17.12 Acme or Trapezoidal Threads

We know that the normal reaction in case of a square threaded screw is

$$R_N = W \cos \alpha,$$

where α is the helix angle.

But in case of Acme or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load (W).

Consider an Acme or trapezoidal thread as shown in Fig. 17.6.

Let $**2\beta$ = Angle of the Acme thread, and

β = Semi-angle of the thread.

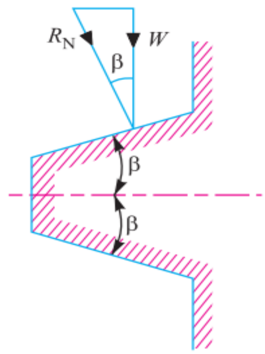

Fig. 17.6. Acme or trapeonidal threads.

* The material of screw is usually steel and the nut is made of cast iron, gun metal, phosphor bronze in order to keep the wear to a mininum.

** For Acme threads, $2\beta = 29°$, and for trapezoidal threads, $2\beta = 30°$.

$$\therefore \quad R_N = \frac{W}{\cos \beta}$$

and frictional force,
$$F = \mu . R_N = \mu \times \frac{W}{\cos \beta} = \mu_1 . W$$

where $\mu / \cos \beta = \mu_1$, known as ***virtual coefficient of friction.***

Notes : 1. When coefficient of friction, $\mu_1 = \frac{\mu}{\cos \beta}$ is considered, then the Acme thread is equivalent to a square thread.

2. All equations of square threaded screw also hold good for Acme threads. In case of Acme threads, μ_1 (*i.e.* $\tan \phi_1$) may be substituted in place of μ (*i.e.* $\tan \phi$). Thus for Acme threads,

$$P = W \tan (\alpha + \phi_1)$$

where ϕ_1 = Virtual friction angle, and $\tan \phi_1 = \mu_1$.

17.13 Stresses in Power Screws

A power screw must have adequate strength to withstand axial load and the applied torque. Following types of stresses are induced in the screw.

1. *Direct tensile or compressive stress due to an axial load.* The direct stress due to the axial load may be determined by dividing the axial load (W) by the minimum cross-sectional area of the screw (A_c) *i.e.* area corresponding to minor or core diameter (d_c).

$\therefore$ Direct stress (tensile or compressive)

$$= \frac{W}{A_c}$$

This is only applicable when the axial load is compressive and the unsupported length of the screw between the load and the nut is short. But when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions. In such cases, the cross-sectional area corresponding to core diameter may be obtained by using Rankine-Gordon formula or J.B. Johnson's formula. According to this,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4 C \pi^2 E} \left(\frac{L}{k} \right)^2 \right]$$

$$\therefore \quad \sigma_c = \frac{W}{A_c} \left[\frac{1}{1 - \frac{\sigma_y}{4 C \pi^2 E} \left(\frac{L}{k} \right)^2} \right]$$

where W_{cr} = Critical load,

σ_y = Yield stress,

L = Length of screw,

k = Least radius of gyration,

C = End-fixity coefficient,

E = Modulus of elasticity, and

σ_c = Stress induced due to load W.

Note : In actual practice, the core diameter is first obtained by considering the screw under simple compression and then checked for critical load or buckling load for stability of the screw.

2. *Torsional shear stress.* Since the screw is subjected to a twisting moment, therefore torsional shear stress is induced. This is obtained by considering the minimum cross-section of the screw. We know that torque transmitted by the screw,

$$T = \frac{\pi}{16} \times \tau (d_c)^3$$

or shear stress induced,

$$\tau = \frac{16\,T}{\pi (d_c)^3}$$

When the screw is subjected to both direct stress and torsional shear stress, then the design must be based on maximum shear stress theory, according to which maximum shear stress on the minor diameter section,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t \text{ or } \sigma_c)^2 + 4\,\tau^2}$$

It may be noted that when the unsupported length of the screw is short, then failure will take place when the maximum shear stress is equal to the shear yield strength of the material. In this case, shear yield strength,

$$\tau_y = \tau_{max} \times \text{Factor of safety}$$

3. *Shear stress due to axial load.* The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to the axial load. Assuming that the load is uniformly distributed over the threads in contact, we have

Shear stress for screw,

$$\tau_{(screw)} = \frac{W}{\pi\, n\,.d_c\,.\,t}$$

and shear stress for nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n\,.d_o\,.\,t}$$

where W = Axial load on the screw,

n = Number of threads in engagement,

d_c = Core or root diameter of the screw,

d_o = Outside or major diameter of nut or screw, and

t = Thickness or width of thread.

4. *Bearing pressure.* In order to reduce wear of the screw and nut, the bearing pressure on the thread surfaces must be within limits. In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between the nut and screw and the nature of lubrication. Assuming that the load is uniformly distributed over the threads in contact, the bearing pressure on the threads is given by

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{{}^{*}W}{\pi\, d.t.n}$$

where d = Mean diameter of screw,

t = Thickness or width of screw = p / 2, and

n = Number of threads in contact with the nut

$$= \frac{\text{Height of the nut}}{\text{Pitch of threads}} = \frac{h}{p}$$

Therefore, from the above expression, the height of nut or the length of thread engagement of the screw and nut may be obtained.

The following table shows some limiting values of bearing pressures.

* We know that $\frac{(d_o)^2 - (d_c)^2}{4} = \frac{d_o + d_c}{2} \times \frac{d_o - d_c}{2} = d \times \frac{p}{2} = d.t$

17.14 Design of Screw Jack

A bottle screw jack for lifting loads is shown in Fig. 17.11. The various parts of the screw jack are as follows:

1. Screwed spindle having square threaded screws,
2. Nut and collar for nut,
3. Head at the top of the screwed spindle for handle,
4. Cup at the top of head for the load, and
5. Body of the screw jack.

In order to design a screw jack for a load W, the following procedure may be adopted:

1. First of all, find the core diameter (d_c) by considering that the screw is under pure compression, *i.e.*

$$W = \sigma_c \times A_c = \sigma_c \times \frac{\pi}{4}(d_c)^2$$

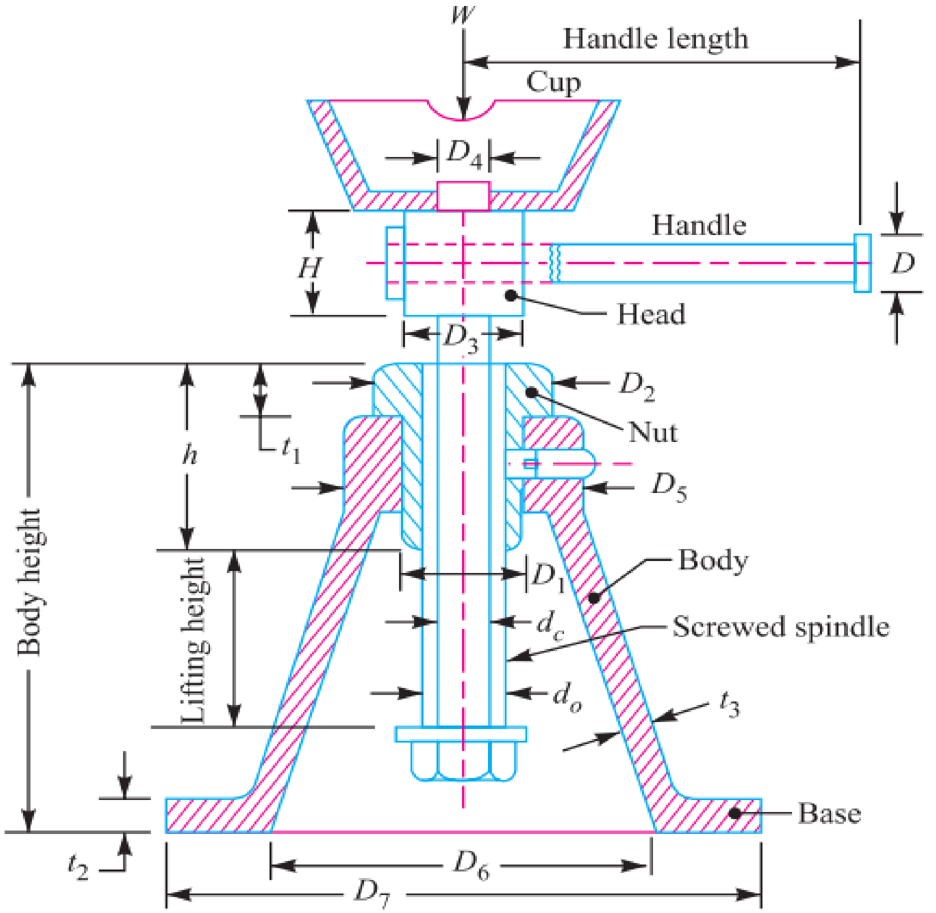

Fig. 17.11. Screw jack.

2. Find the torque (T_1) required to rotate the screw and find the shear stress (τ) due to this torque.

We know that the torque required to lift the load,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2}$$

where P = Effort required at the circumference of the screw, and

d = Mean diameter of the screw.

$\therefore$ Shear stress due to torque T_1,

$$\tau = \frac{16\, T_1}{\pi (d_c)^3}$$

Also find direct compressive stress (σ_c) due to axial load, *i.e.*

$$\sigma_c = \frac{W}{\frac{\pi}{4} (d_c)^2}$$

3. Find the principal stresses as follows:

Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right]$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2}$$

These stresses should be less than the permissible stresses.

4. Find the height of nut (h), considering the bearing pressure on the nut. We know that the bearing pressure on the nut,

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n}$$

where n = Number of threads in contact with screwed spindle.

$\therefore$ Height of nut, $h = n \times p$

where p = Pitch of threads.

5. Check the stressess in the screw and nut as follows :

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t}$$

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o.t}$$

where t = Thickness of screw $= p / 2$

6. Find inner diameter (D_1), outer diameter (D_2) and thickness (t_1) of the nut collar.

The inner diameter (D_1) is found by considering the tearing strength of the nut. We know that

$$W = \frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right] \sigma_t$$

The outer diameter (D_2) is found by considering the crushing strength of the nut collar. We know that

$$W = \frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right] \sigma_c$$

The thickness (t_1) of the nut collar is found by considering the shearing strength of the nut collar. We know that

$$W = \pi\, D_1.t_1.\tau$$

7. Fix the dimensions for the diameter of head (D_3) on the top of the screw and for the cup. Take D_3 = 1.75 d_o. The seat for the cup is made equal to the diameter of head and it is chamfered at the top. The cup is fitted with a pin of diameter $D_4 = D_3 / 4$ approximately. This pin remains a loose fit in the cup.

8. Find the torque required (T_2) to overcome friction at the top of screw. We know that

$$T_2 = \frac{2}{3} \times \mu_1 W \left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2} \right] \quad \text{... (Assuming uniform pressure conditions)}$$

$$= \mu_1 W \left[\frac{R_3 + R_4}{2} \right] = \mu_1 \, W \, R \quad \text{... (Assuming uniform wear conditions)}$$

where R_3 = Radius of head, and

R_4 = Radius of pin.

9. Now the total torque to which the handle will be subjected is given by

$$T = T_1 + T_2$$

Assuming that a person can apply a force of 300 – 400 N intermittently, the length of handle required

$$= T / 300$$

The length of handle may be fixed by giving some allowance for gripping.

10. The diameter of handle (D) may be obtained by considering bending effects. We know that bending moment,

$$M = \frac{\pi}{32} \times \sigma_b \times D^3 \quad \text{... } (\because \sigma_b = \sigma_t \text{ or } \sigma_c)$$

11. The height of head (H) is usually taken as twice the diameter of handle, *i.e.* $H = 2D$.

12. Now check the screw for buckling load.

Effective length or unsupported length of the screw,

$$L = \text{Lift of screw} + \frac{1}{2} \text{ Height of nut}$$

We know that buckling or critical load,

$$W_{cr} = A_c . \sigma_y \left[1 - \frac{\sigma_y}{4 \, C \, \pi^2 \, E} \left(\frac{L}{k} \right)^2 \right]$$

where σ_y = Yield stress,

C = End fixity coefficient. The screw is considered to be a strut with lower end fixed and load end free. For one end fixed and the other end free, $C = 0.25$

k = Radius of gyration = $0.25 \, d_c$

The buckling load as obtained by the above expression must be higher than the load at which the screw is designed.

13. Fix the dimensions for the body of the screw jack.

14. Find efficiency of the screw jack.