Eccentric Loaded Riveted Joint

When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the joint is said to be an ***eccentric loaded riveted joint***, as shown in Fig. 9.23 (*a*). The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or primary shear.

Let P = Eccentric load on the joint, and

e = Eccentricity of the load *i.e.* the distance between the line of action of the load and the centroid of the rivet system *i.e.* G.

The following procedure is adopted for the design of an eccentrically loaded riveted joint.

1. First of all, find the centre of gravity G of the rivet system.

Let A = Cross-sectional area of each rivet,

x_1, x_2, x_3 etc. = Distances of rivets from OY, and

y_1, y_2, y_3 etc. = Distances of rivets from OX.

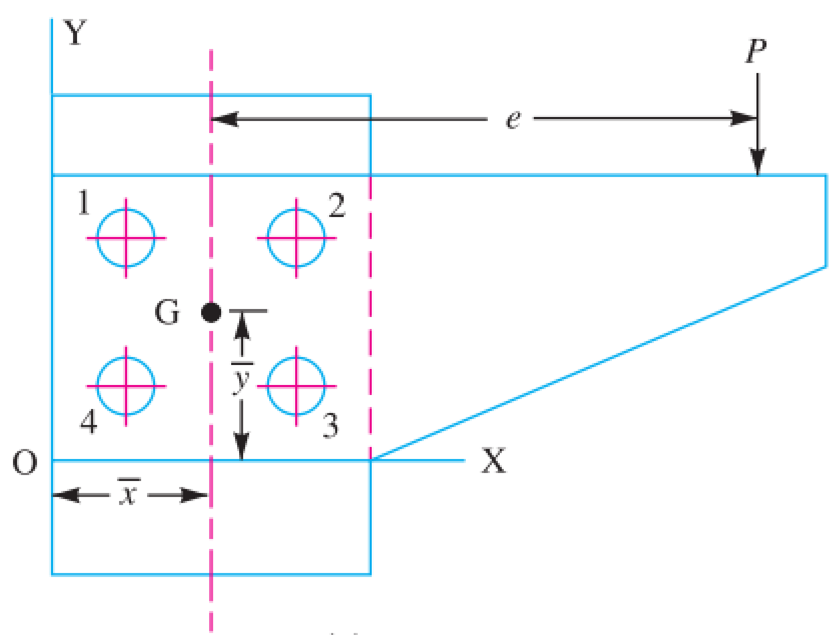
We know that

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + ...}{A_1 + A_2 + A_3 + ...} = \frac{A x_1 + A x_2 + A x_3 + ...}{n . A}$$

$$= \frac{x_1 + x_2 + x_3 + ...}{n} \quad \text{...(where } n = \text{Number of rivets)}$$

Similarly,

$$\bar{y} = \frac{y_1 + y_2 + y_3 + ...}{n}$$



(*a*)

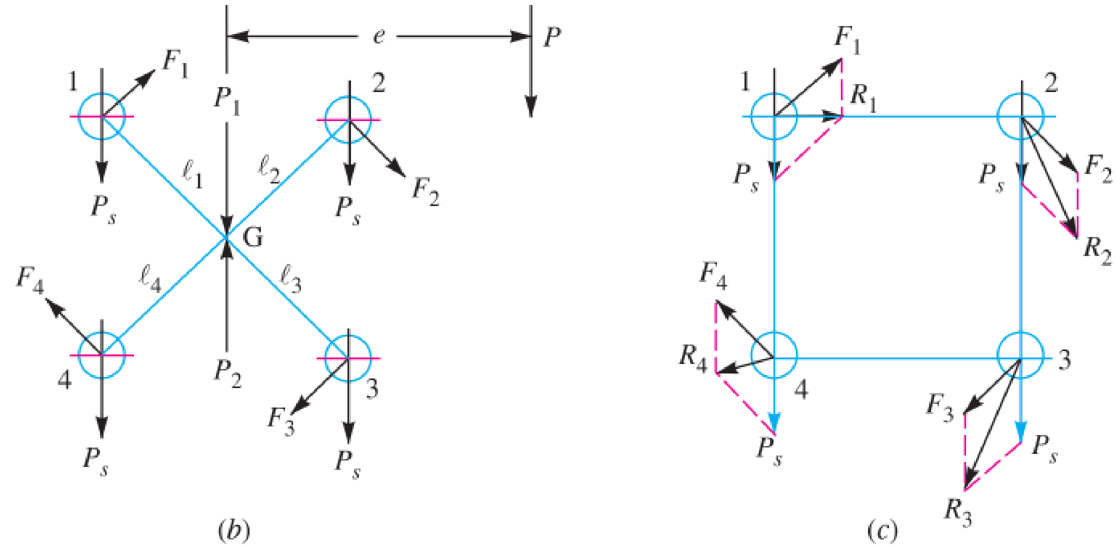


Fig. 9.23. Eccentric loaded riveted joint.

2. Introduce two forces P_1 and P_2 at the centre of gravity '*G*' of the rivet system. These forces are equal and opposite to *P* as shown in Fig. 9.23 (*b*).

3. Assuming that all the rivets are of the same size, the effect of $P_1 = P$ is to produce direct shear load on each rivet of equal magnitude. Therefore, direct shear load on each rivet,

$$P_s = \frac{P}{n}, \text{ acting parallel to the load } P.$$

4. The effect of $P_2 = P$ is to produce a turning moment of magnitude $P \times e$ which tends to rotate the joint about the centre of gravity '*G*' of the rivet system in a clockwise direction. Due to the turning moment, secondary shear load on each rivet is produced. In order to find the secondary shear load, the following two assumptions are made :

(*a*) The secondary shear load is proportional to the radial distance of the rivet under consideration from the centre of gravity of the rivet system.

(*b*) The direction of secondary shear load is perpendicular to the line joining the centre of the rivet to the centre of gravity of the rivet system..

Let $F_1, F_2, F_3 ...$ = Secondary shear loads on the rivets 1, 2, 3...etc.

$l_1, l_2, l_3 ...$ = Radial distance of the rivets 1, 2, 3 ...etc. from the centre of gravity '*G*' of the rivet system.

∴ From assumption (*a*),

$$F_1 \propto l_1 \,;\, F_2 \propto l_2 \text{ and so on}$$

or

$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} = ...$$

∴

$$F_2 = F_1 \times \frac{l_2}{l_1}, \text{ and } F_3 = F_1 \times \frac{l_3}{l_1}$$

We know that the sum of the external turning moment due to the eccentric load and of internal resisting moment of the rivets must be equal to zero.

$$\therefore \quad P.e = F_1.l_1 + F_2.l_2 + F_3.l_3 + ...$$

$$= F_1.l_1 + F_1 \times \frac{l_2}{l_1} \times l_2 + F_1 \times \frac{l_3}{l_1} \times l_3 + ...$$

$$= \frac{F_1}{l_1} [(l_1)^2 + (l_2)^2 + (l_3)^2 + ...]$$

From the above expression, the value of F_1 may be calculated and hence F_2 and F_3 etc. are known. The direction of these forces are at right angles to the lines joining the centre of rivet to the centre of gravity of the rivet system, as shown in Fig. 9.23 (*b*), and should produce the moment in the same direction (*i.e.* clockwise or anticlockwise) about the centre of gravity, as the turning moment ($P \times e$).

5. The primary (or direct) and secondary shear load may be added vectorially to determine the resultant shear load (R) on each rivet as shown in Fig. 9.23 (*c*). It may also be obtained by using the relation

$$R = \sqrt{(P_s)^2 + F^2 + 2 P_s \times F \times \cos \theta}$$

where θ = Angle between the primary or direct shear load (P_s) and secondary shear load (F).

When the secondary shear load on each rivet is equal, then the heavily loaded rivet will be one in which the included angle between the direct shear load and secondary shear load is minimum. The maximum loaded rivet becomes the critical one for determining the strength of the riveted joint. Knowing the permissible shear stress (τ), the diameter of the rivet hole may be obtained by using the relation,

$$\text{Maximum resultant shear load } (R) = \frac{\pi}{4} \times d^2 \times \tau$$

From Table 9.7, the standard diameter of the rivet hole (d) and the rivet diameter may be specified, according to IS : 1929 – 1982 (Reaffirmed 1996).

Notes : 1. In the solution of a problem, the primary and shear loads may be laid off approximately to scale and generally the rivet having the maximum resultant shear load will be apparent by inspection. The values of the load for that rivet may then be calculated.

2. When the thickness of the plate is given, then the diameter of the rivet hole may be checked against crushing.

3. When the eccentric load P is inclined at some angle, then the same procedure as discussed above may be followed to find the size of rivet (See Example 9.18).

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Example 9.14. *An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown in Fig. 9.24.*

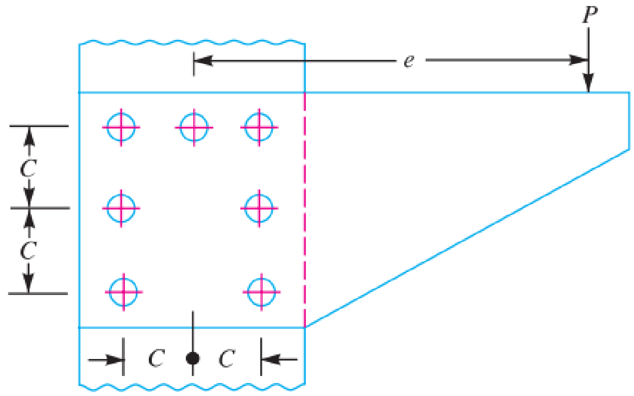

Fig. 9.24

The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 kN ; rivet spacing, C = 100 mm; load arm, e = 400 mm.

Permissible shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the joint.

Solution. Given : t = 25 mm ; P = 50 kN = 50 × 10^3 N ; e = 400 mm ; n = 7 ; τ = 65 MPa = 65 N/mm^2 ; σ_c = 120 MPa = 120 N/mm^2

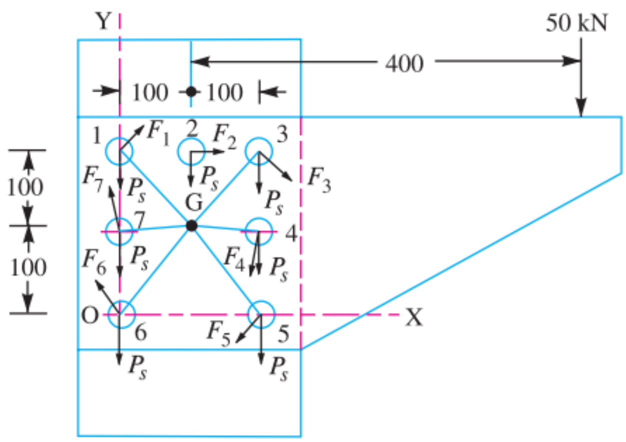

Fig. 9.25

First of all, let us find the centre of gravity (G) of the rivet system.

Let $\bar{x}$ = Distance of centre of gravity from OY,

$\bar{y}$ = Distance of centre of gravity from OX,

$x_1, x_2, x_3...$ = Distances of centre of gravity of each rivet from OY, and

$y_1, y_2, y_3...$ = Distances of centre of gravity of each rivet from OX.

We know that

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n}$$

$$= \frac{100 + 200 + 200 + 200}{7} = 100 \text{ mm} \qquad ...(\because x_1 = x_6 = x_7 = 0)$$

and

$$\bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7}{n}$$

$$= \frac{200 + 200 + 200 + 100 + 100}{7} = 114.3 \text{ mm} \qquad ...(\because y_5 = y_6 = 0)$$

∴ The centre of gravity (*G*) of the rivet system lies at a distance of 100 mm from *OY* and 114.3 mm from *OX*, as shown in Fig. 9.25.

We know that direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{50 \times 10^3}{7} = 7143 \text{ N}$$

The direct shear load acts parallel to the direction of load *P* *i.e.* vertically downward as shown in Fig. 9.25.

Turning moment produced by the load *P* due to eccentricity (*e*)

$$= P \times e = 50 \times 10^3 \times 400 = 20 \times 10^6 \text{ N-mm}$$

This turning moment is resisted by seven rivets as shown in Fig. 9.25.

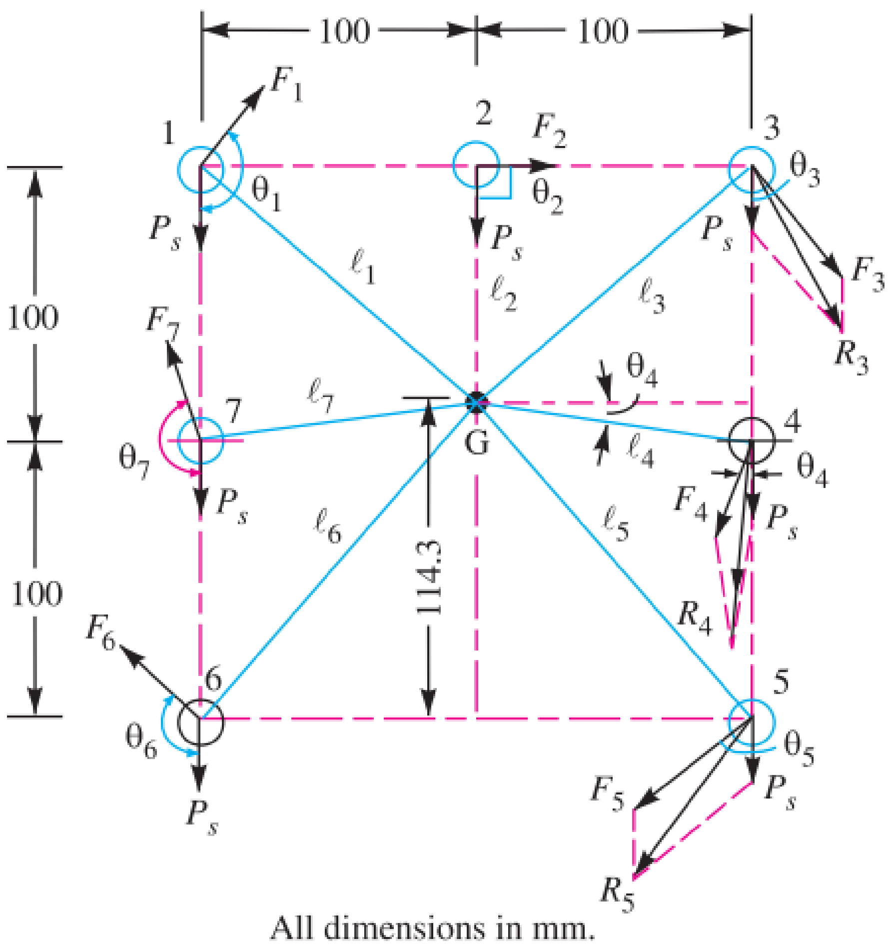

Fig. 9.26

Let F_1, F_2, F_3, F_4, F_5, F_6 and F_7 be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at distances l_1, l_2, l_3, l_4, l_5, l_6 and l_7 respectively from the centre of gravity of the rivet system as shown in Fig. 9.26.

From the geometry of the figure, we find that

$$l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm}$$

$$l_2 = 200 - 114.3 = 85.7 \text{ mm}$$

$$l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm}$$

and

$$l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = 152 \text{ mm}$$

Now equating the turning moment due to eccentricity of the load to the resisting moment of the rivets, we have

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 + (l_7)^2\right]$$

$$= \frac{F_1}{l_1}\left[2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2\right]$$

$$\text{....}(\because l_1 = l_3;\ l_4 = l_7 \text{ and } l_5 = l_6)$$

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2\right]$$

$$20 \times 10^6 \times 131.7 = F_1(34\ 690 + 7345 + 20\ 402 + 46\ 208) = 108\ 645\ F_1$$

$$F_1 = 20 \times 10^6 \times 131.7 / 108\ 645 = 24\ 244 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

$$F_2 = F_1 \times \frac{l_2}{l_1} = 24\ 244 \times \frac{85.7}{131.7} = 15\ 776 \text{ N}$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 24\ 244 \text{ N} \qquad \text{...}(\because l_1 = l_3)$$

$$F_4 = F_1 \times \frac{l_4}{l_1} = 24\ 244 \times \frac{101}{131.7} = 18\ 593 \text{ N}$$

$$F_5 = F_1 \times \frac{l_5}{l_1} = 24\ 244 \times \frac{152}{131.7} = 27\ 981 \text{ N}$$

$$F_6 = F_1 \times \frac{l_6}{l_1} = F_5 = 27\ 981 \text{ N} \qquad \text{...}(\because l_6 = l_5)$$

$$F_7 = F_1 \times \frac{l_7}{l_1} = F_4 = 18\ 593 \text{ N} \qquad \text{...}(\because l_7 = l_4)$$

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 4 and 5 are heavily loaded. Let us now find the angles between the direct and secondary shear load for these three rivets. From the geometry of Fig. 9.26, we find that

$$\cos\theta_3 = \frac{100}{l_3} = \frac{100}{131.7} = 0.76$$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

$$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = 0.658$$

Now resultant shear load on rivet 3,

$$R_3 = \sqrt{(P_s)^2 + (F_3)^2 + 2P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(7143)^2 + (24\ 244)^2 + 2 \times 7143 \times 24\ 244 \times 0.76} = 30\ 033 \text{ N}$$

Resultant shear load on rivet 4,

$$R_4 = \sqrt{(P_s)^2 + (F_4)^2 + 2\ P_s \times F_4 \times \cos\theta_4}$$

$$= \sqrt{(7143)^2 + (18\ 593)^2 + 2 \times 7143 \times 18\ 593 \times 0.99} = 25\ 684 \text{ N}$$

and resultant shear load on rivet 5,

$$R_5 = \sqrt{(P_s)^2 + (F_5)^2 + 2\ P_s \times F_5 \times \cos\theta_5}$$

$$= \sqrt{(7143)^2 + (27\ 981)^2 + 2 \times 7143 \times 27\ 981 \times 0.658} = 33\ 121 \text{ N}$$

The resultant shear load may be determined graphically, as shown in Fig. 9.26.

From above we see that the maximum resultant shear load is on rivet 5. If d is the diameter of rivet hole, then maximum resultant shear load (R_5),

$$33\ 121 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 65 = 51\ d^2$$

$$\therefore \quad d^2 = 33\ 121 / 51 = 649.4 \quad \text{or} \quad d = 25.5 \text{ mm}$$

From Table 9.7, we see that according to IS : 1929–1982 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 25.5 mm and the corresponding diameter of rivet is 24 mm.

Let us now check the joint for crushing stress. We know that

$$\text{Crushing stress} = \frac{\text{Max. load}}{\text{Crushing area}} = \frac{R_5}{d \times t} = \frac{33\ 121}{25.5 \times 25}$$

$$= 51.95 \text{ N/mm}^2 = 51.95 \text{ MPa}$$

Since this stress is well below the given crushing stress of 120 MPa, therefore the design is satisfactory.