Lecture Notes and Exercises on ***STATICS***

1- <u>General Principals:</u>

1.1 Introduction

The subject of statics developed very early in history because it's principles can be formulated simply from measurements of geometry and force. *Statics is the study of bodies that are at rest or move with constant velocity*. We can consider statics as a special case of *dynamics*, in which the *acceleration is zero*.

1.2 Fundamental Concepts

Before we begin our study, it is important to understand the meaning of certain fundamental concepts and principles

Length: Length is used to locate the position of a point in space and thereby describe the size of a physical system.

Time: Although the principles of statics are time independent. This quantity plays an important role in the study of dynamics.

Mass: Mass is a measure of a quantity of matter.

Force: Force is considered as a "push" or "pull" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall. A force is completely characterized by its magnitude, direction, and point of application.

Particle : has a mass, but it size can be neglected.

Rigid Body : A rigid body can be considered as a combination of a large number of Particles

Newton's first law: **A particle originally at rest or moving in a straight line with constant velocity, tends to remain in this State provided the particle is not subjected to an unbalanced force (Fig.1-1).**

$$\sum_{i=1}^{N} F_i = 0$$

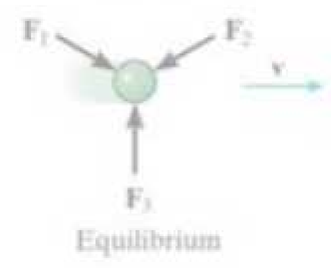


Fig. 1-1

Newton's second law: **A *particle acted* upon by an unbalanced *force "F" experiences* an *acceleration "a"* that has the same direction as the *force* and a *magnitude* that is directly *proportional* to the force (Fig. 1-2). If "F" is applied to a particle or mass "m", this law may be expressed mathematically as**

$$F = m \, . \, a$$

Fig. 1-2

Newton's third Law: The mutual forces of action between two particles are equal, opposite, and collinear (Fig. 1-3).

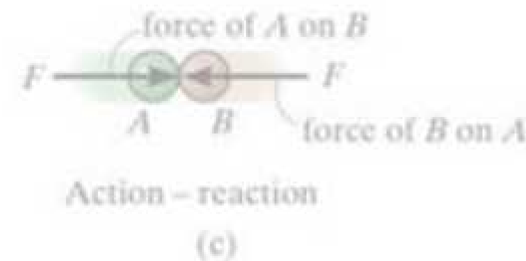


Fig. 1-3

Newton's Law of Gravitational Attraction: Shortly after formulating his three laws of motion. Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically.

$$F = G\frac{m_1 m_2}{r^2}$$

Where

F: Force of gravitational between the two particles.

G: Universal constant of gravitation, according to experimental evidence.

$$G = 66.73\ 10^{-12}\ \frac{m^3}{kg\ s^2}$$

m1, m2: mass of each of the two particles.

r: distance between the two particles.

Weight: Weight refers to the *gravitational attraction* of the *earth* on a body or quantity of mass. The weight of a particle having a mass is stated mathematically.

$$W = mg$$

Measurements give $g = 9.8066\, m/s^2$
Therefore, a body of *mass 1 kg* has a *weight of 9.81* N, a 2 kg body weights 19.62 N, and so on (Fig. 1-4).

Fig. 1-4

Units of Measurement:

• **SI units: The *international System of units*. Abbreviated SI is a *modern version* which has received worldwide recognition. As shown in Tab 1.1. The SI system defines *length in meters (m)*, *time in seconds (s)*, and *mass in kilograms (kg).* In the SI system the unit of force, the *Newton* is a *derived unit*. Thus, 1 Newton (N) is equal to a force required to give 1 kilogram of mass and acceleration of 1 m/s^2**

• US customary: In the **U.S. Customary** system of units (**FPS**) **length** is measured in **feet (ft)**, **time** in **seconds (s)**, and **force** in **pounds (lb). The** unit of **mass**, called a **slug**, **1 slug** is equal to the amount of **matter** accelerated at **1 ft/s²**
when acted **upon by a force of 1 lb** ($1\ slug = 1\ lb\ s2/ft$)

Table 1.1 Systems of Units				
Name	Length	Time	Mass	Force
International Systems of Units	*meter*	*seconds*	*kilogram*	*Newton**
SI	*m*	*s*	*kg*	*N* $\frac{kg.m}{s^2}$
US Customary	*foot*	*second*	*Slug**	*pound*
FPS	*ft*	*s*	$\frac{lb.s^2}{ft}$	*lb*
*Derived unit				

2- **Force Vectors**

2.1 Scalar and vectors

A **scalar :**is any **positive** or **negative physical quantity** that can be completely specified by its **magnitude**.

A **vector:** is any physical quantity that requires both a **magnitude** and **direction** for its complete description. A vector is shown **graphically** by an **arrow**. The **length** of the arrow represents the **magnitude** of the vector, and a fixed axis defines the **direction** of its line of action .The **head** of the arrow indicates the **sense** of **direction of the vector** (Fig 2-1).

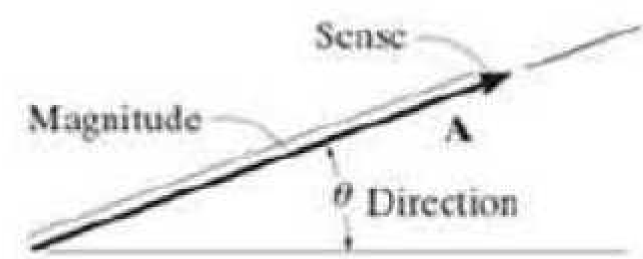


Fig. 2–1

For **handwritten** work, it is often **convenient** to denote a vector quantity by simply drawing an arrow on top it $\vec{A}$.

In **print**, vector quantities are represented by **bold** face letters such as **A**, and its **magnitude** of the vector is **italicized**, ***A***.

2.2 Vector operations

Multiplication and division of vector by a scalar:

If a vector is **multiplied** by a **positive scalar**, its **magnitude is increased** by that amount. When **multiplied by a negative scalar** it will also **change the directional** sense of the vector (Fig 2-2).

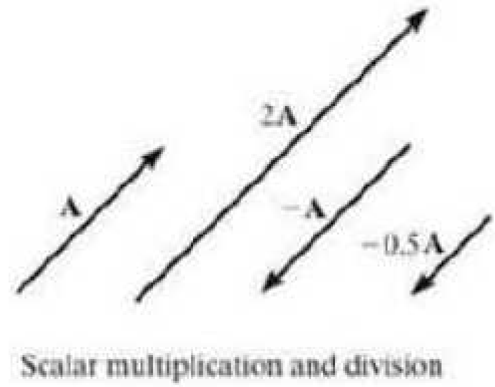


Fig. 2–2

Vector addition:

All vector quantities **obey** the **parallelogram law of addition. Fig 2**-3 and Fig **2-4 an**d Fig 2-**5 illustrates addition of vectors A a**nd **B to** obtain a resultant **R .**

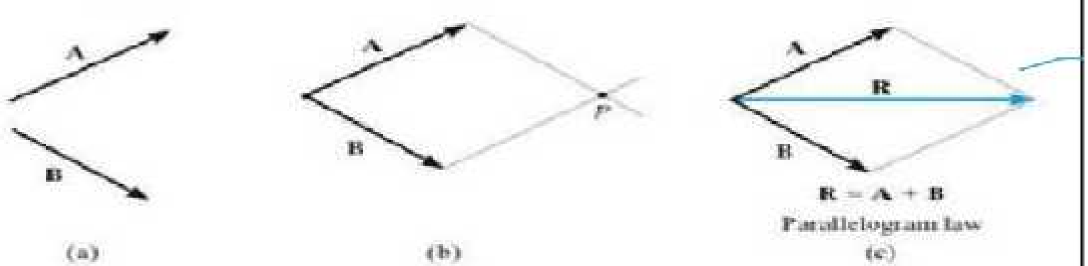


Fig. 2–3

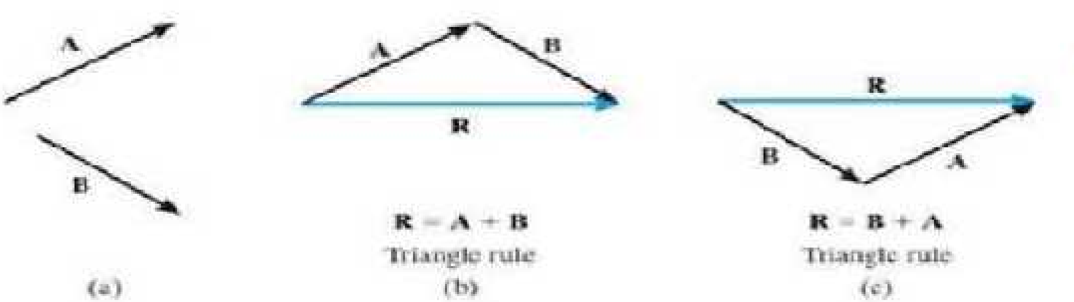


Fig. 2–4

Addition of collinear vectors

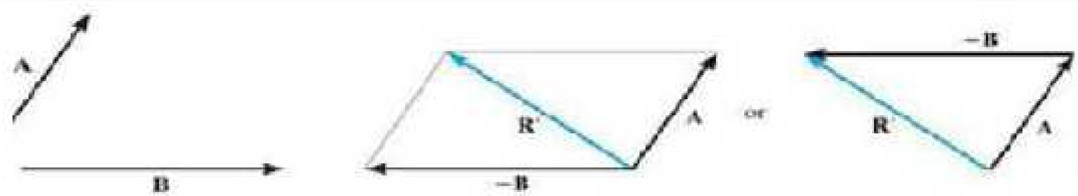
Fig. 2–5

Vector subtraction:

The resultant of the difference between two vectors **A** and **B** of the same type may be expressed as:

$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Fig 2-**6 illustrates subtraction of vectors A** and **B**



Vector subtraction

Fig. 2–6

. 2.3 vector addition of forces:

Experimental evidence has shown that a **force** is a **vector quantity** since it has a specified **magnitude**, **direction**, and **sense** and it **adds according to the parallelogram law**

Finding a resultant force:

The two component forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the pin in Fig 2-7 can be added together to form the **resultant force**

$$\mathbf{F}_R' = \mathbf{F}_1' + \mathbf{F}_2'$$

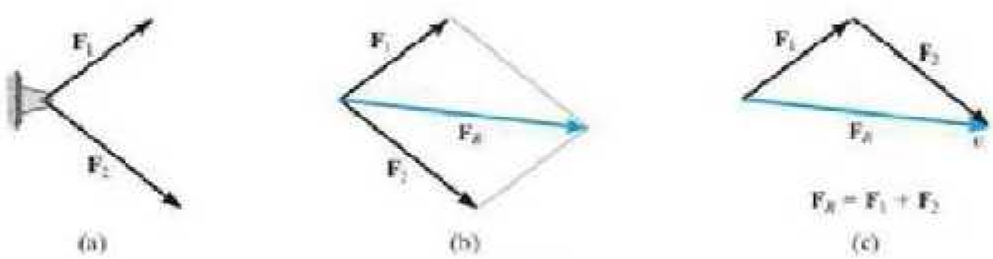


Fig. 2-7

Finding the components of a force:

Sometimes it is necessary to **resolve a force** into two components in order to **study its pulling and pushing effect** in two specific directions Fig 2.8, F is to be resolved into two components along two members, defined by ***u*** and ***v*** (Fig 2.8)

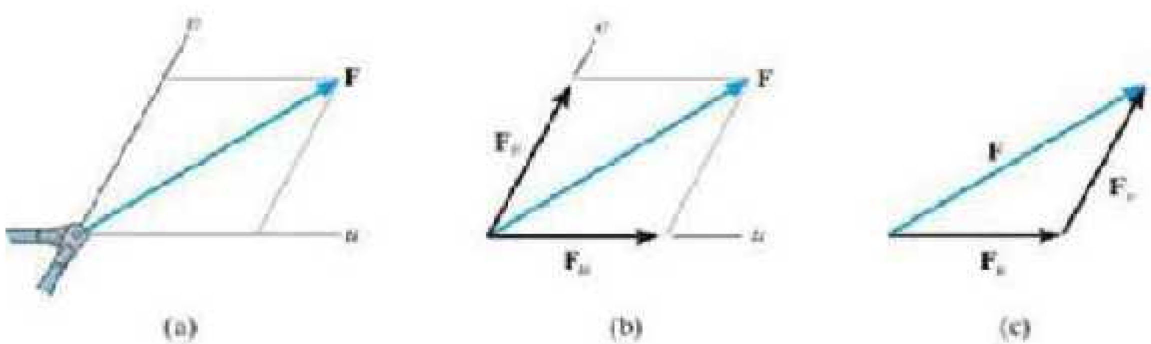


Fig. 2-8

Addition of several forces:

If **more than two forces** are to be **added successive applications of the parallelogram law** can be carried out **in order to obtain** the resultant force. For **example if the three** forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$ **act at** a point o, the **resultant of any two of the forces** is **found** $(\mathbf{F}_1 + \mathbf{F}_2)$ and then this resultant **is added to the third force** yielding **the resultant of all three forces** ($\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$) (Fig 2-9).

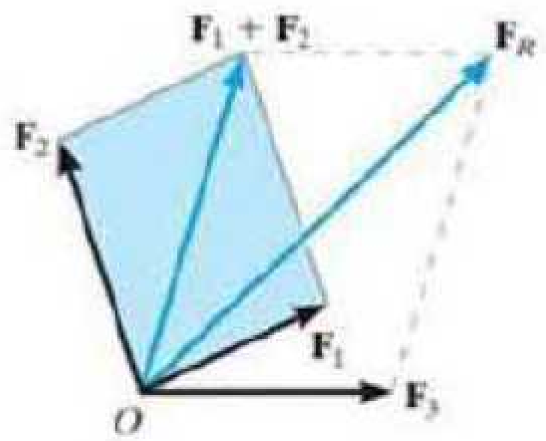


Fig. 2–9

Trigonometry analysis:

Redraw a half portion of the parallelogram to illustrate the **triangular** head to tail addition of the components. From this triangle, the **magnitude of the resultant force** can be determined using **the law of cosines,** and its direction is determined from the **law of sines** .

The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig 2-10

cosine law:

$$C = A2 + B2 - 2AB \cos c$$

sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

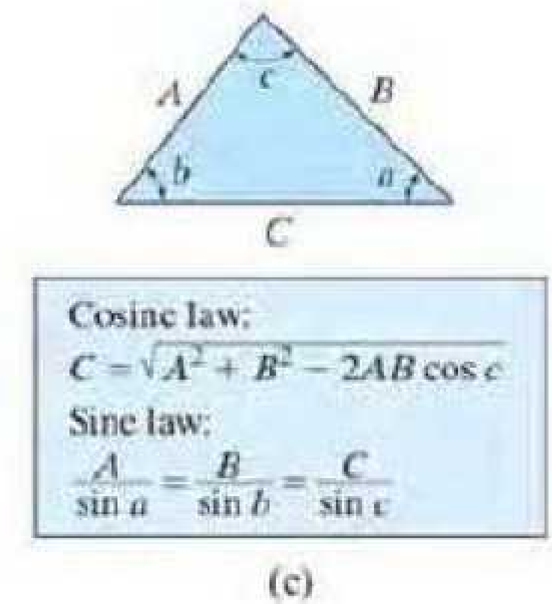


Fig 2-10

Exercise 2.1:

The screw eye in Fig 2-11 is subjected to two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$. Determine the magnitude and direction of the resultant force.

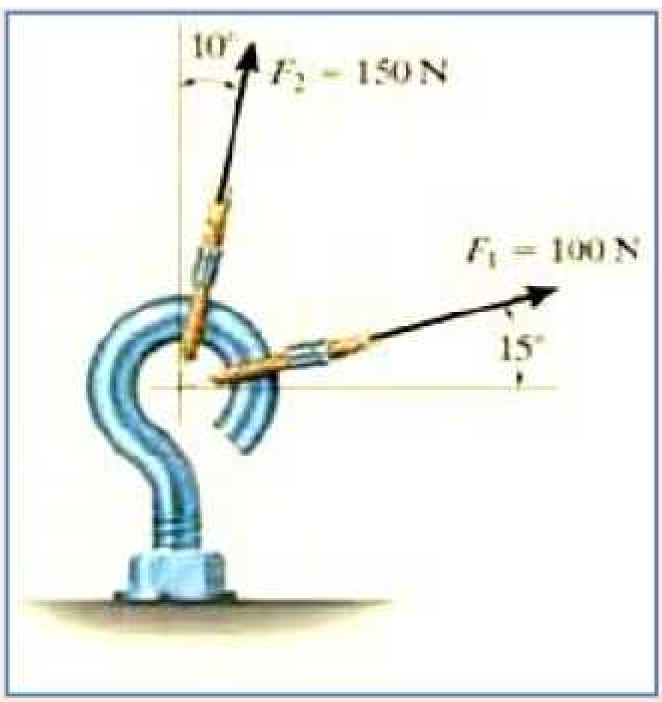
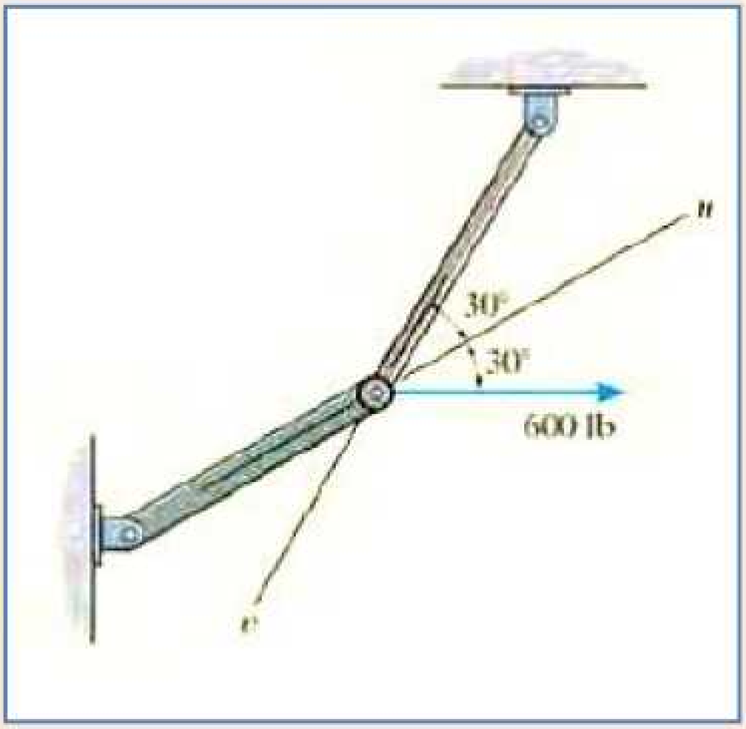


Fig 2-11

Ans: $F_R = 213$ N $\emptyset = 54.7°$

Exercise 2.2:

Resolve the horizontal 600lb force in fig 2.12 into components action along the ***u*** and ***v*** axes and determine the magnitudes of these components



Ans: $F_u = 1039$ lb $F_v = 600$ lb

Fig 2-12

2.4 addition of a system of coplanar forces

When a force in resolved into two components along the ***x*** and ***y*** axes the components are then called **rectangular components**.

The rectangular components of **force F shown** in Fig 2.23 are found using the parallelogram law, so **that**

$$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y}$$
$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

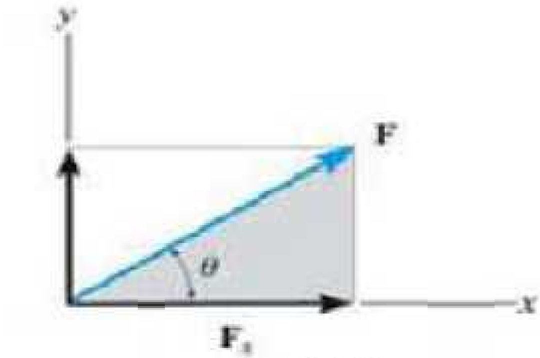


Fig 2-23

instead of using the angle ***θ***, the direction of **F** can also be defined **using a small "slope" triangle**, such as shown in fig 2.24

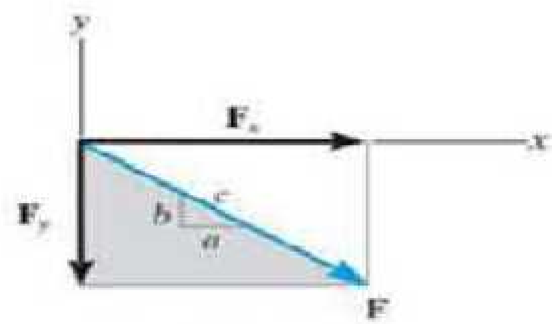


Fig 2-24

$$\frac{F_X}{F} = \frac{a}{c} \Rightarrow F_X = F\left(\frac{a}{c}\right) \qquad \text{And} \qquad \frac{F_y}{F} = \frac{b}{c} \Rightarrow F_y = \left(\frac{b}{c}\right)F$$

It is also possible to represent the ***x*** and ***y*** components of a force in terms of **Cartesian unit vectors i** and **j** (Fig 2.25).

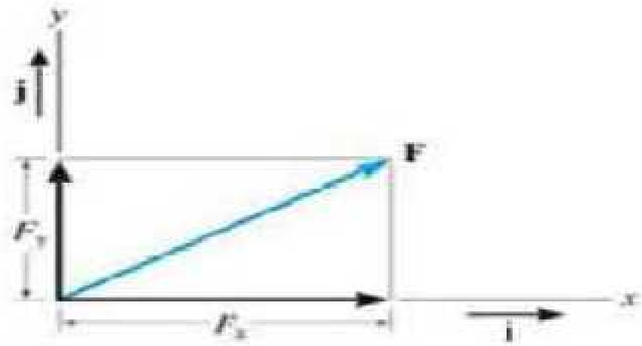


Fig 2-25

We can express **F as** a **Cartesian vector.**

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

In **coplanar force** resultant case, **each force is resolved** into its ***x*** and ***y*** components, and then the respective components are added using **scalar algebra** since they are collinear. For example, consider the three concurrent forces in Fig 2.26.

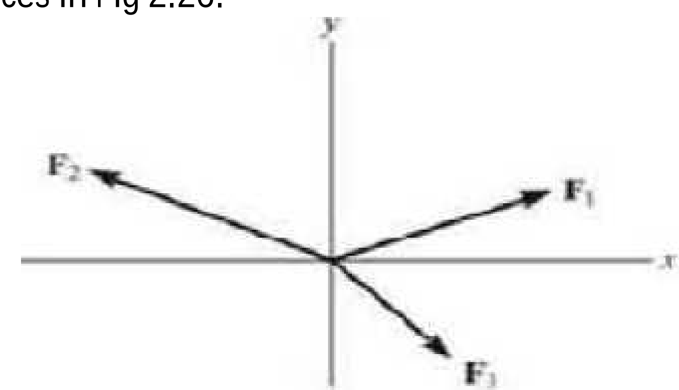


Fig 2-26

Each force is represented as **a Cartesian vector.**

$$\mathbf{F}_1 = F_{1x}\mathbf{i} + F_{1y}\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\mathbf{i} + F_{2y}\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\mathbf{i} - F_{3y}\mathbf{j}$$

The vector resultant **is therefore.**

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = F_{1x} + F_{2x} + F_{3x}\mathbf{i} + F_{1y} + F_{2y} + F_{3y}\mathbf{j}$$
$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = F_{Rx}\mathbf{i} + F_{Ry}\mathbf{j}$$

We can **represent** the components of the resultant force of any number of coplanar forces symbolically by **the algebraic sum the x and y** components of all the forces.

$$F_{Rx} = \Sigma F_x$$
$$F_{Ry} = \Sigma F_y$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant **force can be determined from vector addition as shown in Fig 2**-27.
The magnitude of F_R is then found from the by **Pythagorean theorem**: that is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

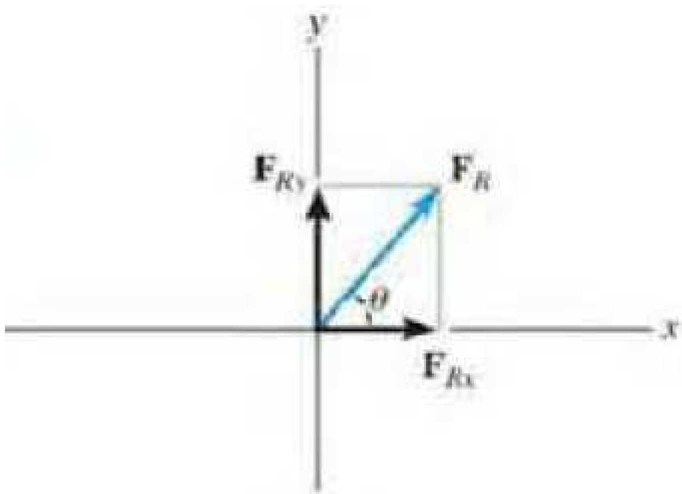


Fig 2-27

Exercise 2.3:

Determine the x and y components of F_1 and F_2 acting on the boom shown in Fig 2.28 express each force as a Cartesian vector.

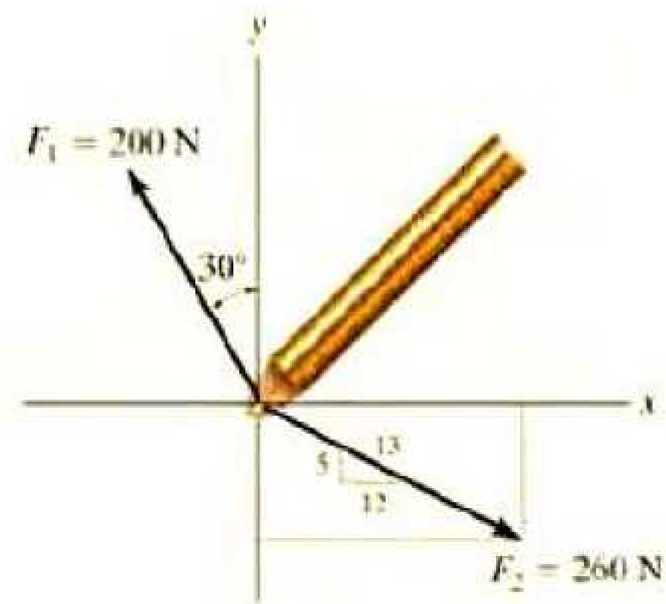


Fig 2-28

Exercise 2.4:

The end of boom O in Fig 2.30 is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.

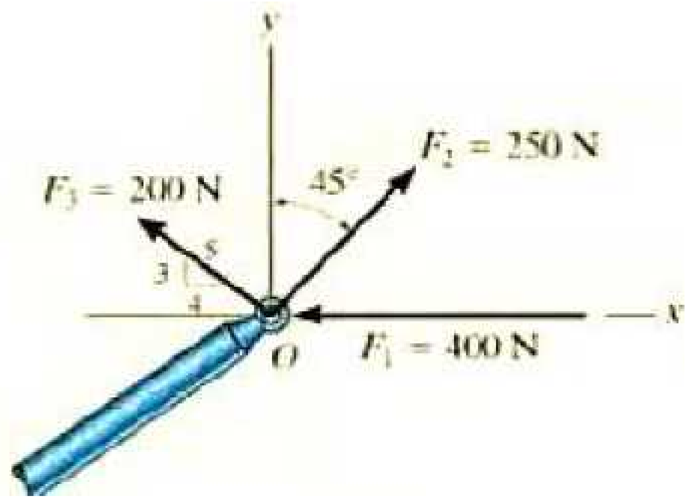


Fig 2-30

Exercise 2.5:

Determine the magnitude and direction of the resultant force.

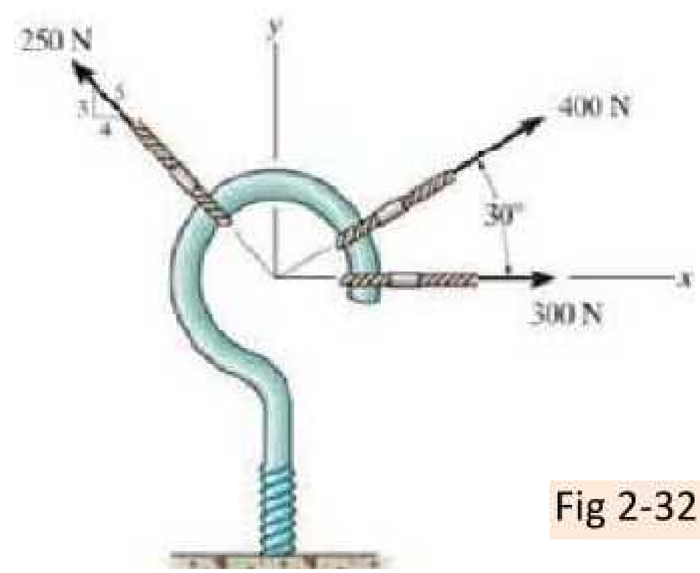


Fig 2-32

Exercise 2.6:

Determine the magnitude and direction measured counterclockwise from the positive **x** axis of the resultant force of the three forces acting on the ring A. Take $\mathbf{F_1}$ = 500N and θ =20°.

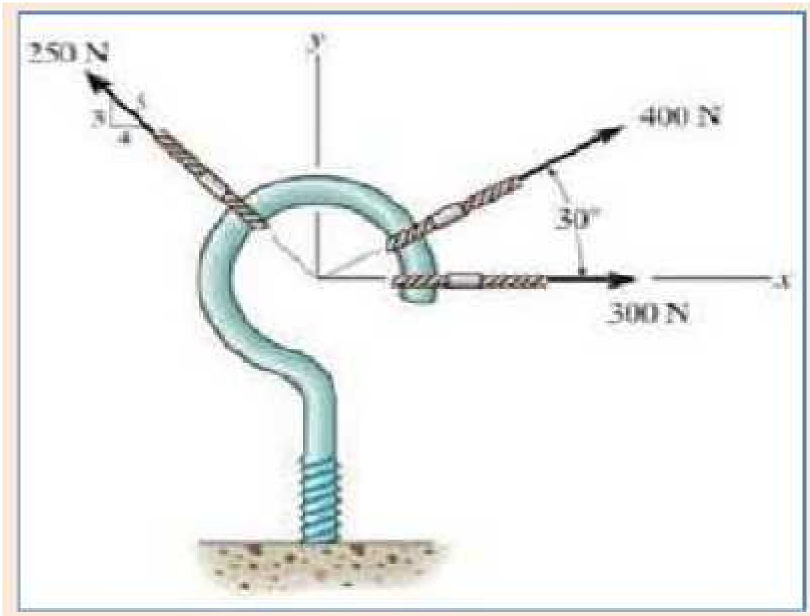


Fig 2-32

2.5 Cartesian vectors

A vector **A** may have three rectangular components along the ***x, y, z*** coordinate axes and **is represented by** the vector sum of its **three rectangular components** (Fig **2-38)**.

$$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y} + \mathbf{A_z}$$

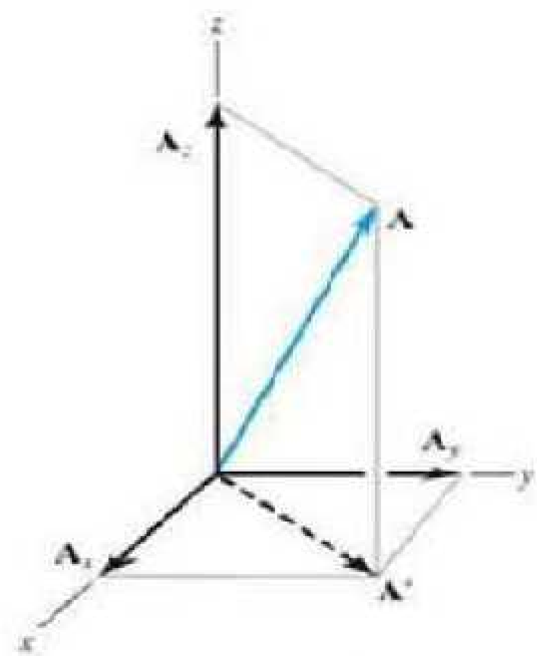


Fig. 2.38

In three dimensions, the set of Cartesian unit **i , j , k is used to d**esignate the directions of the x, y, z axes, respectively. **The positive Carte**sian unit vectors are shown in Fig 2-39.

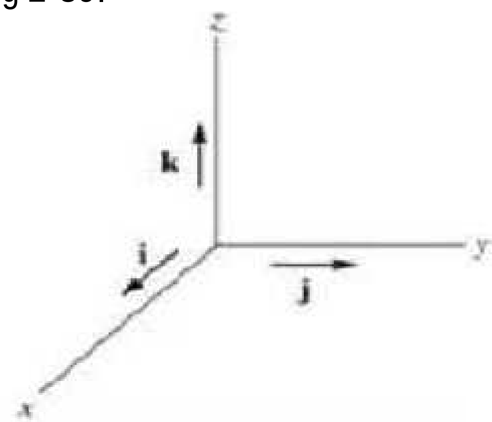


Fig. 2.39

We can write **A in Cartesian vector form as**

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

The **magnitude** of **A is** expressed in Cartesian vector from as

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

The **direction** of **A is** defined by the **coordinate direction** angles **α**, **β**, and γ (Fig 2.40).

$$\cos \alpha = \frac{A_x}{A} \qquad \cos \beta = \frac{A_y}{A} \qquad \cos \gamma = \frac{A_z}{A}$$

With

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

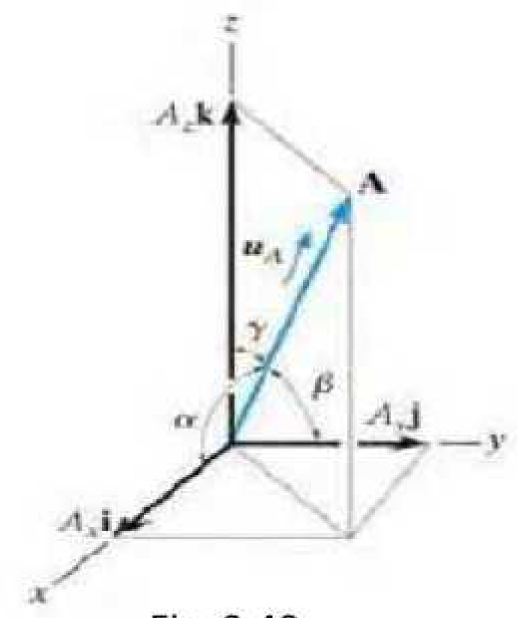


Fig. 2.40

The addition (or subtraction) of two or more vectors are greatly simplified in terms of their Cartesian components. For example, the resultant **R** in Fig 2.41 is written as

$$\mathbf{R} = A_x + B_x\,\mathbf{i} + A_y + B_y\,\mathbf{j} + A_z + B_z\,\mathbf{k}$$

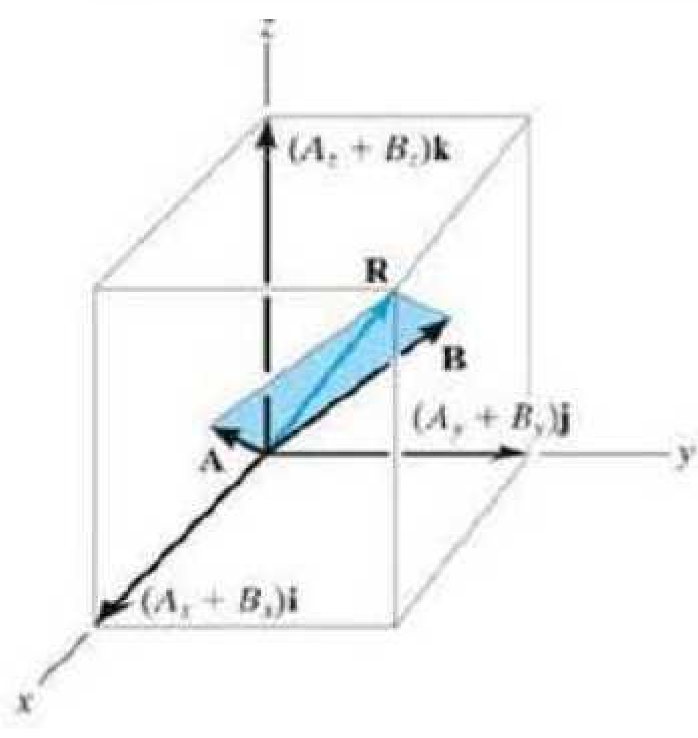


Fig. 2.41

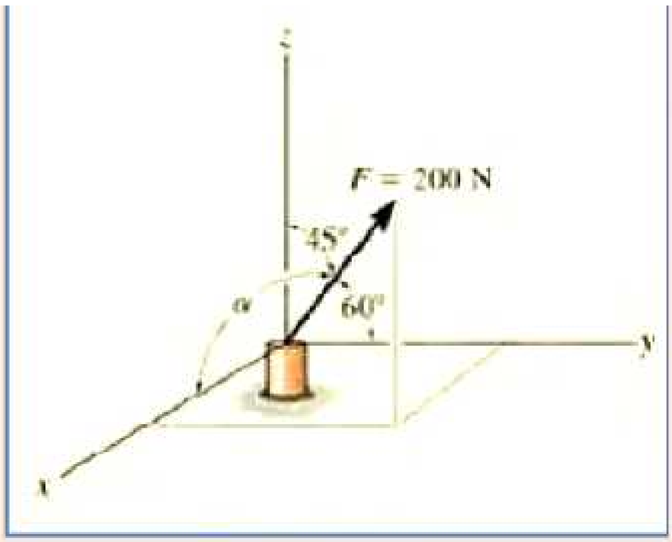
If this is generalized and applied to a system of several concurrent forces, then the force resultant is the vector sum of all the forces in the system and can be written as

$$\mathbf{F}_R = \Sigma \mathbf{F} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k}$$

Exercise 2.7:

Express the force **F** shown in Fig 2.38 as a Cartesian vector.

Fig 2-38



Exercise 2.8:

Determine the magnitude and the coordinate direction angles of the resultant force acting on the ring in Fig 2-39

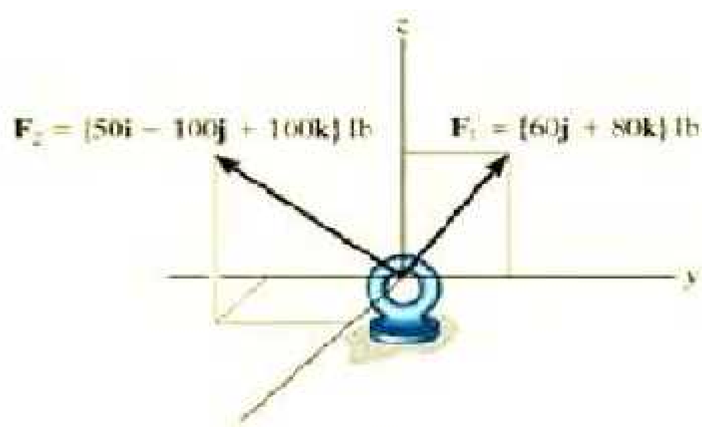


Fig 2-39

Exercise 2.9:

Two forces act on the hook in Fig 2-41 specify the magnitude of **F_2 and its** coordinate direction angles of **F_2** that the resultant **force F_R acts** along the positive **y** axis and has magnitude of 800 N.

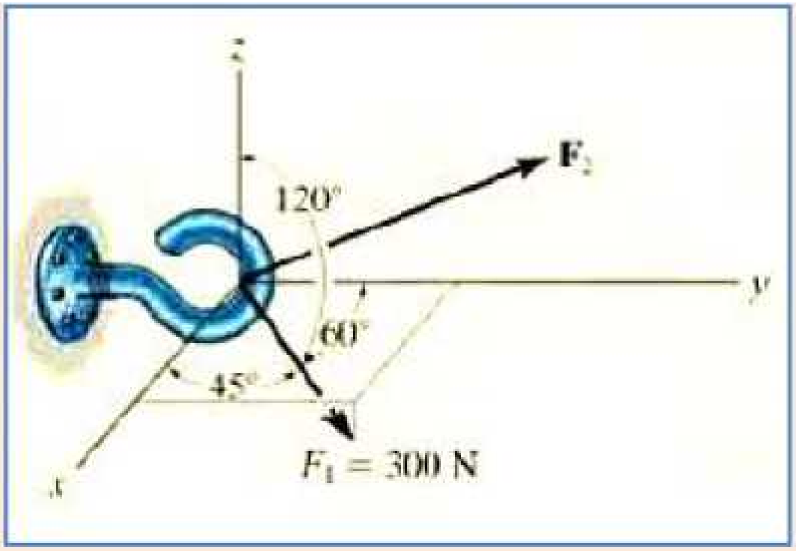


Fig 2-41

Exercise 2.10:

The mast is subject to the three forces shown. Determine the coordinate direction angles α_1, β_1, γ_1 of **F_1** so that the resultant force acting on the mast is **F_R** = {350 **i** } **N.** Take **F_1**=500 N.

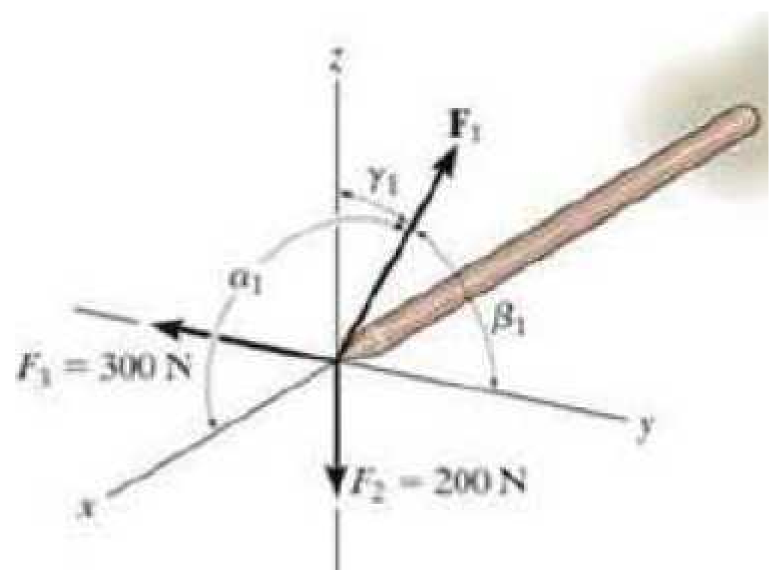


Fig 2-46

2.6 Position Vectors

In the more general case, the position vector may be directed from point A to point B in space, Fig. 2-48. This vector is also designated by the **symbol *r***. As a smaller of convention, we will sometimes refer to this vector with two subscripts to indicate from and to the point where it is directed. Thus, ***r*** can also be designated as $\boldsymbol{r_{AB}}$. Also, note that $\boldsymbol{r_A}$ and $\boldsymbol{r_B}$ in Fig. 2-48, are referenced with only one subscript since they extend from the origin of coordinates.

From Fig. 2-48, by the **head-to-tail vector** addition, using the **triangle** We require:

$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$

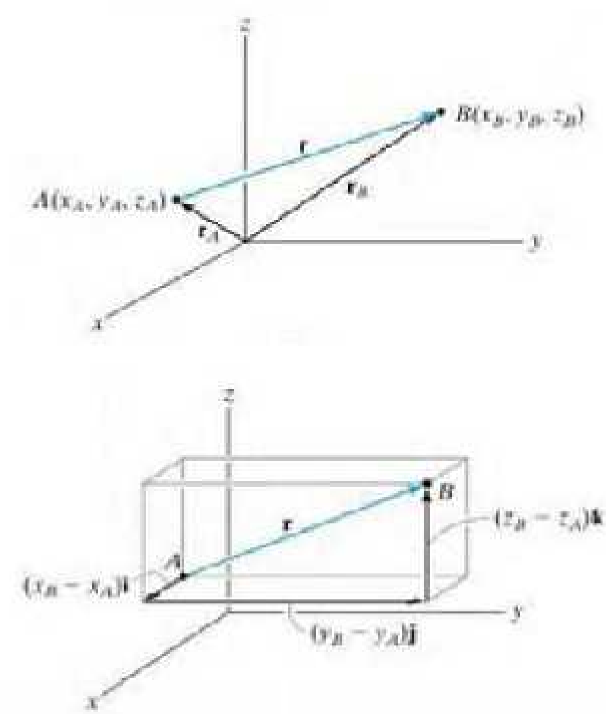


Fig 2-48

Solving **for r and expressing r$_A$ and r$_B$ in Cartesian vector form** yields

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = x_B - x_A\,\mathbf{i} + y_B - y_A\,\mathbf{j} + z_B - z_A\,\mathbf{k}$$

2.7 Dot Product

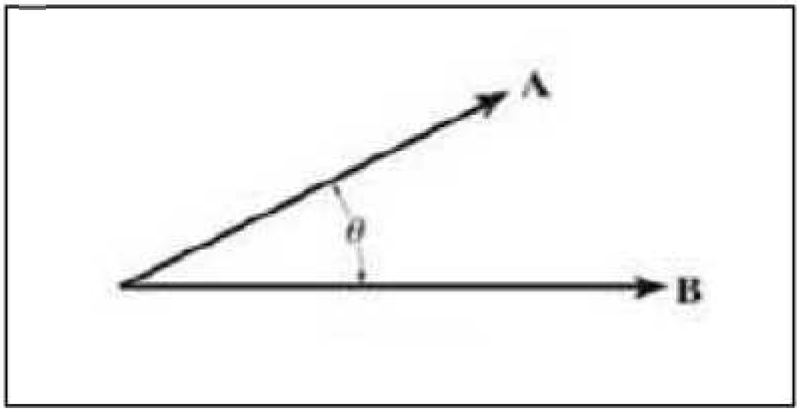
the dot Product of vectors **A** and **B** written **A . B** and read **A** dot **B is** defined as the **product of the magnitudes of A and** B and the cosine of the angle θ between their tails (Fig 2.50).
expressed in equation form.

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta \quad (2.1)$$

Where

$$0° \leq \theta \leq 180°$$

Fig 2-50



equation 2.1 must be used to find the dot product for any two Cartesian unit vectors.
For example:

$\mathbf{i}\cdot\mathbf{i} = 1\,1\cos 0° = 1$ $\mathbf{j}\cdot\mathbf{j} = 1$ $\mathbf{k}\cdot\mathbf{k} = 1$

$\mathbf{i}\cdot\mathbf{j} = 1\,1\cos 90° = 0$ $\mathbf{j}\cdot\mathbf{k} = 0$ $\mathbf{j}\cdot\mathbf{k} = 0$

if we want to find the dot product of two general vectors **A** and **B** that are expressed in Cartesian vector form, then we have

$$\mathbf{A}\cdot\mathbf{B} = A_x\mathbf{i} + A_y\mathbf{j} + A_k\mathbf{k}\; B_x\mathbf{i} + B_y\mathbf{j} + B_k\mathbf{k}$$

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x\mathbf{i}\cdot\mathbf{i} + A_xB_y\mathbf{i}\cdot\mathbf{j} + A_xB_z\mathbf{i}\cdot\mathbf{k} + A_yB_x\mathbf{j}\cdot\mathbf{i} + A_yB_y\mathbf{j}\cdot\mathbf{j} + A_yB_y\mathbf{j}\cdot\mathbf{k} + A_zB_x\mathbf{k}\cdot\mathbf{i} + A_zB_y\mathbf{k}\cdot\mathbf{j} + A_zB_z\mathbf{k}\cdot\mathbf{k}$$

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z \quad \mathbf{(2.2)}$$

We deduce that the angle forces between two vectors can be written as

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}.\vec{\mathbf{B}}}{AB}\right)$$

Where

$$0° \leq \theta \leq 180°$$

STATICS

we notice that if

$$\mathbf{A}\,.\,\mathbf{B} = 0 \;\; \theta = \cos^{-1} 0 = 90°$$

so that **A will be perpendicular** to **B .**

In the case of **line *a* as shown in** figure 2-51, **and if the direction of the** line is specified **by the unit** $\mathbf{u}_a$ **then since** $u_a = 1$**,** we can determine the magnitude **of** $\mathbf{A}_a$ **directly from the dot product**

$$A_a = A\cos\theta$$

$$\mathbf{A}\,.\,\mathbf{u}_a = A.\,1.\cos\theta = A\cos\theta \;\; A_a = \mathbf{A}\,.\,\mathbf{u}_a$$

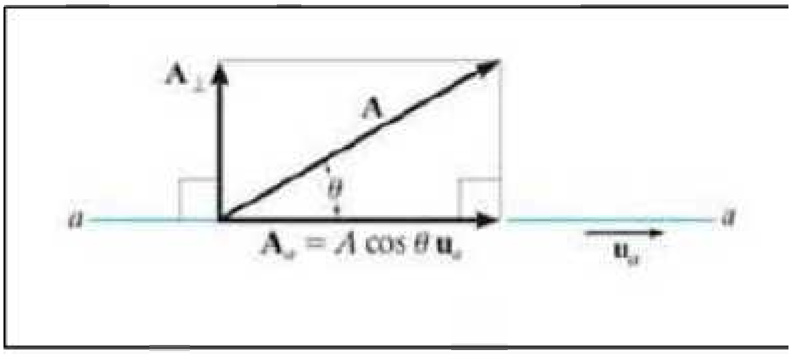


Fig 2-51

Notice that if this result is positive, then $\mathbf{A}_a$ has a directional sense which is the same as $\mathbf{u}_a$, whereas if A_a is a negative scalar, then $\mathbf{A}_a$ has the opposite sense of direction $\mathbf{u}_a$. The component $\mathbf{A}_a$ represented as a vector is therefore

$$A_{\perp} = A\sin\theta \qquad with \quad \theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}.\vec{\mathbf{u}_a}}{A}\right)$$

Alternatively as if A_a is known then by Pythagorean' s theorem we can also write

$$A_{\perp} = \sqrt{A^2 - A_a^2}$$

Force System Resultants

4.1 Moment of a force scalar formulation.

The moment **M_o about point O**, or about an axis passing through O and perpendicular **to the** plane, **is a vector quantity** since it has a specified **magnitude and direction** (fig 4-1).

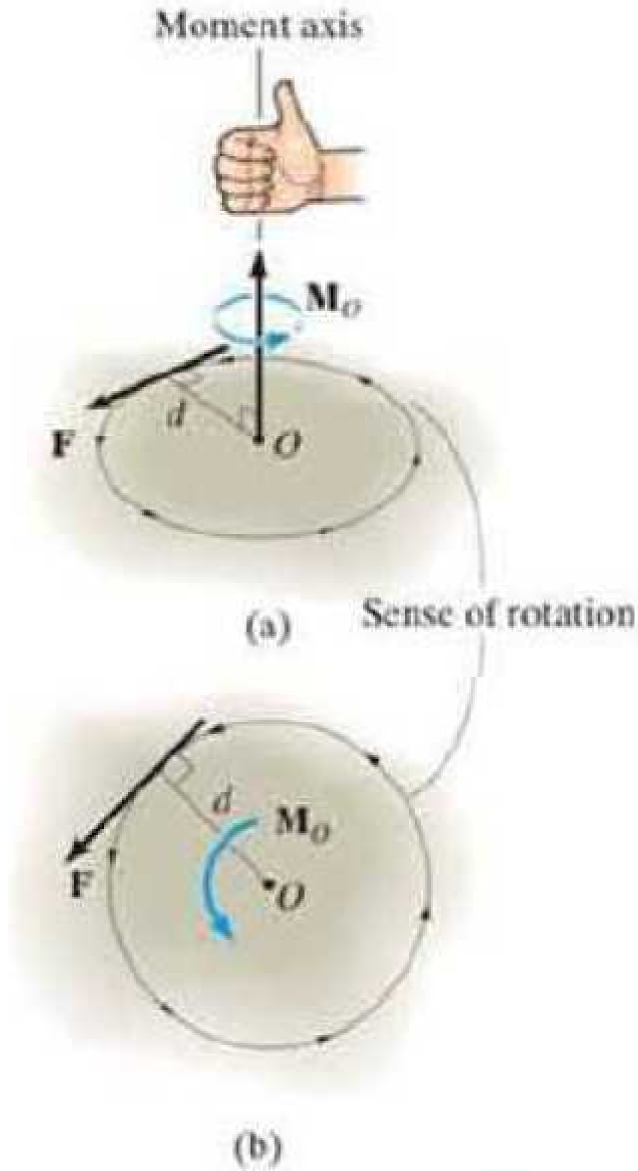


Fig 4-1

The magnitude of **M_o** is

$$M_o = F.d \qquad \textbf{4.1}$$

Where ***d* is the moment arm or perpendicular distance from the axis at point O** to the line of action of the force. **Units** of moment is **N.m** or **lb.ft.** **The direction** of **M_o is defined** by its moment axis which is **perpendicular** to the **plane** that contains the force **F** and its moment **arm *d***. The **right-hand rule** is used establish the sense of the direction of **M_o**.

For two dimensional problems, where **all the** forces lie within the ***x-y*** plane, fig 4-2, the resultant moment **$(M_R)_o$ about point O** (the z axis) can be determined by finding the **algebraic sum of the moments** caused by all the forces in the system. As a **convention** we will generally consider **positive moments** as a **counterclockwise** since they are directed along the positive z axis (out of page). **Clockwise** moments will be **negative**. Using the sign convention, the resultant moment in fig 4-3 is therefore

$$(M_R)_0 = \sum Fd$$

$$(M_R)_0 = F_1d_1 - F_2d_2 + F_3d_3$$

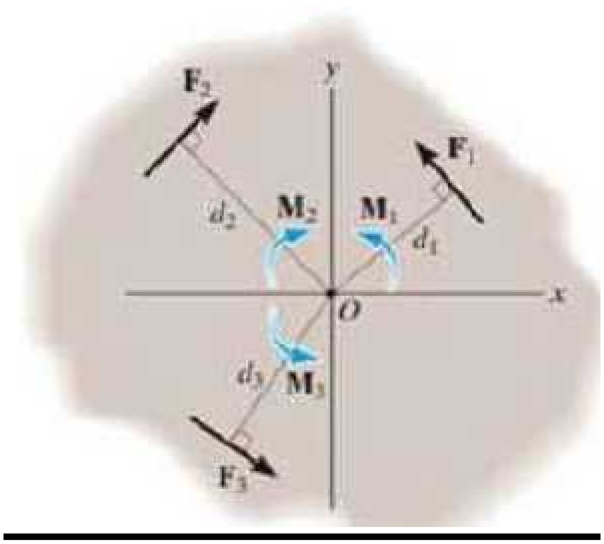


Fig 4-2

4.2 Cross product

The cross **product of two vectors** $\mathbf{A}$ and $\mathbf{B}$ yields the vector C which is written

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} \qquad 4.2$$

And read **C equals A cross B .**

The magnitude of **C is defined as the** product of the magnitudes **A** and **B** and the **sine** of the **angle θ between** their tails (0° ≤ θ ≤ 180°), **thus**

$$C = A\,B\,\sin\theta$$

C has a direction **that is perpendicular to** the **plane** containing **A** and **B such** that **C is specified by the right-hand rule**.

Knowing **both the magnitude and direction of C , we can write**

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (A\,B\,\sin\theta)\mathbf{u}_C \qquad 4.3$$

Where the scalar ***(A B* sin θ) defines the magnitude** of **C**

and the unit vector $\mathbf{u}_C$ **defines the direction** of **C (fig** 4-4).

Laws of operation:

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

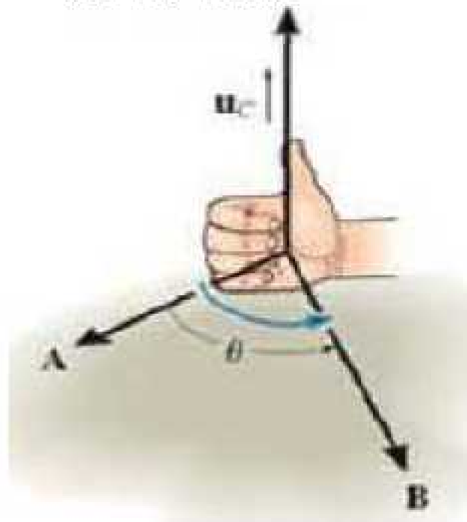
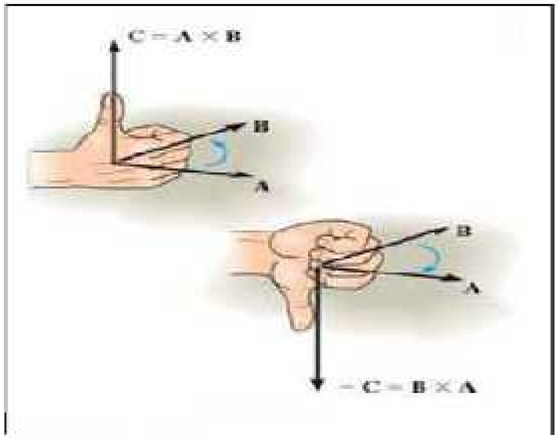


Fig 4-4

$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$ **(commutative law is not valid)**

Fig 4-5



$a\,\mathbf{A} \times \mathbf{B} = a\mathbf{A} \times \mathbf{B} = \mathbf{A} \times a\mathbf{B} = \mathbf{A} \times \mathbf{B}\,a$ (**associative law**)
$\mathbf{A} \times \mathbf{B} + \mathbf{D} = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{D}$ (**distributive law**)

Cartesian vector formulation:

Equation 4.3 may be used to find the cross product of any pair of Cartesian unit vectors. For **example, to find $\mathbf{i} \times \mathbf{j}$, the m**agnitude of the resultant vector is

$$i\ j\ sin90° = 1\ 1\ 1 = 1$$
$$i\ i\ sin0° = 0$$

and its direction is det**ermined using the right-hand rule (fig 4-6), the** resultant vector points **in the + $\mathbf{k}$ direction. Thus $\mathbf{i} \times \mathbf{j} = 1\,\mathbf{k}$.**
In similar maner.

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}.\ \mathbf{i} \times \mathbf{k} = -\mathbf{j}\ \mathbf{i} \times \mathbf{i} = 0$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i}\ \mathbf{j} \times \mathbf{i} = -\mathbf{k}\ \mathbf{j} \times \mathbf{j} = 0$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j}\ \mathbf{k} \times \mathbf{j} = -\mathbf{i}\ \mathbf{k} \times \mathbf{k} = 0$$

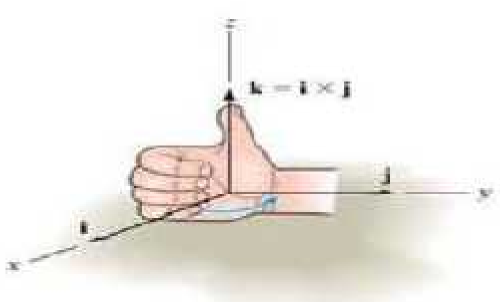


Fig 4-6

A simple scheme shown in fig 4-7 is helpful
for obtaining the same results when the need arises.
Let us now consider the cross product of two
general vectors **A** and **B** .

$\mathbf{A} \times \mathbf{B} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \times B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$

$\mathbf{A} \times \mathbf{B} = (A_x B_x \mathbf{i} \times \mathbf{i} + A_x B_y \mathbf{i} \times \mathbf{j} + A_x B_z(\mathbf{i} \times \mathbf{k})$

$+A_y B_x \mathbf{j} \times \mathbf{i} + A_y B_y \mathbf{j} \times \mathbf{j} + A_y B_z(\mathbf{j} \times \mathbf{k})$

$+A_z B_x \mathbf{k} \times \mathbf{i} + A_z B_y \mathbf{k} \times \mathbf{j} + A_z B_z(\mathbf{k} \times \mathbf{k})$

$\mathbf{A} \times \mathbf{B} = A_y B_z - A_z B_y \mathbf{i} - A_x B_z - A_z B_x \mathbf{j} + A_x B_y - A_y B_x \mathbf{k}$

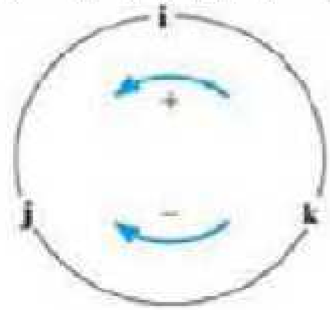


Fig 4-7

This equation may also be written in a more compact determinant form
As

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

<u>4.3 Moment of a force – vector formulation</u>

The moment of a force F about a point O (fig 4-8) can be expressed using the vector cross product namely

$$\mathbf{M_0} = \mathbf{r} \times \mathbf{F} \qquad 4.4$$

Here **r** represents a position vector direct from O to any point on the line of action of **F** .
The magnitude of the cross product is defined from Eq. 4-3 as

$$M_0 = r\,F\,\sin\theta$$

were θ is measured between the tails of **r and F** The direction and sense of **Mo in Eq**. 4-4 are determined by the right-hand rule as it applies to the cross product (fig 4-9).

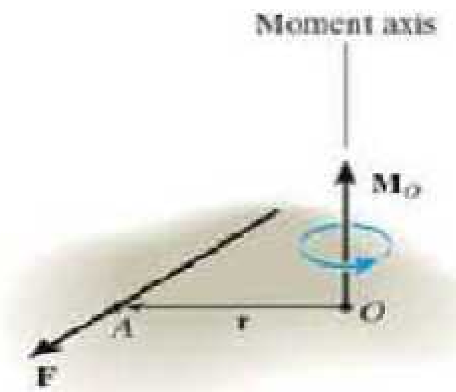


Fig 4-8

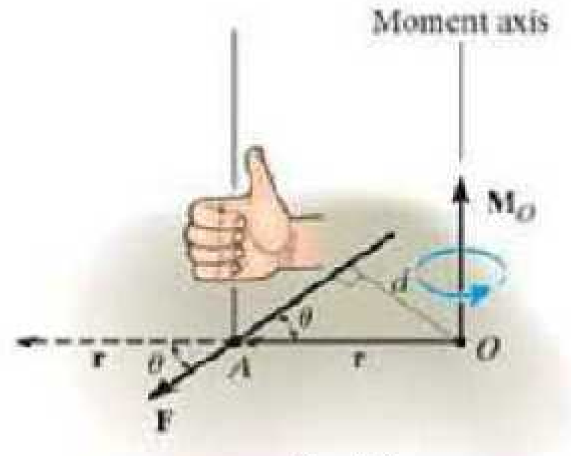


Fig 4-9

Cartesian vector formulation:

If we establish ***x, y, z*** coordinate axes, then the position vector **r and** force **F** can be expressed as Cartesian vectors (fig 4-10)

$$\overrightarrow{\mathbf{M_0}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where r_x, r_y, r_z represent the ***x, y, z*** components of the position vector **drawn from point** O to any point on the line of action of the force.
F_x, F_y, F_z represent the ***x, y, z*** of the force vector.

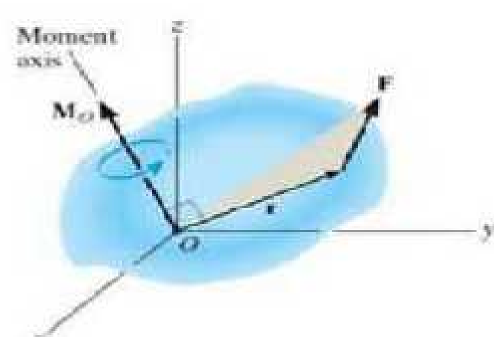


Fig 4-10

Resultant Moment of a system of forces:

If a body is acted upon by a system of forces (fig 4-11), the resultant moment of the forces about point O can be determined by vector addition of the moment of each force. This resultant can be written symbolically as

$$\overrightarrow{\mathbf{M}_{\mathbf{R_o}}} = \sum (\vec{\mathbf{r}} \times \vec{\mathbf{F}})$$

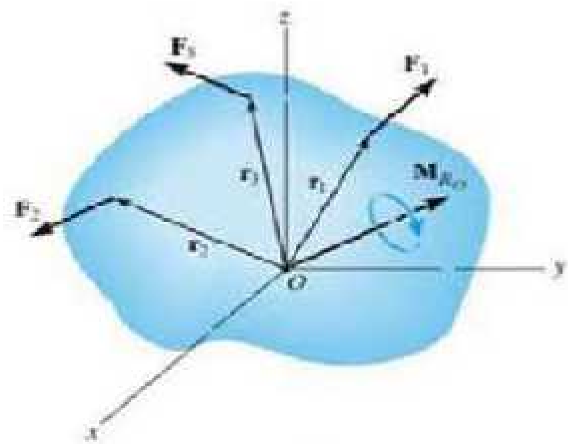


Fig 4-11

Exercise 4.1:

Two forces act on the rod shown in fig 4-13. Determine the resultant moment they create about the flange at O. Express the result as a Cartesian vector.

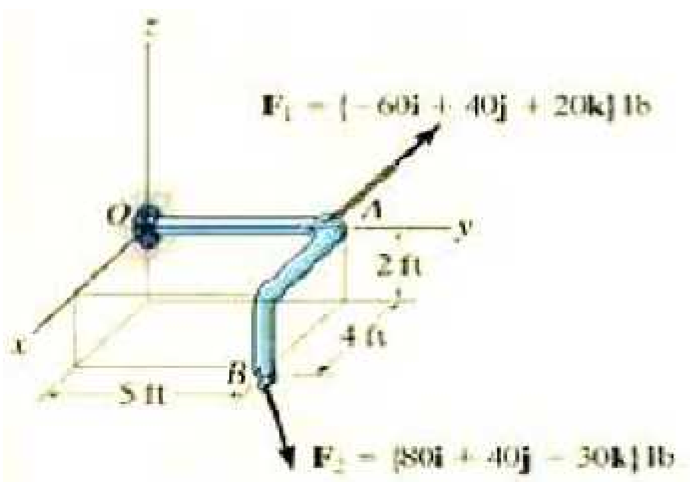


Fig 4-13

4.4 Principle of moments

The principle of moments is referred to the French mathematician Varignon (1654-1722). It states that the moment of a force about a point is equal to the sum of the moments of the components of the force about the point. If we consider the case of fig 4-14, we have.

$$M_o = r \times F = r \times F_1 + F_2 = r \times F_1 + r \times F_2$$

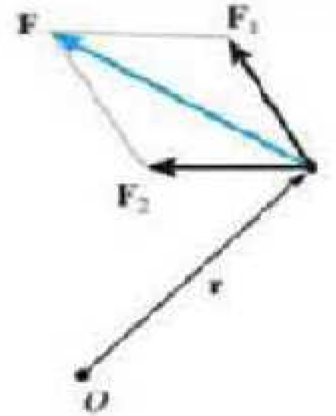


Fig 4-14

Exercise 4.2:

Determine the moment of the force in fig 4-15 about the point O.

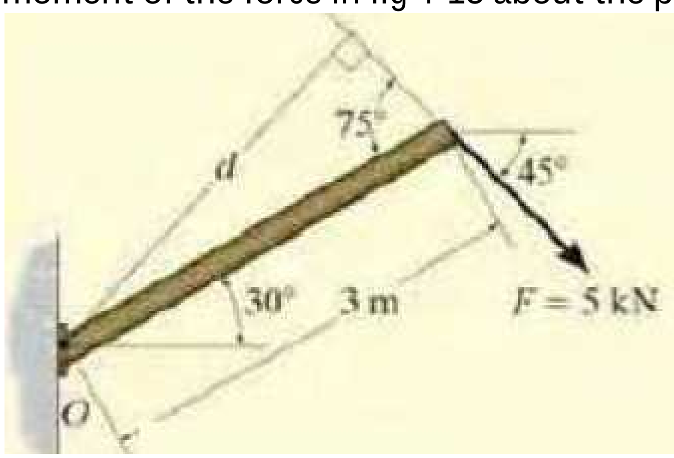


Fig 4-15

Exercise 4.3:

Determine the moment of the force in fig 4-16 about point O. Express the result as a Cartesian vector

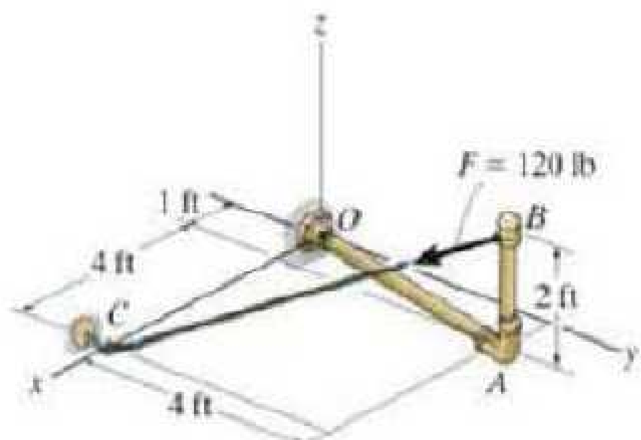


Fig 4-16

Exercise 4.4:

Force **F** acts at the end of the angle bracket shown in fig 4-17. Determine the moment of the force about point O.

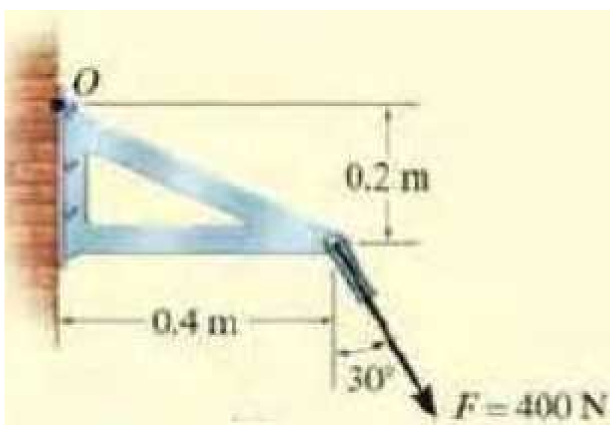


Fig 4-17

4.6 Moment of a couple

a couple is defined as a two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance ***d*** (fig 4-24). The moment produced by a couple is called a couple moment.

Scalar Formulation

The moment of a couple M (fig 4-25), is defined as having a magnitude of

$$M = F d$$

Where ***F*** is the magnitude of one of the forces and ***d*** is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right hand rule. M will act perpendicular to the plane containing these forces.

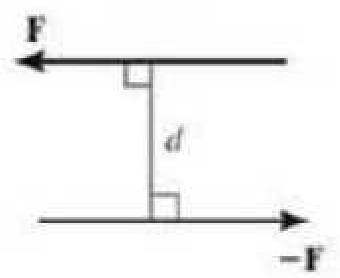


Fig 4-24

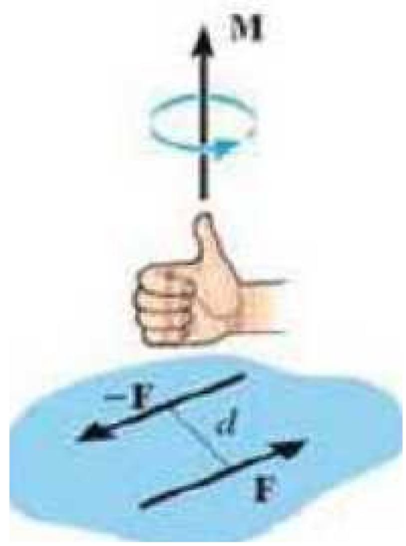


Fig 4-25

Vector Formulation

The moment of a couple can also be expressed by the vector Cross product as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Resultant couple moment

Since couple moments are vectors, their resultant can be determined by vector addition. If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as

$$\overrightarrow{\mathbf{M_R}} = \sum(\vec{\mathbf{r}} \times \vec{\mathbf{F}})$$

Exercise 4.5:

Determine the resultant couple moment of the three couples acting on the plate in fig 4-26.

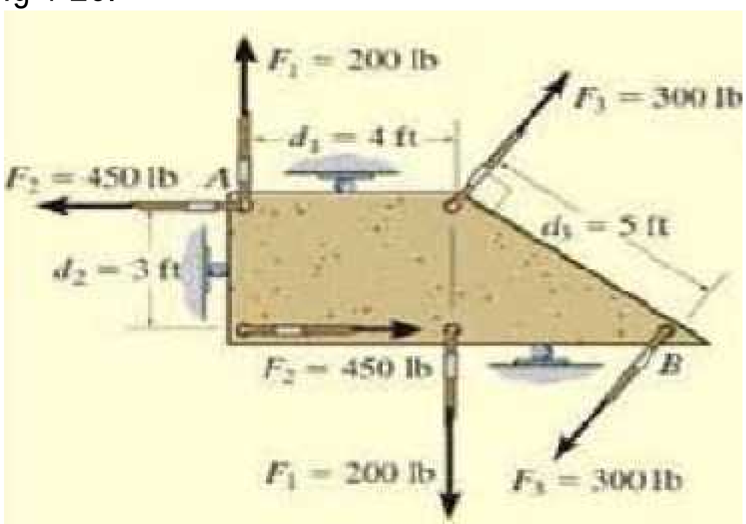


Fig 4-26

Exercise 4.6:

Determine the couple moment acting on the pipe shown in fig 4-28
Segment AB is directed30° below the **x-y** plane. Take OA=8 in and AB=6 in

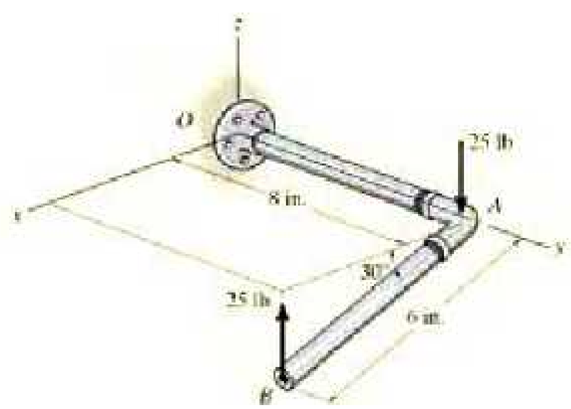


Fig 4-28

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Exercise 4.7:

Two couples act on the beam as shown. Determine the magnitude of F **so that the resultant couple moment is 300 lb.ft** counterclockwise. Where on the beam does the resultant couple act?

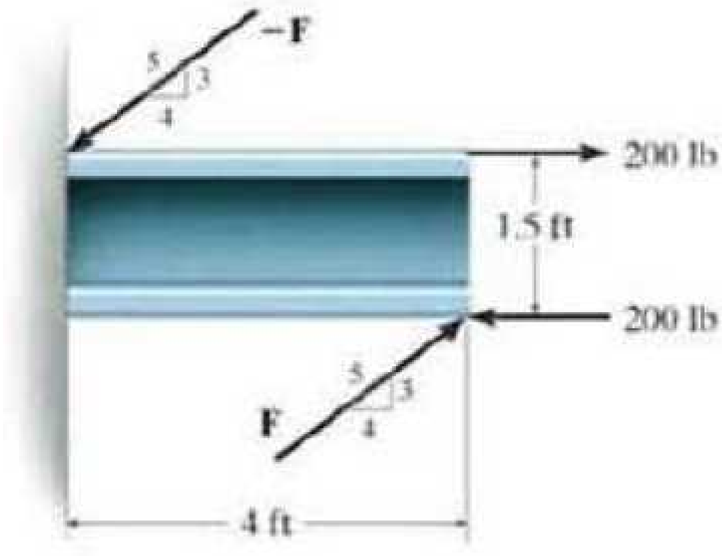

Fig 4-31