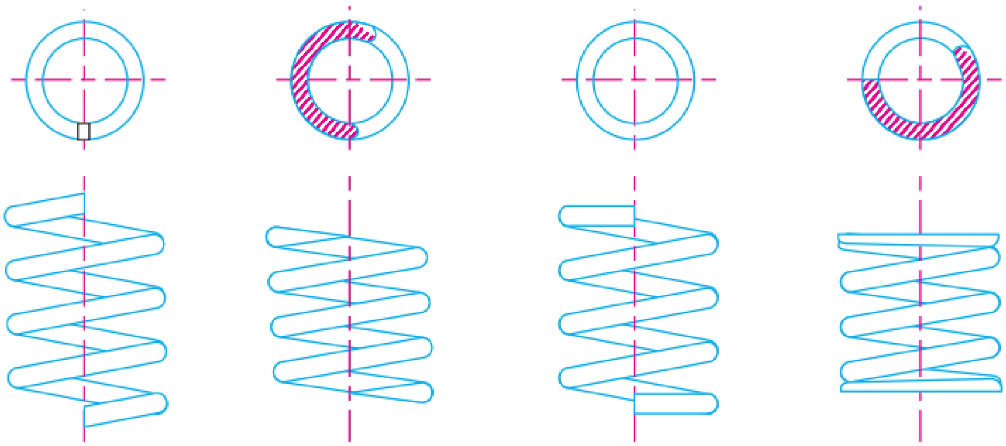


Fig 23.7. End connections for compression helical spring.

Table 23.3. Total number of turns, solid length and free length for different types of end connections.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\, d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\, d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\, d$	$p \times n + 2d$

where n = Number of active turns,

p = Pitch of the coils, and

d = Diameter of the spring wire.

23.18 Concentric or Composite Springs

A concentric or composite spring is used for one of the following purposes :

1. To obtain greater spring force within a given space.

2. To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. 23.22 (*a*) and are compressed equally. Such springs are used in automobile clutches, valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems.

Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. 23.22 (*b*). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force.

The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor (K), it is desirable to have the same spring index (C).

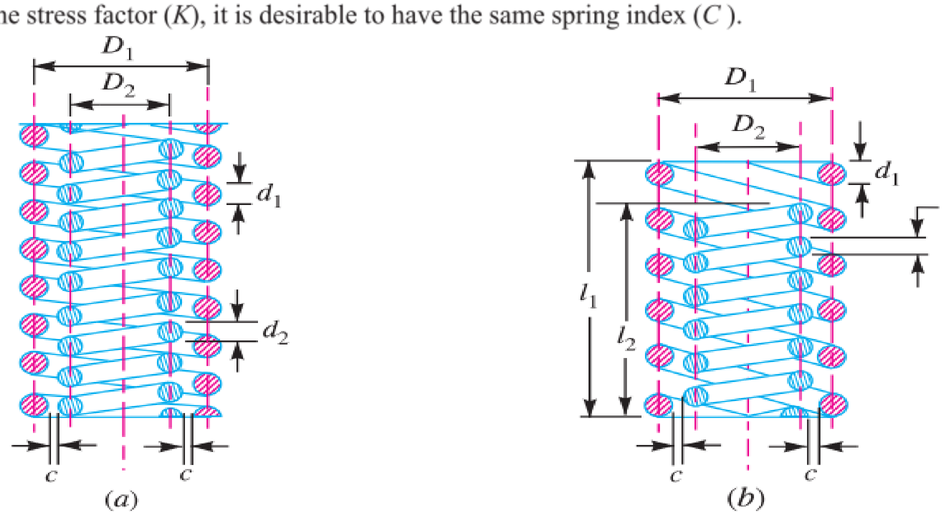


Fig. 23.22. Concentric springs.

Consider a concentric spring as shown in Fig. 23.22 (*a*).

Let W = Axial load,

W_1 = Load shared by outer spring,

W_2 = Load shared by inner spring,

d_1 = Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of inner spring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of inner spring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of inner spring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3} \qquad \text{...}(i)$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

or
$$\frac{8W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \qquad \text{...}(ii)$$

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e.*

$$n_1.d_1 = n_2.d_2$$

∴ The equation **(*ii*)** may be written as

$$\frac{W_1\,(D_1)^3}{(d_1)^5} = \frac{W_2\,(D_2)^3}{(d_2)^5} \qquad \text{...}(iii)$$

Now dividing equation **(*iii*)** by equation **(*i*)**, we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \qquad \text{...}(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same.

From equations **(*i*)** and **(*iv*)**, we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \qquad \text{..}(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as $\frac{d_1 - d_2}{2}$.

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or $$\frac{D_1 - D_2}{2} = d_1 \qquad ...(vi)$$

From equation **(*iv*)**, we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation **(*vi*)**, we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$\therefore \qquad d_1\,(C-2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2} \qquad ...(vii)$$

Example 23.19. *A concentric spring for an aircraft engine valve is to exert a maximum force of 5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solid length and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both the springs is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and (c) the number of active coils in each spring.*

Assume G = 80 kN/mm² and diametral clearance to be equal to the difference between the wire diameters.

Solution. Given : $W = 5000$ N ; $\delta = 40$ mm ; $\tau_1 = \tau_2 = 850$ MPa $= 850$ N/mm^2 ; $C = 6$; $G = 80$ kN/mm$^2 = 80 \times 10^3$ N/mm^2

The concentric spring is shown in Fig. 23.22 (*a*).

(a) Load shared by each spring

Let W_1 and W_2 = Load shared by outer and inner spring respectively,

d_1 and d_2 = Diameter of spring wires for outer and inner springs respectively, and

D_1 and D_2 = Mean diameter of the outer and inner springs respectively.

* The net clearance between the two springs is given by

$$2\,c = (D_1 - D_2) - (d_1 + d_2)$$

Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

or $$D_1 - D_2 = 2\,d_1$$

We know that $$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

$$\therefore \qquad C.d_1 - C.d_2 = 2\,d_1$$

or $$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \qquad ...(i)$$

We also know that $$\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \qquad ...(ii)$$

and $$W_1 + W_2 = W = 5000 \text{ N} \qquad ...(iii)$$

From equations **(*ii*)** and **(*iii*)**, we find that

$W_1 = 3462$ N, and $W_2 = 1538$ N **Ans.**

(*b*) *Main dimensions of both the springs*

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stress induced in the outer spring (τ_1),

$$850 = K_1 \times \frac{8\, W_1 . C}{\pi (d_1)^2} = 1.2525 \times \frac{8 \times 3462 \times 6}{\pi (d_1)^2} = \frac{66\ 243}{(d_1)^2}$$

$\therefore$ $(d_1)^2 = 66\ 243 / 850 = 78$ or $d_1 = 8.83$ say 10 mm **Ans.**

and $D_1 = C.d_1 = 6\, d_1 = 6 \times 10 = 60$ mm **Ans.**

Similarly, maximum shear stress induced in the inner spring (τ_2),

$$850 = K_2 \times \frac{8 W_2 . C}{\pi (d_2)^2} = 1.2525 \times \frac{8 \times 1538 \times 6}{\pi (d_2)^2} = \frac{29\ 428}{(d_2)^2}$$

$\therefore$ $(d_2)^2 = 29\ 428 / 850 = 34.6$ or *$d_2 = 5.88$ say 6 mm **Ans.**

and $D_2 = C.d_2 = 6 \times 6 = 36$ mm **Ans.**

(*c*) *Number of active coils in each spring*

Let n_1 and n_2 = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring (δ),

$$40 = \frac{8\, W_1 . C^3 . n_1}{G . d_1} = \frac{8 \times 3462 \times 6^3 \times n_1}{80 \times 10^3 \times 10} = 7.48\, n_1$$

$\therefore$ $n_1 = 40 / 7.48 = 5.35$ say 6 **Ans.**

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

$\therefore$ Solid length of the outer spring,

$$L_{S1} = n_1' . d_1 = 8 \times 10 = 80 \text{ mm}$$

Let n_2' be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2' . d_2 = n_1' . d_1$$

* The value of d_2 may also be obtained from equation (*i*), *i.e.*

$$\frac{d_1}{d_2} = 1.5 \quad \text{or} \quad d_2 = \frac{d_1}{1.5} = \frac{8.83}{1.5} = 5.887 \text{ say } 6 \text{ mm}$$

or $n_2' = \dfrac{n_1' . d_1}{d_2} = \dfrac{8 \times 10}{6} = 13.3$ say 14

and $n_2 = 14 - 2 = 12$ **Ans.** ... ($\because n_2' = n_2 + 2$)

Since both the springs have the same free length, therefore

Free length of outer spring

= Free length of inner spring

= $L_{S1} + \delta + 0.15\, \delta = 80 + 40 + 0.15 \times 40 = 126$ mm **Ans.**

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

= $D_1 + d_1 = 60 + 10 = 70$ mm **Ans.**

Inner diameter of the outer spring

= $D_1 - d_1 = 60 - 10 = 50$ mm **Ans.**

Outer diameter of the inner spring

= $D_2 + d_2 = 36 + 6 = 42$ mm **Ans.**

Inner diameter of the inner spring

= $D_2 - d_2 = 36 - 6 = 30$ mm **Ans.**

23.19 Helical Torsion Springs

The helical torsion springs as shown in Fig. 23.23, may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc.

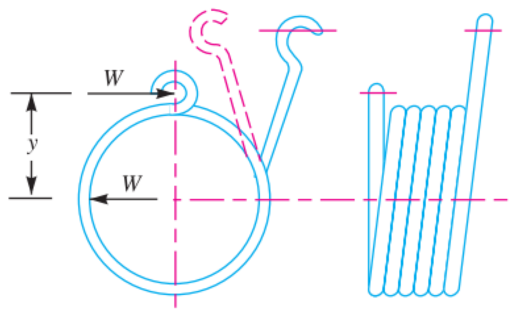


Fig. 23.23. Helical torsion spring.

A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is

$$\sigma_b = K \times \frac{32\ M}{\pi\ d^3} = K \times \frac{32\ W.y}{\pi\ d^3}$$

where K = Wahl's stress factor = $\dfrac{4C^2 - C - 1}{4C^2 - 4C}$,

C = Spring index,

M = Bending moment = $W \times y$,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

and total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\ d^4 / 64} = \frac{64\ M.D.n}{E.d^4}$$

where l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\dfrac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

and deflection,
$$\delta = \theta \times y = \frac{64\ M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\ M}{t.b^2} = K \times \frac{6\ W \times y}{t.b^2}$$

where
$$K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$$

* We know that $M / I = E / R$, where R is the radius of curvature.

$$\therefore \quad R = \frac{E.I}{M} \text{ or } \frac{l}{\theta} = \frac{E.I}{M} \text{ or } \theta = \frac{M.l}{E.I} \quad \ldots \left(\because R = \frac{l}{\theta}\right)$$

Angular deflection, $\theta = \frac{12\,\pi\,M.D.n}{E.t.b^3}$; and $\delta = \theta.y = \frac{12\,\pi\,M.D.n}{E.t.b^3} \times y$

In case the spring is made of square wire with each side equal to b, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6\,M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\,\pi\,M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\,\pi\,M.D.n}{E.b^4} \times y$$

Note : Since the diameter of the spring D reduces as the coils wind up under the applied load, therefore a clearance must be provided when the spring wire is to be wound round a mandrel. A small clearance must also be provided between the adjacent coils in order to prevent sliding friction.

Example 23.21. *A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200 kN/mm². The number of effective turns may be taken as 5.5.*

Solution. Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m $= 6000$ N-mm ; $C = 10$; $E = 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2 ; $n = 5.5$

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

$\therefore$ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \quad \textbf{Ans.}$$

Structural Analysis

6.1 Simple Trusses

A truss is a **structure composed of slender members joined** together at their end points. The members commonly used in construction consist of **wooden struts** or **metal bars**. In particular, **planar trusses** lie in a single plane and are often used to support **roofs** and **bridges**. The truss shown in Fig. 6-1a is an example of a typical roof-supporting truss. In this figure, the **roof load** is **transmitted** to the truss **at the joints**. Since this loading acts in the same plane as the truss, Fig. 6-1b, the analysis of the forces developed in the truss members will be two-dimensional.

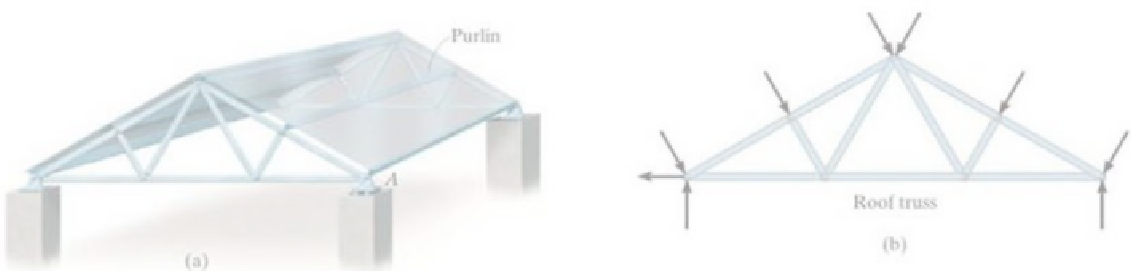


Fig. 6–1

In the case of a bridge. such as shown in Fig. 6-2a. **the load** on the deck is first **transmitted to stringers**, then to **floor beams**, and **finally to the joints** of the two supporting side trusses. Like the roof truss the bridge truss loading is also **coplanar**, Fig. 6-2b.

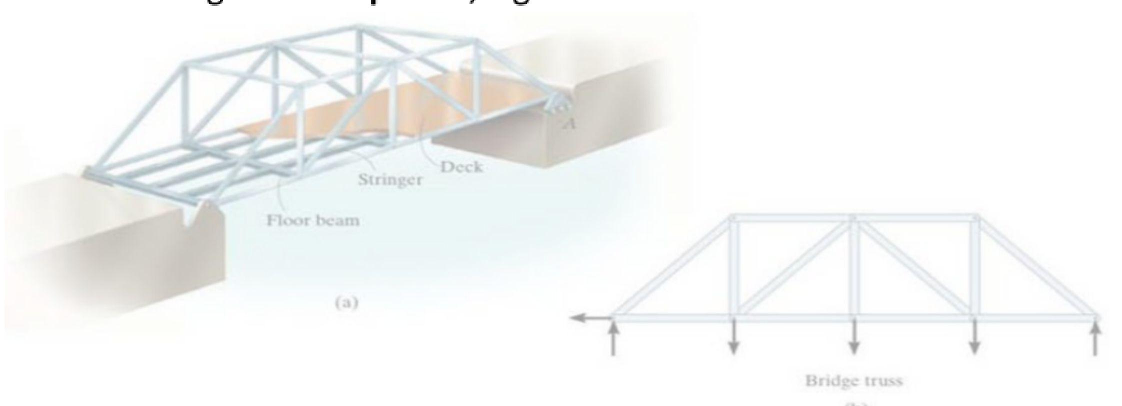


Fig. 6-2

Assumptions for Design:

To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to given loading. **To do this we will make two important assumptions:**

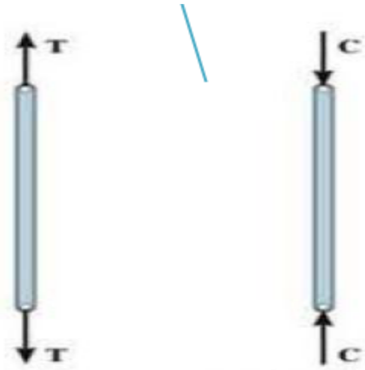
- **All Loadings are applied at the joints.**

- **the members are joined together by smooth pins.** The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate as shown in Fig 6-3a. or by simply passing a large bolt or pin through each or the members. Fig 6-3b.

(a)

(b)

Fig. 6–3



Tension
(a)

Compression
(b)

Fig. 6–4

If the **force tends to elongate** the member, it is a **tensile** force (**T**), Fig. 6-4a, whereas if tends **to shorten** the member, it is a **compressive** force (**C**), Fig. 6-4b.

Simple Truss:

If three members are pin connected at their ends they form a **triangular truss** that will be **rigid**, Fig. 6-5. Attaching two more members and connecting these members to a new joint D forms a larger truss. Fig. 6-6. This procedure can be repeated as many times as desired to form an even larger truss. If a truss can be constructed by expanding the basic triangular truss in this way. it is called a **simple truss**.

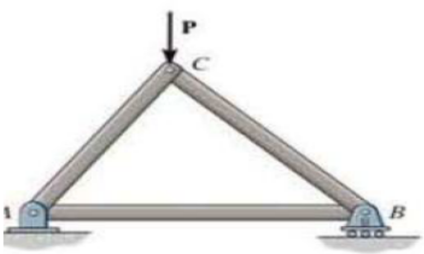


Fig. 6–5

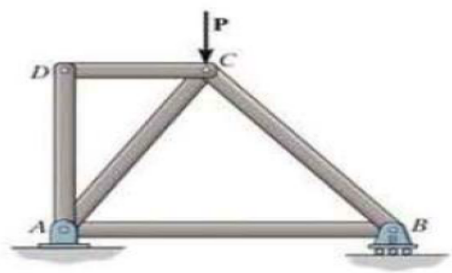


Fig. 6–6

6.2 The Method of Joints

in order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints. This method is based on the fact that **if the entire truss is in equilibrium**, then each of **its joints is also in equilibrium**. Therefore, if the free–body diagram of each joint is drawn, **the force equilibrium equations can then be used to obtain the member forces acting on each joint.** Since the members of a **plane truss** are straight two-force members lying in a single plane, each joint is subjected to a force system that is **coplanar and concurrent**. As a result, only $\sum F_x = 0$ and $\sum F_y = 0$ need to be satisfied for equilibrium.

For example, **consider the pin at joint B** of the truss in Fig. 6-7a. **Three forces** act on the pin, namely, the **500 N force** and the **forces exerted by members BA and BC**. The free body diagram of the pin is shown in Fig. 6-7b. Here $\overrightarrow{\mathbf{F_{BA}}}$ is "**pulling**" on the pin, which means that member BA is in tension; whereas $\overrightarrow{\mathbf{F_{BC}}}$ is "**pushing**" on the pin, and consequently member **BC** is in **compression**. These effects are clearly demonstrated by isolating the joint with small segments of the member connected to the pin, Fig. 6-7c. The pushing or pulling on these small segments indicates the effect of the member being either in compression or tension.

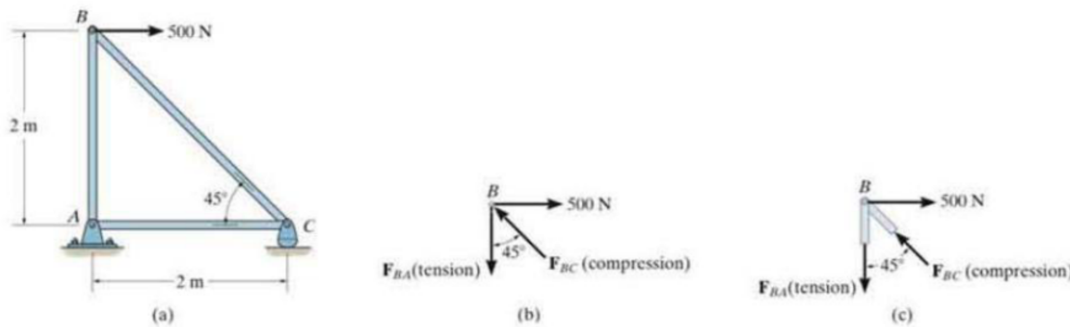
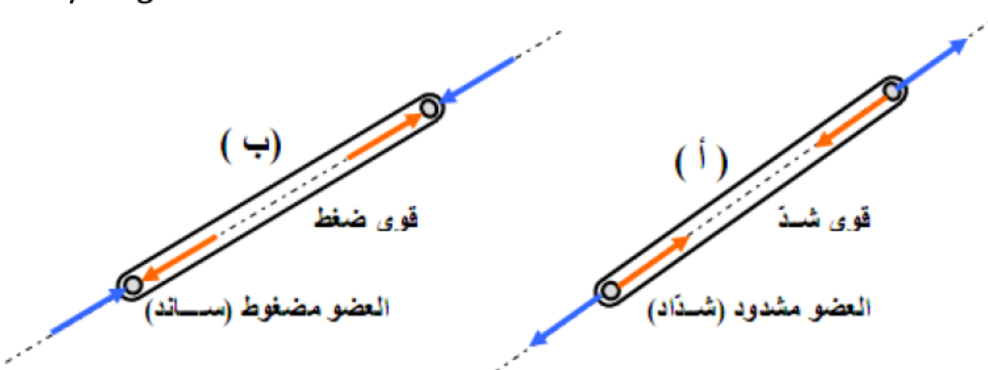


Fig. 6-7

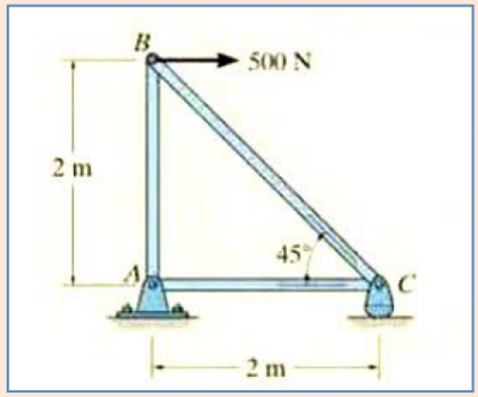
Always assume the unknown member forces acting on the joint's Free-body diagram to be in tension: i.e., the forces "pull" on the pin. If this is done, then numerical solution of the equilibrium equations will yield **positive scalars** for members in **tension** and **negative scalars** for members in **compression**, Once an unknown member force is found, use its correct magnitude and sense (T or C) on subsequent joint free body diagrams.



Exercise 6.1:

Determine the force in each member of the truss shown in Fig. 6-8. and indicate whether the members arc in tension or compression.

Fig. 6-8

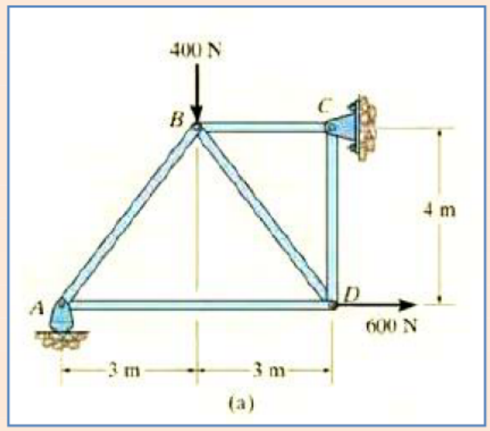


Ans: $F_{BC} = 707.1$ N $F_{CA} = 500$ N $F_{BA} = 500$ N $C_y = 500$ N

Exercise 6.3:

Determine the force in each member of the truss shown in Fig. 6-10. Indicate whether the members are in tension or compression.

Fig. 6-10



Ans: $F_{AB} = 750$ N (C) $F_{AD} = 450$ N (T) $F_{BD} = 250$ N (T) $F_{DC} = 200$ N (C) $F_{CB} = 600$ N (C) dulwahab Amran

6.4 The Method of Sections

When we need to find the force in only a **few members** of a Truss. We can analyze the Truss, using the **method of sections**. It is based on the principle that if **the truss is in equilibrium** then any segment of the truss is also in equilibrium. For example, consider The two truss members shown on the Fig.6-14. If the forces within The members are to be determined, then **an imaginary section**, indicated by the blue line, can be used to cut each member into two parts and thereby "expose" each internal force as "external" to The free-body diagrams shown on The right. Clearly, it can be seen that equilibrium requires that the member in tension (T) be subjected to a "pull", whereas the member in compression (C) is subjected to a "push".

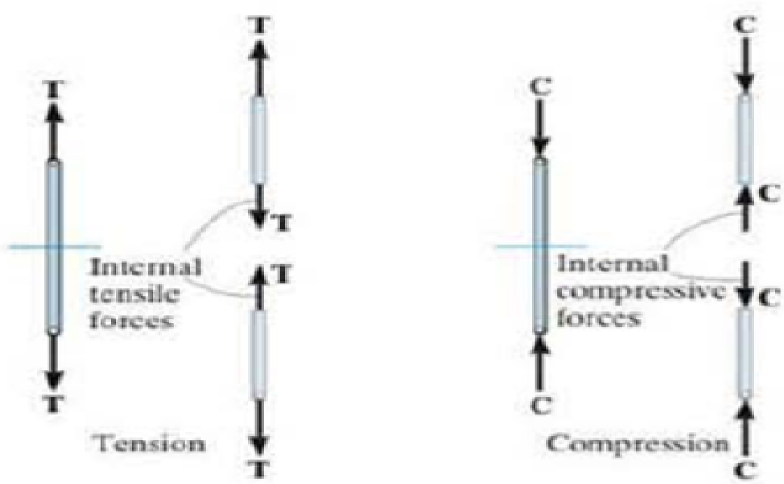


Fig. 6-14

The method of sections can also be used to **"cut"** or section The members of an entire truss. If the section passes through the truss and the free-body diagram of either of its two parts is drawn, we can then apply the **equations of equilibrium** to that part to determine The member forces at the **"cut section".** Since only three independent equilibrium equations ($\sum F_x = 0,\ \ \sum F_y = 0,\ \ \sum M_0 = 0$) can be applied to the free-body diagram of any segment, then we should try to **select a section** that, in general, passes through not more than three members in which the forces are unknown.

For example, consider the truss in Fig. 6-15a. If the forces in members BC, GC, and GF are to be determined, then section aa would be appropriate. The free -body diagrams of the two segments are shown in Figs 6-15b and 6-15c. Note that the line of action of each member force is specified from the geometry of the truss, since the force in a member is along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part - Newton's third law. Members BC and GC are assumed to be in tension since they are

subjected to a "pull", whereas GF in compression since it is subjected to a "push".

The three unknown member forces $\mathbf{F_{BC}}$, $\mathbf{F_{GC}}$ **and** $\mathbf{F_{GF}}$ **can be obtained by applying the three equilibrium equations** to the free-body diagram in Fig. 6-15b. If however, the free-body diagram in Fig. 6-15c is considered, the three support reactions D_x, D_y and E_x will have to be known, because only three equations of equilibrium are available. (This, of course, is done in the usual manner by considering a free-body diagram of the entire truss.)

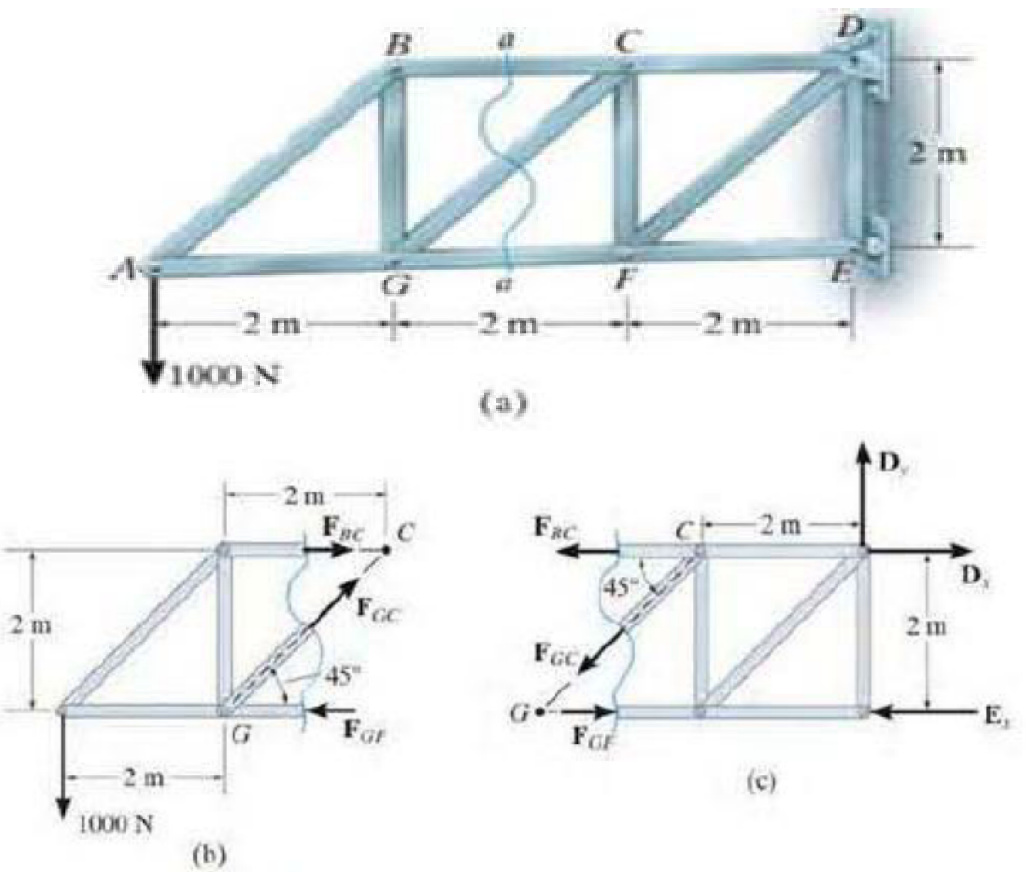


Fig. 6-15

Exercise 6.5:

Determine the force in members GE, GC, and BC of the truss shown in Fig. 6-16.

Indicate whether the members are in tension or compression.

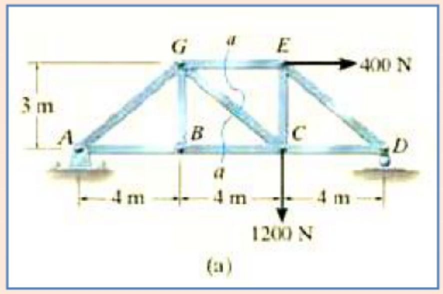


Fig. 6-16

Ans: $F_{BC} = 800\ N\ (T)$ $F_{GE} = 800\ N\ (C)$ $F_{GC} = 500\ N\ (T)$

Exercise 6.6:

Determine the force in member CF of the truss shown in Fig. 6-17.

Indicate whether the member is in tension or compression. Assume each member is pin connected.

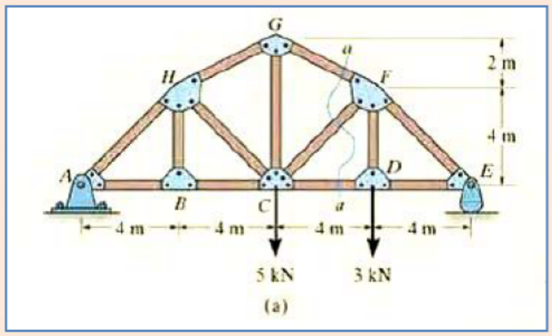


Fig. 6-17

Ans: $F_{CF} = 0.589\ kN\ (C)$

Exercise 6.7:

Determine the force in member EB of the roof truss shown in Fig. 6-18. Indicate whether the member is in tension or compression.

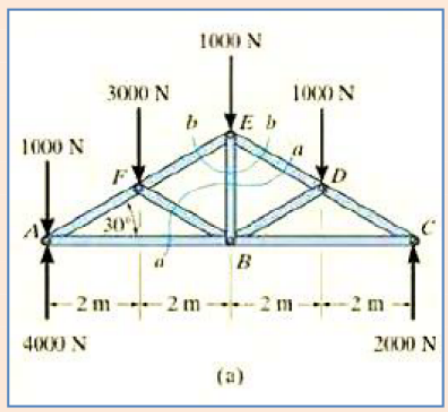


Fig. 6-18

Ans: $F_{CF} = 0.589\ kN\ (C)$