Example 23.1. *A compression coil spring made of an alloy steel is having the following specifications :*

Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given : $D = 50$ mm ; $d = 5$ mm ; *$n = 20$; $W = 500$ N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

$\therefore$ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

nd maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi\, d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$$= 534.7 \text{ MPa}$$ **Ans.**

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm^2 ; $G = 84$ kN/mm^2 $= 84 \times 10^3$ N/mm^2

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$\therefore$ Spring index, $$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\, d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\, W$$

$\therefore$ $$W = 350 / 0.848 = 412.7 \text{ N}$$ **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm}$$ **Ans.**

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\, W$$

$$\therefore \quad W = 350 / 0.913 = 383.4 \text{ N } \textbf{Ans.}$$

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$

Example 23.4. *A mechanism used in printing machinery consists of a tension spring assembled with a preload of 30 N. The wire diameter of spring is 2 mm with a spring index of 6. The spring has 18 active coils. The spring wire is hard drawn and oil tempered having following material properties:*

Design shear stress = 680 MPa

Modulus of rigidity = 80 kN/mm²

Determine : 1. the initial torsional shear stress in the wire; 2. spring rate; and 3. the force to cause the body of the spring to its yield strength.

Solution. Given : $W_i = 30$ N ; $d = 2$ mm ; $C = D/d = 6$; $n = 18$; $\tau = 680$ MPa $= 680$ N/mm^2 ; $G = 80$ kN/mm^2 $= 80 \times 10^3$ N/mm^2

1. *Initial torsional shear stress in the wire*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

∴ Initial torsional shear stress in the wire,

$$\tau_i = K \times \frac{8\,W_i \times C}{\pi\, d^2} = 1.2525 \times \frac{8 \times 30 \times 6}{\pi \times 2^2} = 143.5 \text{ N/mm}^2$$

$$= 143.5 \text{ MPa } \textbf{Ans.}$$

2. *Spring rate*

We know that spring rate (or stiffness of the spring),

$$= \frac{G.d}{8\,C^3.n} = \frac{80 \times 10^3 \times 2}{8 \times 6^3 \times 18} = 5.144 \text{ N/mm } \textbf{Ans.}$$

3. Force to cause the body of the spring to its yield strength

Let W = Force to cause the body of the spring to its yield strength.

We know that design or maximum shear stress (τ),

$$680 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.2525 \times \frac{8W \times 6}{\pi \times 2^2} = 4.78\,W$$

$\therefore$ $W = 680 / 4.78 = 142.25$ N **Ans.**

23.13 Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta \quad \ldots(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation (*i*), we have

$$U = \frac{1}{2} \times \frac{\pi\,d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right)$$

Note : When a load (say P) falls on a spring through a height h, then the energy absorbed in a spring is given by

$$U = P\,(h + \delta) = \tfrac{1}{2}\,W.\delta$$

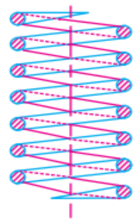
where W = Equivalent static load *i.e.* the gradually applied load which shall produce the same effect as by the falling load P, and

δ = Deflection produced in the spring.

23.2 Types of Springs

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

1. *Helical springs.* The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig. 23.1 (*a*) and ***tension helical spring*** as shown in Fig. 23.1 (*b*).



(*a*) Compression helical spring.

(*b*) Tension helical spring.

Fig. 23.1. Helical springs.

The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

In ***open coiled helical springs,*** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(*a*) These are easy to manufacture.

(*b*) These are available in wide range.

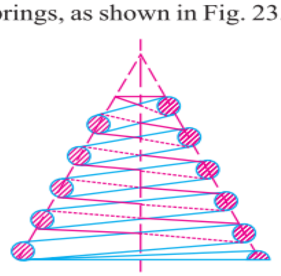
(*c*) These are reliable.

(*d*) These have constant spring rate.

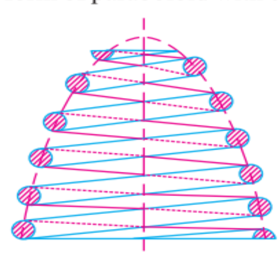
(*e*) Their performance can be predicted more accurately.

(*f*) Their characteristics can be varied by changing dimensions.

2. *Conical and volute springs.* The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. 23.2 (*a*), is wound with a uniform pitch whereas the volute springs, as shown in Fig. 23.2 (*b*), are wound in the form of paraboloid with constant pitch



(*a*) Conical spring.



(*b*) Volute spring.

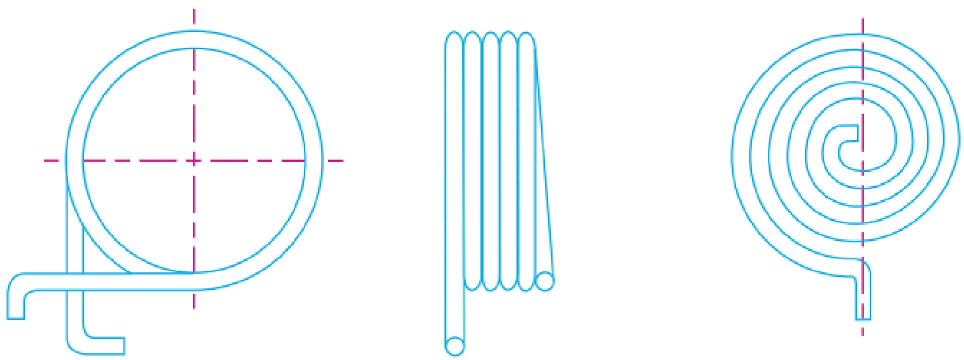
Fig. 23.2. Conical and volute springs.

and lead angles. The springs may be made either partially or completely telescoping. In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate. This characteristic is sometimes utilised in vibration problems where springs are used to support a body that has a varying mass.

The major stresses produced in conical and volute springs are also shear stresses due to twisting.

3. *Torsion springs.* These springs may be of ***helical*** or ***spiral*** type as shown in Fig. 23.3. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.

The major stresses produced in torsion springs are tensile and compressive due to bending.



(*a*) Helical torsion spring.

(*b*) Spiral torsion spring.

Fig. 23.3. Torsion springs.

4. *Laminated or leaf springs.* The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. 23.4. These are mostly used in automobiles.

The major stresses produced in leaf springs are tensile and compressive stresses.

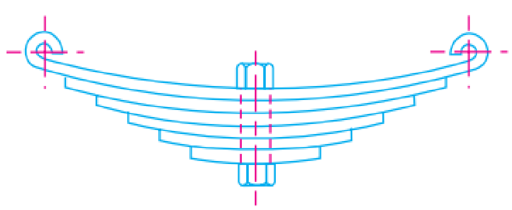


Fig. 23.4. Laminated or leaf springs.

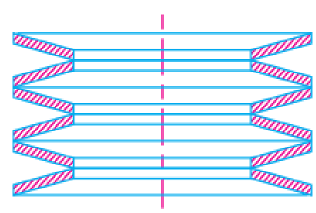


Fig. 23.5. Disc or bellevile springs.

5. *Disc or bellevile springs.* These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. 23.5. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

Table 23.1. Values of allowable shear stress, Modulus of elasticity and Modulus of rigidity for various spring materials.

Material	*Allowable shear stress (τ) MPa*			*Modulus of rigidity (G) kN/m²*	*Modulus of elasticity (E) kN/mm²*
	Severe service	*Average service*	*Light service*		
1. Carbon steel					
(*a*) Upto to 2.125 mm dia.	420	525	651	80	210
(*b*) 2.125 to 4.625 mm	385	483	595	80	210
(*c*) 4.625 to 8.00 mm	336	420	525	80	210
(*d*) 8.00 to 13.25 mm	294	364	455	80	210
(*e*) 13.25 to 24.25 mm	252	315	392	80	210
(*f*) 24.25 to 38.00 mm	224	280	350	80	210
2. Music wire	392	490	612	80	210
3. Oil tempered wire	336	420	525	80	210
4. Hard-drawn spring wire	280	350	437.5	80	210
5. Stainless-steel wire	280	350	437.5	70	196
6. Monel metal	196	245	306	44	105
7. Phosphor bronze	196	245	306	44	105
8. Brass	140	175	219	35	100

The standard size of spring wire may be selected from the following table :

Table 23.2. Standard wire gauge (SWG) number and corresponding diameter of spring wire.

SWG	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

Table 23.3. Total number of turns, solid length and free length for different types of end connections.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\,d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\,d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\,d$	$p \times n + 2d$

where n = Number of active turns,

p = Pitch of the coils, and

d = Diameter of the spring wire.