When the axial load is taken up by a thrust collar as shown in Fig. 17.2 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \frac{2}{3} \times \mu_1 \times W \left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right]$$

... (Assuming uniform pressure conditions)

$$= \mu_1 \times W \left(\frac{R_1 + R_2}{2} \right) = \mu_1 \; W R$$

....(Assuming uniform wear conditions)

where R_1 and R_2 = Outside and inside radii of collar,

R = Mean radius of collar $= \frac{R_1 + R_2}{2}$, and

μ_1 = Coefficient of friction for the collar.

(*b*) Thrust collar.

$\therefore$ Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2$$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1 \times l$$

Example 17.4. *A vertical two start square threaded screw of a 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever which is 400 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for collar bearing is 0.20.*

Solution. Given : d = 100 mm ; p = 20 mm ; W = 18 kN = 18 × 10^3N ; D_1 = 250 mm or R_1 = 125 mm ; D_2 = 100 mm or R_2 = 50 mm ; l = 400 mm ; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.20$

Force required at the end of lever

Let P = Force required at the end of lever.

Since the screw is a two start square threaded screw, therefore lead of the screw

$$= 2\,p = 2 \times 20 = 40 \text{ mm}$$

We know that $\tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{40}{\pi \times 100} = 0.127$

1. ***For raising the load***

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right]$$

$$= 18 \times 10^3 \left[\frac{0.127 + 0.15}{1 - 0.127 \times 0.15} \right] = 5083 \text{ N}$$

and mean radius of the collar,

$$R = \frac{R_1 + R_2}{2} = \frac{125 + 50}{2} = 87.5 \text{ mm}$$

$\therefore$ Total torque required at the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 \; WR$$

$$= 5083 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 569\,150 \text{ N-mm}$$

We know that torque required at the end of lever (T),

$569\,150 = P_1 \times l = P_1 \times 400$ or $P_1 = 569\,150/400 = 1423$ N **Ans.**

2. *For lowering the load*

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi\tan\alpha}\right]$$

$$= 18 \times 10^3 \left[\frac{0.15 - 0.127}{1 + 0.15 \times 0.127}\right] = 406.3 \text{ N}$$

and the total torque required the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 W R$$

$$= 406.3 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 335\ 315 \text{ N-mm}$$

We know that torque required at the end of lever (T),

$$335\ 315 = P_1 \times l = P_1 \times 400 \quad \text{or} \quad P_1 = 335\ 315 / 400 = 838.3 \text{ N} \textbf{ Ans.}$$

Example 17.15. *A screw jack is to lift a load of 80 kN through a height of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm². Design and draw the screw jack. The design should include the design of 1. screw, 2. nut, 3. handle and cup,and 4. body.*

Solution. Given : W = 80 kN = 80 × 10^3N ; H_1 = 400 mm = 0.4 m ; $\sigma_{et} = \sigma_{ec}$ = 200 MPa = 200 N/mm^2 ; τ_e = 120 MPa = 120 N/mm^2 ; $\sigma_{et(nut)}$ = 100 MPa = 100 N/mm^2 ; $\sigma_{ec\ (nut)}$ = 90 MPa = 90 N/mm^2 ; $\tau_{e(nut)}$ = 80 MPa = 80N/mm^2 ; p_b = 18 N/mm^2

The various parts of a screw jack are designed as discussed below:

1. Design of screw for spindle

Let d_c = Core diameter of the screw.

Since the screw is under compression, therefore load (W),

$$80 \times 10^3 = \frac{\pi}{4}(d_c)^2 \times \frac{\sigma_{ec}}{F.S.} = \frac{\pi}{4}(d_c)^2 \frac{200}{2} = 78.55\,(d_c)^2$$

... (Taking factor of safety, $F.S.$ = 2)

$\therefore$ $(d_c)^2 = 80 \times 10^3 / 78.55 = 1018.5$ or d_c = 32 mm

For square threads of normal series, the following dimensions of the screw are selected from Table 17.2.

*Core diameter, d_c = 38 mm **Ans.**

Nominal or outside diameter of spindle,

d_o = 46 mm **Ans.**

Pitch of threads, p = 8 mm **Ans.**

Now let us check for principal stresses:

We know that the mean diameter of screw,

$$d = \frac{d_o + d_c}{2} = \frac{46 + 38}{2} = 42 \text{ mm}$$

and $$\tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 42} = 0.0606$$

Assuming coefficient of friction between screw and nut,

$$\mu = \tan \phi = 0.14$$

∴ Torque required to rotate the screw in the nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2} = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}\right] \frac{d}{2}$$

$$= 80 \times 10^3 \left[\frac{0.0606 + 0.14}{1 - 0.0606 \times 0.14}\right] \frac{42}{2} = 340 \times 10^3 \text{ N-mm}$$

Now compressive stress due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}(d_c)^2} = \frac{80 \times 10^3}{\frac{\pi}{4}(38)^2} = 70.53 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\, T_1}{\pi (d_c)^3} = \frac{16 \times 340 \times 10^3}{\pi (38)^3} = 31.55 \text{ N/mm}^2$$

∴ Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \frac{1}{2}\left[70.53 + \sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2}[70.53 + 94.63] = 82.58 \text{ N/mm}^2$$

* From Table 17.2, we see that next higher value of 32 mm for the core diameter is 33 mm. By taking $d_c = 33$ mm, gives higher principal stresses than the permissible values. So core diameter is chosen as 38 mm.

The given value of σ_c is equal to $\frac{\sigma_{ec}}{F.S.}$, *i.e.* $\frac{200}{2} = 100 \text{ N/mm}^2$

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2} \times 94.63 = 47.315 \text{ N/mm}^2$$

The given value of τ is equal to $\frac{\tau_e}{F.S.}$, *i.e.* $\frac{120}{2} = 60 \text{ N/mm}^2$.

Since these maximum stresses are within limits, therefore design of screw for spindle is safe.

2. *Design for nut*

Let n = Number of threads in contact with the screwed spindle,

h = Height of nut = $n \times p$, and

t = Thickness of screw = $p / 2 = 8 / 2 = 4$ mm

Assume that the load is distributed uniformly over the cross-sectional area of nut.

We know that the bearing pressure (p_b),

$$18 = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{80 \times 10^3}{\frac{\pi}{4}\left[(46)^2 - (38)^2\right] n} = \frac{151.6}{n}$$

∴ $n = 151.6 / 18 = 8.4$ say 10 threads **Ans.**

and height of nut, $h = n \times p = 10 \times 8 = 80$ mm **Ans.**

Now, let us check the stresses induced in the screw and nut.

We know that shear stress in the screw,

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t} = \frac{80 \times 10^3}{\pi \times 10 \times 38 \times 4} = 16.15 \text{ N/mm}^2$$

... ($\because$ $t = p/2 = 4$ mm)

and shear stress in the nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o.t} = \frac{80 \times 10^3}{\pi \times 10 \times 46 \times 4} = 13.84 \text{ N/mm}^2$$

Since these stresses are within permissible limit, therefore design for nut is safe.

Let D_1 = Outer diameter of nut,

D_2 = Outside diameter for nut collar, and

t_1 = Thickness of nut collar.

First of all considering the tearing strength of nut, we have

$$W = \frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right]\sigma_t$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_1)^2 - (46)^2\right]\frac{100}{2} = 39.3\left[(D_1)^2 - 2116\right] \quad ...\left[\because \sigma_t = \frac{\sigma_{et(nut)}}{F.S.}\right]$$

or $(D_1)^2 - 2116 = 80 \times 10^3 / 39.3 = 2036$

$\therefore$ $(D_1)^2 = 2036 + 2116 = 4152$ or $D_1 = 65$ mm **Ans.**

Now considering the crushing of the collar of the nut, we have

$$W = \frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right]\sigma_c$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_2)^2 - (65)^2\right]\frac{90}{2} = 35.3\left[(D_2)^2 - 4225\right] \quad ...\left[\sigma_c = \frac{\sigma_{ec(nut)}}{F.S.}\right]$$

or $(D_2)^2 - 4225 = 80 \times 10^3 / 35.3 = 2266$

$\therefore$ $(D_2)^2 = 2266 + 4225 = 6491$ or $D_2 = 80.6$ say 82 mm **Ans.**

Considering the shearing of the collar of the nut, we have

$$W = \pi\, D_1 \times t_1 \times \tau$$

$$80 \times 10^3 = \pi \times 65 \times t_1 \times \frac{80}{2} = 8170\, t_1 \quad ...\left[\tau = \frac{\tau_{e(nut)}}{F.S.}\right]$$

$\therefore$ $t_1 = 80 \times 10^3/8170 = 9.8$ say 10 mm **Ans.**

3. *Design for handle and cup*

The diameter of the head (D_3) on the top of the screwed rod is usually taken as 1.75 times the outside diameter of the screw (d_o).

$\therefore$ $D_3 = 1.75\, d_o = 1.75 \times 46 = 80.5$ say 82 mm **Ans.**

The head is provided with two holes at the right angles to receive the handle for rotating the screw. The seat for the cup is made equal to the diameter of head, *i.e.* 82 mm and it is given chamfer at the top. The cup prevents the load from rotating. The cup is fitted to the head with a pin of diameter $D_4 = 20$ mm. The pin remains loose fit in the cup. Other dimensions for the cup may be taken as follows :

Height of cup = 50 mm **Ans.**

Thickness of cup = 10 mm **Ans.**

Diameter at the top of cup = 160 mm **Ans.**

Now let us find out the torque required (T_2) to overcome friction at the top of the screw.

Assuming uniform pressure conditions, we have

$$T_2 = \frac{2}{3} \times \mu_1 W \left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2} \right]$$

$$= \frac{2}{3} \times 0.14 \times 80 \times 10^3 \left[\frac{\left(\frac{82}{2}\right)^3 - \left(\frac{20}{2}\right)^3}{\left(\frac{82}{2}\right)^2 - \left(\frac{20}{2}\right)^2} \right] \quad \text{... (Assuming } \mu_1 = \mu)$$

$$= 7.47 \times 10^3 \left[\frac{(41)^3 - (10)^3}{(41)^2 - (10)^2} \right] = 321 \times 10^3 \text{ N-mm}$$

$\therefore$ Total torque to which the handle is subjected,

$$T = T_1 + T_2 = 340 \times 10^3 + 321 \times 10^3 = 661 \times 10^3 \text{ N-mm}$$

Assuming that a force of 300 N is applied by a person intermittently, therefore length of handle required

$$= 661 \times 10^3 / 300 = 2203 \text{ mm}$$

Allowing some length for gripping, we shall take the length of handle as 2250 mm.

A little consideration will show that an excessive force applied at the end of lever will cause bending. Considering bending effect, the maximum bending moment on the handle,

$$M = \text{Force applied} \times \text{Length of lever}$$

$$= 300 \times 2250 = 675 \times 10^3 \text{ N-mm}$$

Let D = Diameter of the handle.

Assuming that the material of the handle is same as that of screw, therefore taking bending stress $\sigma_b = \sigma_t = \sigma_{et} / 2 = 100 \text{ N/mm}^2$.

We know that the bending moment (M),

$$675 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times D^3 = \frac{\pi}{32} \times 100 \times D^3 = 9.82\ D^3$$

$\therefore$ $D^3 = 675 \times 10^3 / 9.82 = 68.74 \times 10^3$ or $D = 40.96$ say 42 mm **Ans.**

The height of head (H) is taken as $2D$.

$\therefore$ $H = 2\,D = 2 \times 42 = 84$ mm **Ans.**

Now let us check the screw for buckling load.

We know that the effective length for the buckling of screw,

$$L = \text{Lift of screw} + \frac{1}{2} \text{ Height of nut} = H_1 + h/2$$

$$= 400 + 80 / 2 = 440 \text{ mm}$$

When the screw reaches the maximum lift, it can be regarded as a strut whose lower end is fixed and the load end is free. We know that critical load,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4C\,\pi^2\,E} \left(\frac{L}{k} \right)^2 \right]$$

For one end fixed and other end free, $C = 0.25$.

Also $k = 0.25\,d_c = 0.25 \times 38 = 9.5$ mm

$$\therefore \quad W_{cr} = \frac{\pi}{4}(38)^2\ 200 \left[1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3} \left(\frac{440}{9.5} \right)^2 \right] \quad \text{... (Taking } \sigma_y = \sigma_{et})$$

$$= 226\,852\,(1 - 0.207) = 179\,894 \text{ N}$$

Since the critical load is more than the load at which the screw is designed (*i.e.* 80×10^3 N), therefore there is no chance of the screw to buckle.

4. *Design of body*

The various dimensions of the body may be fixed as follows:

Diameter of the body at the top,

$$D_5 = 1.5\ D_2 = 1.5 \times 82 = 123 \text{ mm} \quad \textbf{Ans.}$$

Thickness of the body,

$$t_3 = 0.25\ d_o = 0.25 \times 46 = 11.5 \text{ say } 12 \text{ mm} \quad \textbf{Ans.}$$

Inside diameter at the bottom,

$$D_6 = 2.25\ D_2 = 2.25 \times 82 = 185 \text{ mm} \quad \textbf{Ans.}$$

Outer diameter at the bottom,

$$D_7 = 1.75\ D_6 = 1.75 \times 185 = 320 \text{ mm} \quad \textbf{Ans.}$$

Thickness of base, $t_2 = 2\ t_1 = 2 \times 10 = 20$ mm **Ans.**

Height of the body = Max. lift + Height of nut + 100 mm extra

= 400 + 80 + 100 = 580 mm **Ans.**

The body is made tapered in order to achieve stability of jack.

Let us now find out the efficiency of the screw jack. We know that the torque required to rotate the screw with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 80 \times 10^3 \times 0.0606 \times \frac{42}{2} = 101\ 808 \text{ N-mm}$$

$\therefore$ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{101808}{661 \times 10^3} = 0.154 \text{ or } 15.4\% \quad \textbf{Ans.}$$