrt[n]{y-1}} = (\frac{T_{max}}{T_{min}})^{-1/\sqrt[n]{y-1}} \dots \dots (6)$$

From equal (3) & (6)

$$\frac{\mathbf{V}_1}{\mathbf{V}_4} = \frac{\mathbf{V}_2}{\mathbf{V}_3} \qquad \dots \dots (7)$$
$$\therefore \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathbf{V}_3}{\mathbf{V}_4}$$

Subst.equ (7) into equ(4)

$$Q_R = \operatorname{RTim} \ln \frac{V_2}{V_1}$$
$$\eta = 1 - \frac{Q_R}{Q_H} = 1 - \frac{\operatorname{RTmin} \ln \frac{V_2}{V_1}}{\operatorname{RTmax} \ln \frac{V_2}{V_1}},$$



Temperature must be in Kelvin or (\dot{R})

-Entropy

The inequality of clausius:-

Consider the two reversible and irreversible heat operating between the same high and low temperature reservoirs.



 $\eta_{\rm th} = 1 - \frac{Q_R}{Q_H}$ for any engine

For a reversible heat engine

$$\eta_{\rm th} = 1 - \frac{T_R}{T_H} , \ W_{rev} = Q_H - Q_R$$
$$Q_R \quad T_R \qquad Q_H \quad Q_R$$

 $\frac{c_R}{Q_H} = \frac{c_R}{T_H} \longrightarrow \frac{c_H}{T_H} = \frac{c_R}{T_R}$

 $\frac{Q_H}{T_H} - \frac{Q_R}{T_R} = 0$ for a reversible heat engine.

$$\oint_{H}^{R} \left(\frac{\delta Q}{T}\right)_{\text{rev.}} = \frac{1}{TH} \oint \delta Q_{H} - \frac{1}{TR} \oint \delta Q_{R} = \frac{Q_{H}}{T_{H}} - \frac{Q_{R}}{T_{R}} = 0$$

$$\therefore \oint (\frac{s_Q}{T})_{\text{rev}} = 0 \dots *$$

For an irreversible heat engine :- $W_{irr} < W_{rev}$.

$$(Q_H - Q_{\tilde{R}})_{irr} < (Q_H - Q_R)_{rev.}$$

.. .

$$\therefore Q_{\overline{R}} > Q_{R} , Q_{\overline{R}} = Q_{R} + Q_{diff}$$

$$Q_{\overline{R}} - Q_{R} = Q_{diff}$$

$$\oint (\frac{dQ}{T})_{irr} = \frac{Q_{H}}{T_{H}} - \frac{Q_{\overline{R}}}{T_{R}} = \frac{Q_{H}}{T_{H}} - \frac{Q_{R}}{T_{R}} - \frac{Q_{diff}}{T_{R}}$$

$$= 0 - \frac{Q_{diff}}{T_{R}} = - \frac{Q_{diff}}{T_{R}}$$

$$\therefore \oint (\frac{\delta Q}{T})_{irr.} < 0 \dots * *$$

Combining the results of two equations (*) & (**) to obtain Clausies in equality :-

reversible	= 0		c.dO.
(limiting)		$\left(\frac{\pi}{T}\right) \leq 0$ for any cycle	$\oint(\frac{\pi q}{T})$
irreversible	<0		
(possible)			
(impossible)	>0		

Entropy :- It is a state property described the relative disorder of the motion of the molecules of a substance .As the system disorder increased , the entropy of the system increased .on the other hand a minimum value (may by zero) for entropy if the system ordered completely .Boltzman suggests the following formula to evaluate entropy as :-

 $S = K \ln (\mu) = S_0$

K: Boltzman constant , μ : thermodynamic property

Let a system (control mass) under goes a reversible process from state (1) to a state (2) along a path (A), and let the cycle be completely along path (B) in a reversible process .



$$\oint \frac{\delta Q}{T} = 0 = \oint_1^2 \left(\frac{\delta Q}{T}\right)_A + \oint_2^1 \left(\frac{\delta Q}{T}\right)_B \dots \dots \dots \dots \dots (1)$$

Now consider another reversible cycle ,which proceeds along path (c) and completed along.

Path (B)

$$\oint \left(\frac{\delta Q}{T}\right) = 0 = \int_1^2 \left(\frac{\delta Q}{T}\right)_C + \int_2^1 \left(\frac{\delta Q}{T}\right)_B$$

From the two equations (1,2), we have

$$\int_{1}^{2} \left(\frac{\delta Q}{T}\right)_{A} = \int_{1}^{2} \left(\frac{\delta Q}{T}\right)_{C}$$

We conclude that this quantity is independent of the path and it is a function of the end states only, it is a property called ((entropy)) (S).

$$dS = \left(\frac{\delta Q}{T}\right)_{rev.} \qquad \text{extensive property}$$
$$s_2 - s_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev.} \qquad (kJ/kg.k)$$
$$\Delta s = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev.}$$

Evaluation of Entropy :-

For a reversible process :-

$$\int ds = \int (\frac{\delta Q}{T})_{\rm rev.}$$

Or
$$\Delta s = \int \left(\frac{\delta Q}{T}\right)_{\text{rev.}}$$

-For closed system :-

 $\delta Q - \delta w = du$, $\delta Q = Tds \& \delta w = pdv$

Tds - pdv = du

-For open system

 $\delta Q - \delta w = du$, $\delta Q = Tds$, $\delta w = -vdv$

Tds + Vdp = dh

-For Isentropic process (reversible adiabatic)

Constant entropy (S=cons.)

 $\Delta s = 0$, Q=0 & Pv= const.

-For a system at T_1 with surrounding at T_2

With $T_1 > T_2$

 $\Delta s_{suy.} = \frac{-Q}{T_1}$ $\Delta s_{sur.} = \frac{-Q}{T_2}$



$$(\Delta S)_{\rm sys} + (\Delta S)_{\rm surv.} \geq 0$$

 $-\frac{Q}{T_1} + \frac{Q}{T_2} = \frac{Q}{T_2} - \frac{Q}{T_2} \qquad \qquad \therefore \qquad (\Delta S) \quad \text{is zero}$

The process will continue spontaneously irreversible as far

as :
$$(\Delta s)_{\text{Tot}}$$
 is + (ve)

 $(\Delta S)_{\text{Tot}} = 0$ when $T_1 = T_2$

The mathematical expression of the second law in term of entropy as property can be stated simply as :

 $(\Delta s)_{\text{Tot.}}$

-The Thermodynamic property Relation :-

We derive two important thermodynamic relations for a simple compressible substance .

$$Tds = du + pdv \dots (1)$$
$$Tds = dH - vdp \dots (2)$$

These equations are derived for a reversible process of a simple compressible substance :

 $\delta Q = du + \delta w$ $\delta Q = Tds$, $\delta W = pdv$ Tds = du + pdv *

Since Enthalpy is defined as :

H=u+ pv dH=du+pdv+vdp (**) substituting equ(**) into equ(*) Tds= dH+vdp(2)

Ex:- An insulated rigid tank contains (2 kg) of a saturated liquid –vapor mixture with quality (0.25) of water at a pressure (100 kpa) .An electrical heater inside is turned on and kept on until all the liquid vaporised .Determine the entropy change during this process ?

P= 100 kpa x = 0.25 $V_1 = xvg+(1-x)V_f = 0.25(1.9641)+(0.75)(0.001)$ $= 0.4243 \text{ m}^3/\text{kg}$ $S_1 = (0.25(S_g)+0.75(S_{f_g}))$ = (0.25)(1.3028)+0.75(6.0562) = 2.81618 kJ/kg $V_2 = V_1$ Sat.vap $S_2 = 6.8649 \text{ KJ/kg}$

 $\Delta S = m(S_2 - S_1) = 2(6.8649 - 2.8168) = 8.1 \text{ KJ/k}$

-Entropy change of an Ideal gas :-

Two very useful for competing the entropy change :-

```
Tds= du +pd∨
Tds= dH-∨dp
```

59

Mech.Eng.Dept.

$$du = CvdT$$

$$ds = Cv\frac{dT}{T} + \frac{P}{T} dV \qquad \frac{P}{T} = \frac{R}{V}$$

$$ds = Cv\frac{dT}{T} + R\frac{dV}{V}$$

$$S_{2} - S_{1} = \int_{1}^{2} Cv\frac{dT}{T} + R \ln \frac{V_{2}}{V_{1}}$$

With constant Cv:

$$\Delta S = \operatorname{Cv} \ln \frac{T_2}{T_1} + \operatorname{R} \ln \frac{v_2}{v_1} \dots *$$

$$Tds = dH-Vdp$$

dH=CpdT
$$\frac{V}{T} = \frac{R}{P}$$

ds = Cp $\frac{dT}{T} - \frac{V}{T}$ dp
ds = Cp $\frac{dT}{T} - R \frac{dp}{p}$
 $\Delta s = \int_{T_1}^{T_2} Cp \frac{dT}{T} - R \ln \frac{P_2}{P_1}$

With constant Cp :

$$\Delta s = \operatorname{Cp} \ln \frac{T_2}{T_1} - \operatorname{R} \ln \frac{P_2}{P_1}$$

-For isentropic (reversible adiabatic) process

$$\Delta s = 0$$
Cp $\ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_2} = 0$

$$\ln \frac{T_2}{T_1} = \frac{R}{Cp} \ln \frac{P_2}{P_1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{R/Cp} \quad \frac{R}{Cp} = \frac{Cp - Cv}{Cp} = 1 - \frac{Cv}{Cp} = 1 - (1/\gamma) = \gamma/(\gamma - 1)$$
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\gamma/(\gamma - 1)}$$

-For constant pressure process :

$$\Delta s = \operatorname{Cp} \ln \frac{T_2}{T_1}$$

-For constant volume process

$$\Delta s = \operatorname{Cv} \ln \frac{T_2}{T_1}$$

Q: Air is compressed from (90 kpa) to 400 kpa in a pistion cylinder in a reversible and isothermal manner at (293 k), Determine the entropy of air and the work done.

Sol:-

$$\Delta S = Cp \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$
$$= - (0.287) \ln \frac{400}{90} = - 0.428 \text{ kJ/kg.k}$$

For a reversible isothermal process:

$$q=T\Delta S = (293)(-0.428) = -125-4 \text{ kJ/kg}$$

 $q_{out} = 125.4 \text{ kJ/kg}$
 $d_{aut} = Q-W$
 $W=Q=125.4 \text{ kJ/kg}$

H.W:- show that for a polytrophic process ideal gas :-

PVⁿ =const. ,
$$v = \frac{Cp}{Cv}$$

$$\Delta S_{12} = \frac{R(v-n)}{(v-1)(1-n)} \ln \frac{T_2}{T_1}$$

Q1: A carnot refrigerator operates between the refrigerated space temperature (3 \square) and the environment temperature (22 \square) with (2

KW) as power consumption .Determine the rate of heat remove from the refrigerator space.

$$C.0.P_{carnot} = \frac{T_R}{T_H - T_R} = \frac{(3+273)}{(22+273) - (3+273)}$$

= 14.5
$$C.0.P_R = \frac{Q_R}{W} \implies \dot{Q}_R = \text{C.O.P}_R.W$$

= (14.5)(2)= 29 KW
= 1740 kJ/min

Q2:- An air conditioner operates steadily between the cooled space and the outdoor temperatures as show in figure below, the power consumption is (5 hp), Determine the maximum rate of heat removed from the air condition space.

$$(C.O.P)_{R.rev.} = \frac{T_R}{T_H - T_R}$$

$$= \frac{72 + 460}{(90 + 460) - (72 + 460)}$$

$$= 29.6$$

$$(C.O.P) = \frac{\dot{Q}_R}{W} = \dot{Q}_R = C.O.P.W$$

$$= (29.6)(5 \text{ hp}) \frac{42.41 \text{ Btu/min}}{1 \text{ hp}}$$

= 6277 Btu/min

Q3:A carnot heat engine used to drive a carnot refrigerator as shown in figure below ,Determine the maximum rate of heat removal from the refrigerated space and the total rate of heat rejection to the ambient air.

Sol:-

$$\mathfrak{m}_{th,max.} = 1 - \frac{T_R}{T_H}$$

$$=1- \frac{27+273}{900+273} = 0.744$$

W =
$$\mathfrak{m}_{th}\dot{Q}_H$$
 =(0.744)(800)
= 595.2 kJ/kg

For refrigerator :-

 $C.0.P_{R,rev.} = \frac{T_R}{T_H - T_R} = \frac{-5 + 273}{(27 + 273) - (-5 + 273)} = 8.37$ $\dot{Q}_{R,R} = (C.O.P)W = (8.37)(595.2) = 4982 \text{ kJ/min}$ $\dot{Q}_{R,HE} = \dot{Q}_{H,HE} - W = 800 - 595.2 = 204.8 \text{ kJ/min}$ $\dot{Q}_{H,R} = \dot{Q}_{R,R} + W = 4982 + 595.2 = 5577.2 \text{ kJ/min}$

Q4:- A (0.5 m³) rigid tank is initially filled with a saturated mixture of (R -134 a) at 200 kpa with quality (0.4), Heat is transferred from a source to the tank with temperature (35) until pressure rises to (400 kpa), Determine the entropy change of the refrigerant ,entropy change of the source and the total entropy for this process.

Sol:-

P₁= 200 KPa

$$x_1 = 0.4$$

 $= (0.4)(221.43)+(0.6)(36.69)$
 $= 110.612 \text{ kJ/kg}$
 $S_1 = xS_g+(1-x)S_f$
 $= (0.4)(0.9253)+(0.6)(0.1481)=0.17772 \frac{KJ}{kg.k}$
 $V_1 = xV_g+(1-x)V_f= (0.4)(0.0993)+(0.6)(0.7532\times10^{-3})$
 $= 0.0404 \text{ m}^3/\text{kg}$
P₂= 400 kpa
Dr. Mohammed D Salman
 $V_1 = xV_g+(1-x)V_f$

×

 $V_2 = V_1$

 $0.0404 = 0.0509x + 0.7904 \times 10^{-3}(1 - x)$

x= 0.7857 *x*= 0.7857

 $u_2 = x u_g + (1 - x) u_f = (0.7857)(231.97) + (1 - 0.7857)(61.69)$

= 195.47 kJ/kg S₂=(0.7857)(0.9145)+(1-0.7857)(0.2399)= 0.7699 kJ/kg

$$m = \frac{v}{v} = 12.37 \text{ KJ/kg}$$

 $(\Delta S)_{\text{syst.}} = m(S_2 - S_1) = 12.37(0.7699 - 0.17772) = 7.325 \frac{\kappa J}{k}$

Q_{in}= m(u₂- u₁)=12.37(195.47-110.612)= 1043.75 Kj

 $Q_{\text{source}} = -Q_{\text{in}} = -1043.75 \text{ KJ}$, $\Delta S_{\text{source}} = \frac{-Q_{\text{source}}}{T}$

$$=\frac{-1043}{308} = -3.38 \frac{kJ}{\kappa}$$
$$\Delta S_{\text{total}} = (\Delta S)_{\text{syst}} + (\Delta S)_{\text{source}} = 3.936 \text{ KJ/Kg}$$

Q5:- A cylinder is initially filled with saturated water vapor (1.2 kg) at 200 , Heat is transferred to the steam and it expands isothermally until pressure reaches (800 kpa) ,Determine the heat transfer and the work output for the process.

Sol:

$$\begin{array}{c}
 T_{1} = 200 \\
 Sat.vapor
\end{array} \\
\left. \begin{array}{c}
 u_{1} = u_{g} = 2595.3 \text{ kJ/kg} \\
 S_{1} = S_{g} = 6.4323 \text{ kJ/kg} \\
 \end{array} \\
P_{2} = 800 \text{ KPa} \\
 T_{2} = T_{1} = 200 \end{array} \right\}$$

$$u_{2} = 2631.1 \text{ kJ/kg} \\
 S_{2} = 6.8177 \text{ kJ/kg}$$

 $Q = T\Delta S = Tm(S_2 - S_1) = (473)(1.2)(6.8177 - 6.4323)$

$$= 219.9 \text{ KJ}$$

$$\Delta u = \text{Q- W}$$

$$W = \text{Q- m}(u_2 - u_1)$$

$$= 219.9 - (1.2)(2631.1 - 2595.3)$$

$$= 175.6 \text{ kJ}$$

((Ideal gas))

Q:- Oxygen gas is compressed from initial state $(25 \boxed{k}, 0.8 \frac{m^2}{kg})$ to $(287 \boxed{k}, 0.1 \frac{m^3}{kg})$, Determine the entropy change for the process. at $T_{av} = \frac{298+560}{2} = 429 \text{ K}$ $\boxed{k}Cv_{av} = 0.690 \frac{kJ}{kg.k}$ $\Delta S = Cv_{av}$. $\ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$ $= 0.69 \ln \frac{260}{298} + 0.2598 \ln \frac{0.1}{0.8}$ = -0.105 kJ/kg

Q1: A cylinder contains Nitrogen gas at 300 K, it is compressed polytrophic according to the relation ($PV^{1.3}$ = const.), Determine the entropy change of Nitrogen gas for this process.

Sol:

Cv= 0.743 kJ/kg.k , R= 0.297 kJ/kg.k

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$
 I $T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{1.3-1} = (300)(2)^{0.3} = 369.3 \text{ K}$
 $\Delta S = 0.743 \ln \frac{369.3}{300} + 0.297 \ln(0.5) = -0.0514 \frac{kJ}{K}$

- Second Law For a control volume :-

$$\frac{ds_{c.m}}{dt} = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen.}$$

Entropy change for a control mass in a

rate from

$$\frac{ds_{c.v}}{dt} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v.}}{T} + \dot{S}_{gen}$$
$$\frac{ds_{c.v.}}{dt} \ge \sum m_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v.}}{T}$$



irreversibility

when S_{gen} is the entropy generation in the process due to irreversibility occurring inside the system (control mass)

-For steady state steady flow consider :-

$$\frac{ds_{c.v.}}{dt} = 0 \quad \text{, uniform mass flow rate} \quad \dot{m}_e = \dot{m}_i = \dot{m}$$
$$\dot{m} (s_e - s_i) \quad \text{if } \Sigma \frac{\dot{q}_{c.v.}}{dt}$$

For a adiabatic process $S_e \ge S_i$

For iseatropic process $S_e = S_i$

-<u>Efficiency</u> :-

a-For reversible adiabatic efficiency of a turbine :-

first law :- $h_i + \frac{Vi^2}{2} = h_e + \frac{ve^2}{2} + W$

second law :- $S_e = S_i$

$$\eta_{T} = \frac{acheal\ turbine\ work}{isentropic\ work}$$



 $W_{isentropic} = h_i - h_{e(isentropic)} kJ/kg$ = $\dot{m}(h_i - h_{e(isen.)}) KJ/sec.$ $\eta_T = \frac{h_i - h_e}{h_i - h_{e(isen.)}}$

Ex:- steam enters a steam turbine at a pressure of 1 MPa , a temperature of 300 and a velocity of 50 m/sec. the steam leaves the turbine at a pressure of 150 KPa and a velocity of 200 m/sec.Determine the work per kilogram of steam flowing through the turbine assuming the process to be reversible and adiabatic .

First law : $h_i + \frac{Vi^2}{2} = h_e + \frac{Ve^2}{2} + W$ at P := 1 MPa , Ti= 300 $h_i = 3051.2 \text{ kJ/kg}$ S_i=7.1229 kJ/kg k isentropic (reversible adiabatic) proves , S_e = S_i P_e= 0.15 MPa & S_e = 7.1229 kJ/kg k 7.1229= 1.4336 + 5.7987 x_e $x_e = 0.9827$ $h_e = h_f + x_e h_{fg} = 467.1 + 0.9827(2226.5)$ = 2655.0 kJ/kg $\therefore W_{isent.} = 3051.2 + \frac{50\times50}{2\times1000} - 2655 - \frac{(200)^2}{2\times1000} = 377.5 \text{ kJ/kg}$

If actual work is 300 kJ/kg

$$\eta_{T} = \frac{W_{a}}{W_{S}} = \frac{300}{377.5}$$

b-compressor and pump adiabatic (isentropic) efficiency

-pump

$$\eta_{P} = \frac{isentropic \ pump \ work}{actual \ pump \ worl} = \frac{Wisent.}{Wact} = \frac{h_i - h_{e,s}}{h_i - h_e}$$

Wisentropic= $V_i(P_e - P_i) = h_e - h_i$

$$\eta_{\rm C} = \frac{isentropic work}{actual worl} = \frac{h_i - h_{e,s}}{h_i - h_e}$$

Ex:-Argon enters an adiabatic compressor with an isentropic efficiency of 80 % with condition shown in figure below ,Determine the exit temperature of argon and the work input to the compressor.

Sol:-

$$T_{2,s} = T_1 \left(\frac{P_{2,s}}{P_1} \right)^{\gamma - 1/\gamma} = 550 \left(\frac{200}{20} \right)^{0.667/1.667}$$

= 1381.9 R

K.E=
$$\frac{V2^2 - V1^2}{2} = \frac{(240)^2 - (60)^2}{2} \left(\frac{1 Btu/1bm}{25037 ft^2/sec.^2}\right)$$

= 1.08 Btu/1 bm

$$\eta_{\rm C} = \frac{Wisent.}{Wactual} = , \text{ First law } -W_{C.V.} = \operatorname{Cp}(T_e - T_i) + \frac{\operatorname{Ve}^2 - \operatorname{Vi}^2}{2}$$
$$\eta_{\rm C} = \frac{-Wis.}{-Wa} = \frac{Cp(T_e - T_i)s + \frac{Ve^2 - Vi^2}{2}}{Cp(T_e - T_i) + \frac{Ve^2 - Vi^2}{2}} \implies 0.8 = \frac{0.1253(1381.9 - 550) + 1.08}{0.1253(T_e - 550) + 1.08}$$

Now with $P_e = 0.1 \text{ MPa}$, $h_e = 2705.3 \frac{kJ}{kg}$ Superheated steam $T_e = 114.4$ $rac{k}{kg}$ Superheated steam $T_e = 114.4$ $rac{k}{kg}$ Superheated steam $S_e = 7.4373 \text{ kJ/kg}$ $W_{c.v.} = \eta_C \cdot \dot{m} (h_i - h_{e,s})$

= (0.8)(0.125)(3149.5 - 2594.3) = 55.52 KW (kJ/sec.)

Q1:- Refrigerant – 134 a enters an adiabatic compressor with an isentropic efficiency of (0.8) at a pressure (120 KPa) (saturated vapor) with (0.3 m^3 /min) and leaves at a pressure (1 MPa),Determine the exit temperature and the power input to the compressor ,assuming the process is an isentropic process.

Sol:-

- $\begin{array}{c} P_{i}=120 \text{ KPa} \\ \text{sat.vapor} \end{array} \right\} \hspace{1cm} hi=hg=236.97 \text{ kJ/kg} \\ Si=Sg=0.94779 \text{ kJ/kg.k} \\ V_{1}=Vg \ 120 \text{ kpa}=0.16212 \text{ m}^{3}/\text{kg} \\ P_{e}=1 \text{ MPa} \\ S_{2}=S_{1}=0.94779 \end{array} \right\} \hspace{1cm} h_{e,s}=281.21 \text{ kJ/kg} \\ \end{array}$
- $\eta_{\rm C} = \frac{h_{e,s} h_i}{h_e h_i} \longrightarrow \qquad h_e = h_i + (h_{e,s} h_i) / \eta_{\rm C}$ $= 236.97 + \frac{(281.21 236.97)}{0.8}$

 $P_{e}=1 \text{ MPa}$ $h_{e}=292.26 \text{ kJ/kg}$ $T_{e}=60.9$

$$\dot{m} = \frac{0.3}{V_1} = \frac{0.3m^3/sec.}{0.16212m^3/kg} = 0.0308 \text{ kg/sec.}$$

Air standard power cycle :-

This is using a gas (air) as working fluid .

1- Otto cycle :-

The air standard otto cycle is an ideal gas cycle that approximately a spark –ignition internal combustion engine. The cycle shown in Pv and T-S diagrams:-



-Process 1-2 isentropic compression PV=const.-Process 2-3 reversible constant volume heating by Q_H -Process 3-4 isentropic expansion PV=const.-Process 4-1 reversible constant volume cooling Q_R $Q_H = m_{C,V}$ $(T_3 - T_2)$, $Q_R = m_{C,V}$ $(T_4 - T_1)$ $\eta_{th.} = \frac{Q_H - Q_R}{Q_H} = 1 - \frac{Q_R}{Q_H} = 1 - \frac{mcv(T_4 - T_1)}{mcv(T_3 - T_2)}$ $= 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_2/T_2 - 1)}$

For isentropic process :-

 $\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma - 1} = (\frac{V_4}{V_3})^{\gamma - 1} = \frac{T_3}{T_4} \implies \frac{T_3}{T_4} = \frac{T_2}{T_1} \implies \frac{T_3}{T_2} = \frac{T_4}{T_1}$ $\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - (r_v)^{1 - \gamma} = 1 - \frac{1}{(r_v)^{\gamma - 1}}$ Which r_v : compression ratio $= \frac{V_1}{V_2} = \frac{V_4}{V_3}$

The *mean effective pressure* (MEP) is another quantity that is often used when rating piston-cylinder engines; it is the pressure that, if acting on the piston during the power stroke, would produce an amount of work equal to that actually done during the entire cycle. Thus: $MEP = \frac{Wcycl.}{V2 - V1}$

Ex:- The compression ratio in an air -standard otto cycle is (8). At the beginning of the compression stroke the pressure is 0.1 MPa and the temperature is 15 The heat transfer to the air per cycle is 1800 kJ/kg air ,Determine the pressure and temperature at the end of each process of

the cycle ,the thermal efficiency and the mean effective pressure.

Sol:

Process 1-2

 $S_2 = S_1$ second law

$$\frac{T_2}{T_1} = (\frac{V_1}{V_2})^{\gamma-1} = (r_v)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right) = (r_v)$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{-1} = (r_v)^{v-1}$$

$$V_1 = \frac{RT}{P} = \frac{0.287 \times 288}{100} = 0.827 \quad \frac{m^3}{kg}$$

$$\frac{T_2}{T_1} = (8)^{0.4} \quad \text{Im} \quad T_2 = 662 \text{ K}$$

$$\frac{P_2}{P_1} = (8)^{1.4} \quad \text{Im} \quad P_2 = 1.838 \text{ Mpa}$$

$$V_2 = \frac{0.827}{8} = 0.1034 \text{ m}^3/\text{kg}$$

$$Q_H = Q_{23} = CV(T_3 - T_2) = \Delta u$$

$$T_3 - T_2 = \frac{1800}{0.7165} = 2512 \quad \text{Im} \quad T_3 = 3174 \text{ k}$$

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} = \frac{3174}{662} = 4.795 \quad \text{Im} P_3 = 8.813 \text{ MPa}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_2}\right)^{v-1} = (8)^{0.4} \quad \rightarrow T_4 = 1380 \text{ K}$$

$$\left(\frac{P_2}{P_4}\right) = \left(\frac{V_4}{V_2}\right) = 8^{1.4} \rightarrow P_4 = 0.4795 \text{ MPa}$$

 $\eta = 1 - \frac{1}{(r_V)^{\gamma - 1}} = 56.5\%$

another method to compute (η) :

$$Q_{R} = C_{v} (T_{1} - T_{4}) = 0.7165 (288.2 - 1380) = -782.3 \text{ KJ/Kg}$$
$$\eta = 1 - \frac{QH}{QR} = 56.5\%$$
$$W_{net} = 1800 - 782.3 = 1017.7 \text{ KJ/Kg}$$
$$MEP = \frac{Wcycl.}{V2 - V1} = \frac{1017.7}{0.827 - 0.1034} = 1406 \text{ Kpa}$$

2-<u>The Diesel Cycle</u> :-

The ideal compression ignition or Diesel cycle in which heat added at constant pressure, the cycle shown in T-S and P-V diagrams.



Process 1-2: isentropic compression .

Process 2-3 : reversible constant pressure heating Q_H

Process 3-4 : isentropic expansion .

Process 4-1: reversible constant volume cooling Q_R

$$Q_H = mcp(T_3 - T_2)$$
, $Q_R = mcv(T_4 - T_1)$
Wnet=
$$Q_H - Q_R$$

 $\eta_{th} = \frac{Wnet}{Q_H} = \frac{Q_H - Q_R}{Q_R} = 1 - \frac{Q_R}{Q_H}$
 $= 1 - \frac{Cv(T_4 - T_3)}{Cp(T_3 - T_2)}$
 $= 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$

****** *Prove that :*

$$\eta = 1 - \frac{1}{r^{k-1}} \left(\frac{r_c^k - 1}{k(r_c - 1)} \right)$$
 where $r_c = V_3 / V_2$

EX:- An air- standard Diesel cycle has a compression ratio of 18, and the heat transfered to the working fluid per cycle is 1800 .At the beginning of the beginning of the compression process the pressure is 0.1 MPa and the temp. is 15, Determine the pressure and temp. at each point in the cycle , the thermal efficiency and the mean effective pressure.

Sol :

$$V_{1} = \frac{RT}{P} = \frac{0.287 \times 288}{100} = 0.827 \quad \frac{m^{3}}{kg}$$

$$V_{2} = \frac{V_{1}}{18} \quad \boxed{V}_{2} = 0.04595 \quad \frac{m^{3}}{kg}$$

$$\frac{T_{2}}{T_{1}} = \left(\frac{V_{1}}{V_{2}}\right)^{y-1} = (18)^{0.4} = 3.1777 = T_{2} = 915.8 \text{ K}$$

$$\frac{P_{2}}{P_{1}} = \left(\frac{V_{1}}{V_{2}}\right) = (18)^{1.4} \quad \boxed{I} = 5.72 \text{ MPa}$$

$$Q_{H} = Q_{23} = \text{Cp}(T_{3} - T_{2}) = 1800 \text{ kJ/kg}$$

$$T_{3} - T_{2} \quad \frac{1800}{1.0035} = 1794 \text{ K} \quad \boxed{I} = 2710 \text{ K}$$

Dr. Mohammed D. Salman

$$\frac{V_2}{V_2} = \frac{T_3}{T_2} = \frac{2710}{915.8} \bigvee V_3 = 0.13598 \qquad \text{m}^3/\text{kg}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1} = \left(\frac{0.827}{0.13598}\right)^{0.4} = 2.0588 \qquad \textbf{I}_4 = 1316 \text{ k}$$

$$Q_R = Q_{41} = \text{Cv}(T_1 - T_4) = 0.7165(288-1316)$$

= - 736.6 kJ/kg

 $W_{net} = 1800 - 736.6 = 1036.4 \text{ kJ/kg}$

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{1036.4}{1800} = 59.1 \%$$

3- Bryton Cycle:

The gas turbine is another mechanical system that produces power. It may operate on an open cycle when used as a truck engine, or on a closed cycle when used in a power plant. In open cycle operation, air enters the compressor, passes through a constant-pressure combustion chamber, then through a turbine and finally exits as products of combustion to the atmosphere.

$$\eta = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{C_p (T_4 - T_1)}{C_p (T_3 - T_2)} = 1 - \frac{T_1}{T_2} \left(\frac{T_4 / T_1 - 1}{T_3 / T_2 - 1} \right)$$



Figure 7.4 The Brayton cycle components. (a) Open cycle. (b) Closed cycle.

Using the isentropic relations

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} \qquad \frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{k/(k-1)}$$

and observing that $P_2 = P_3$ and $P_1 = P_4$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$
 or $\frac{T_4}{T_1} = \frac{T_3}{T_2}$

Hence, the thermal efficiency can be written as

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{(k-1)/k}$$

In terms of the pressure ratio $r_p = P_2/P_1$, the thermal efficiency is

$$\eta = 1 - r_p^{(1-k)/k}$$

Dr. Mohammed D. Salman

Another important feature of the gas turbine that seriously limits thermal effi ciency is the high work requirement of the compressor, measured by the

Back work ratio, $BWR = W_{comp}./W_{turb}$.

The compressor may require up to 80 percent of the turbine's output (a back work ratio of 0.8), leaving only 20 percent for net work output. This relatively high limit is experienced when the efficiencies of the compressor and turbine are too low. Solved problems illustrate this point. *EX:* Air enters the compressor of a gas turbine at 100 kPa and 25°C. For a pressure ratio of 5 and a maximum temperature of 850°C, determine the back work ratio (BWR) and the thermal efficiency for this Brayton cycle. Assume constant specific heat.

Solution

To find the back work ratio we observe that

$$BWR = \frac{w_{comp}}{w_{turb}} = \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)} = \frac{T_2 - T_1}{T_3 - T_4}$$

The temperatures are $T_1 = 298$ K, $T_3 = 1123$ K, and

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = 298 \times 5^{0.2857} = 472$$
$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{(k-1)/k} = 1123 \times 5^{-0.2857} = 709 \text{ K}$$

The back work ratio is then

BWR =
$$\frac{w_{\text{comp}}}{w_{\text{turb}}} = \frac{472.0 - 298}{1123 - 709.1} = 0.420$$
 or 42.0%

The thermal efficiency is

$$\eta = 1 - r^{(1-k)/k} = 1 - 5^{-0.2857} = 0.369 \quad \text{or} \quad 36.9\%$$

Dr. Mohammed D. Salman