Torsional and Bending Stresses in Machine Parts

5.1 Introduction

Sometimes machine parts are subjected to pure torsion or bending or combination of both torsion and bending stresses. We shall now discuss these stresses in detail in the following pages.

5.2 Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion.*** The stress set up by torsion is known as ***torsional shear stress.*** It is zero at the centroidal axis and maximum at the outer surface.

Consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown in Fig. 5.1. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \qquad ...(i)$$

where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.

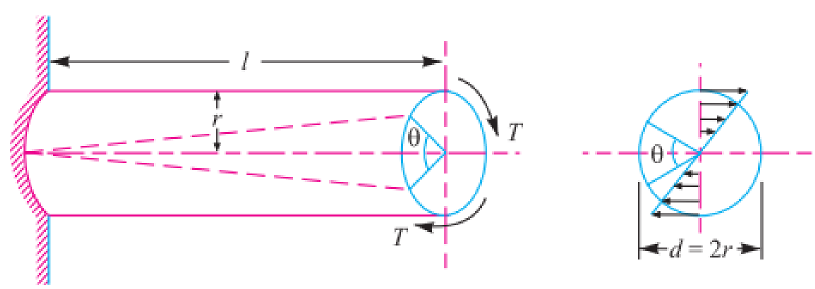


Fig. 5.1. Torsional shear stress.

The equation (*i*) is known as ***torsion equation.*** It is based on the following assumptions:

1. The material of the shaft is uniform throughout.
2. The twist along the length of the shaft is uniform.
3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.
4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.
5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Notes : 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (*i*), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{XX} + I_{YY} = \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

$$\therefore \quad T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J = \frac{\pi}{32} \, [(d_o)^4 - (d_i)^4] \text{ and } r = \frac{d_o}{2}$$

$$\therefore \quad T = \tau \times \frac{\pi}{32} [(d_o)^4 - (d_4)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right]$$

$$= \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4) \qquad \ldots \left(\text{Substituting, } k = \frac{d_i}{d_o} \right)$$

3. The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2 \pi N . T}{60} = T . \omega \qquad \ldots \left(\because \omega = \frac{2 \pi N}{60} \right)$$

where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Example 5.1. *A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2 \pi N . T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3 / 16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \quad \text{or} \quad d = 81.5 \text{ mm}$$ **Ans.**

Example 5.2. *A steel shaft 35 mm in diameter and 1.2 m long held rigidly at one end has a hand wheel 500 mm in diameter keyed to the other end. The modulus of rigidity of steel is 80 GPa.*

1. What load applied to tangent to the rim of the wheel produce a torsional shear of 60 MPa?

2. How many degrees will the wheel turn when this load is applied?

Solution. Given : d = 35 mm or r = 17.5 mm ; l = 1.2 m = 1200 mm ; D = 500 mm or R = 250 mm ; C = 80 GPa = 80 kN/mm² = 80×10^3 N/mm² ; τ = 60 MPa = 60 N/mm²

1. *Load applied to the tangent to the rim of the wheel*

Let W = Load applied (in newton) to tangent to the rim of the wheel.

We know that torque applied to the hand wheel,

$$T = W.R = W \times 250 = 250\, W \text{ N-mm}$$

and polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32}(35)^4 = 147.34 \times 10^3 \text{ mm}^4$$

We know that $\frac{T}{J} = \frac{\tau}{r}$

$$\therefore \quad \frac{250\, W}{147.34 \times 10^3} = \frac{60}{17.5} \quad \text{or} \quad W = \frac{60 \times 147.34 \times 10^3}{17.5 \times 250} = 2020 \text{ N}$$ **Ans.**

2. *Number of degrees which the wheel will turn when load W = 2020 N is applied*

Let θ = Required number of degrees.

We know that $\frac{T}{J} = \frac{C.\theta}{l}$

$$\therefore \quad \theta = \frac{T.l}{C.J} = \frac{250 \times 2020 \times 1200}{80 \times 10^3 \times 147.34 \times 10^3} = 0.05°$$ **Ans.**

Example 5.3. *A shaft is transmitting 97.5 kW at 180 r.p.m. If the allowable shear stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more that 1° in a length of 3 metres. Take C = 80 GPa.*

Solution. Given : $P = 97.5$ kW $= 97.5 \times 10^3$ W ; N = 180 r.p.m. ; τ = 60 MPa = 60 N/mm² ; $\theta = 1° = \pi / 180 = 0.0174$ rad ; l = 3 m = 3000 mm ; C = 80 GPa = 80×10^9 N/m² = 80×10^3 N/mm²

Let T = Torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted by the shaft (P),

$$97.5 \times 10^3 = \frac{2\pi\, N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\, T$$

$$\therefore \quad T = 97.5 \times 10^3/18.852 = 5172 \text{ N-m} = 5172 \times 10^3 \text{ N-mm}$$

Now let us find the diameter of the shaft based on the strength and stiffness.

1. ***Considering strength of the shaft***

We know that the torque transmitted (*T*),

$$5172 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$\therefore$ $d^3 = 5172 \times 10^3/11.78 = 439 \times 10^3$ or $d = 76$ mm ...(*i*)

2. ***Considering stiffness of the shaft***

Polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = 0.0982\, d^4$$

We know that $\frac{T}{J} = \frac{C.\theta}{l}$

$$\frac{5172 \times 10^3}{0.0982\, d^4} = \frac{80 \times 10^3 \times 0.0174}{3000} \quad \text{or} \quad \frac{52.7 \times 10^6}{d^4} = 0.464$$

$\therefore$ $d^4 = 52.7 \times 10^6/0.464 = 113.6 \times 10^6$ or $d = 103$ mm ...(*ii*)

Taking larger of the two values, we shall provide $d = 103$ say 105 mm **Ans.**

Example 5.4. *A hollow shaft is required to transmit 600 kW at 110 r.p.m., the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MPa and twist in a length of 3 metres not to exceed 1.4 degrees. Find the external diameter of the shaft, if the internal diameter to the external diameter is 3/8. Take modulus of rigidity as 84 GPa.*

Solution. Given : $P = 600$ kW $= 600 \times 10^3$ W ; $N = 110$ r.p.m. ; $T_{max} = 1.2\, T_{mean}$; $\tau = 63$ MPa $= 63$ N/mm^2 ; $l = 3$ m $= 3000$ mm ; $\theta = 1.4 \times \pi / 180 = 0.024$ rad ; $k = d_i / d_o = 3/8$; $C = 84$ GPa $= 84 \times 10^9$ N/m^2 $= 84 \times 10^3$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft,

d_o = External diameter of the shaft, and

d_i = Internal diameter of the shaft.

We know that power transmitted by the shaft (*P*),

$$600 \times 10^3 = \frac{2\,\pi\, N . T_{mean}}{60} = \frac{2\pi \times 110 \times T_{mean}}{60} = 11.52\, T_{mean}$$

$\therefore$ $T_{mean} = 600 \times 10^3/11.52 = 52 \times 10^3$ N-m $= 52 \times 10^6$ N-mm

and maximum torque transmitted by the shaft,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 52 \times 10^6 = 62.4 \times 10^6 \text{ N-mm}$$

Now let us find the diameter of the shaft considering strength and stiffness.

1. *Considering strength of the shaft*

We know that maximum torque transmitted by the shaft,

$$T_{max} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$62.4 \times 10^6 = \frac{\pi}{16} \times 63 \times (d_o)^3 \left[1 - \left(\frac{3}{8}\right)^4\right] = 12.12\,(d_o)^3$$

$$\therefore \quad (d_o)^3 = 62.4 \times 10^6/12.12 = 5.15 \times 10^6 \text{ or } d_o = 172.7 \text{ mm} \quad ...(i)$$

2. *Considering stiffness of the shaft*

We know that polar moment of inertia of a hollow circular section,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{32}(d_o)^4\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{32}(d_o)^4\,(1 - k^4) = \frac{\pi}{32}(d_o)^4\left[1 - \left(\frac{3}{8}\right)^4\right] = 0.0962\,(d_o)^4$$

We also know that

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{62.4 \times 10^6}{0.0962\,(d_o)^4} = \frac{84 \times 10^3 \times 0.024}{3000} \text{ or } \frac{648.6 \times 10^6}{(d_o)^4} = 0.672$$

$$\therefore \quad (d_o)^4 = 648.6 \times 10^6/0.672 = 964 \times 10^6 \text{ or } d_o = 176.2 \text{ mm} \quad ...(ii)$$

Taking larger of the two values, we shall provide

d_o = 176.2 say 180 mm **Ans.**

5.3 Shafts in Series and Parallel

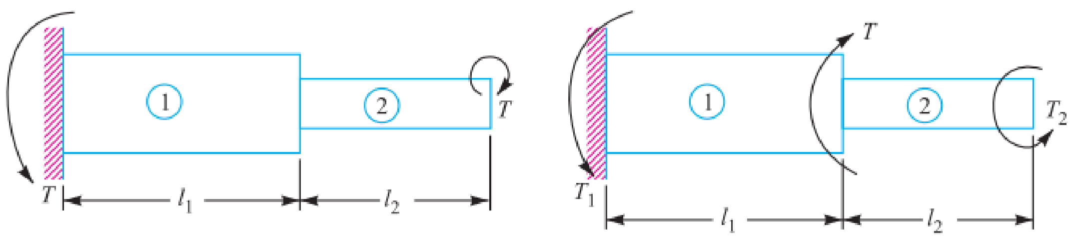
When two shafts of different diameters are connected together to form one shaft, it is then known as ***composite shaft.*** If the driving torque is applied at one end and the resisting torque at the other end, then the shafts are said to be connected in series as shown in Fig. 5.2 (*a*). In such cases, each shaft transmits the same torque and the total angle of twist is equal to the sum of the angle of twists of the two shafts.

Mathematically, total angle of twist,

$$\theta = \theta_1 + \theta_2 = \frac{T.l_1}{C_1 J_1} + \frac{T.l_2}{C_2 J_2}$$

If the shafts are made of the same material, then $C_1 = C_2 = C$.

$$\therefore \qquad \theta = \frac{T.l_1}{CJ_1} + \frac{T.l_2}{CJ_2} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2}\right]$$



(*a*) Shafts in series. (*b*) Shafts in parallel.

Fig. 5.2. Shafts in series and parallel.

When the driving torque (T) is applied at the junction of the two shafts, and the resisting torques T_1 and T_2 at the other ends of the shafts, then the shafts are said to be connected in parallel, as shown in Fig. 5.2 (*b*). In such cases, the angle of twist is same for both the shafts, *i.e.*

$$\theta_1 = \theta_2$$

or

$$\frac{T_1 l_1}{C_1 J_1} = \frac{T_2 l_2}{C_2 J_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{C_1}{C_2} \times \frac{J_1}{J_2}$$

and

$$T = T_1 + T_2$$

If the shafts are made of the same material, then $C_1 = C_2$.

$$\therefore \qquad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{J_1}{J_2}$$

Example 5.5. *A steel shaft ABCD having a total length of 3.5 m consists of three lengths having different sections as follows:*

AB is hollow having outside and inside diameters of 100 mm and 62.5 mm respectively, and BC and CD are solid. BC has a diameter of 100 mm and CD has a diameter of 87.5 mm. If the angle of twist is the same for each section, determine the length of each section. Find the value of the applied torque and the total angle of twist, if the maximum shear stress in the hollow portion is 47.5 MPa and shear modulus, C = 82.5 GPa.

Solution. Given: $L = 3.5$ m ; $d_o = 100$ mm ; $d_i = 62.5$ mm ; $d_2 = 100$ mm ; $d_3 = 87.5$ mm ; $\tau = 47.5$ MPa $= 47.5$ N/mm^2 ; $C = 82.5$ GPa $= 82.5 \times 10^3$ N/mm^2

The shaft *ABCD* is shown in Fig. 5.3.

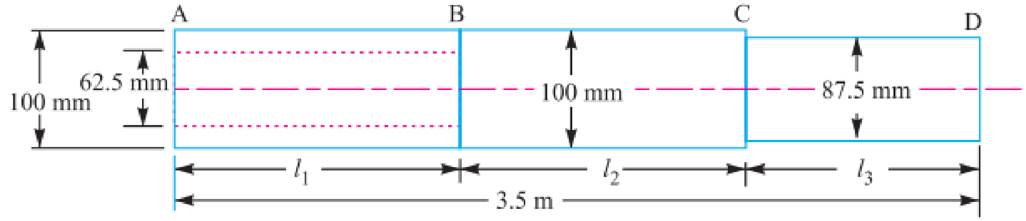


Fig. 5.3

Length of each section

Let l_1, l_2 and l_3 = Length of sections *AB*, *BC* and *CD* respectively.

We know that polar moment of inertia of the hollow shaft *AB*,

$$J_1 = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(100)^4 - (62.5)^4] = 8.32 \times 10^6 \text{ mm}^4$$

Polar moment of inertia of the solid shaft *BC*,

$$J_2 = \frac{\pi}{32}(d_2)^4 = \frac{\pi}{32}(100)^4 = 9.82 \times 10^6 \text{ mm}^4$$

and polar moment of inertia of the solid shaft *CD*,

$$J_3 = \frac{\pi}{32}(d_3)^4 = \frac{\pi}{32}(87.5)^4 = 5.75 \times 10^6 \text{ mm}^4$$

We also know that angle of twist,

$$\theta = T \,.\, l / C \,.\, J$$

Assuming the torque T and shear modulus C to be same for all the sections, we have

Angle of twist for hollow shaft *AB*,

$$\theta_1 = T \,.\, l_1 / C \,.\, J_1$$

Similarly, angle of twist for solid shaft *BC*,

$$\theta_2 = T \,.\, l_2 / C \,.\, J_2$$

and angle of twist for solid shaft *CD*,

$$\theta_3 = T \,.\, l_3 / C \,.\, J_3$$

Since the angle of twist is same for each section, therefore

$$\theta_1 = \theta_2$$

$$\frac{T \,.\, l_1}{C \,.\, J_1} = \frac{T \,.\, l_2}{C \,.\, J_2} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{J_1}{J_2} = \frac{8.32 \times 10^6}{9.82 \times 10^6} = 0.847 \qquad ...(i)$$

Also $\theta_1 = \theta_3$

$$\frac{T \,.\, l_1}{C \,.\, J_1} = \frac{T \,.\, l_3}{C \,.\, J_3} \quad \text{or} \quad \frac{l_1}{l_3} = \frac{J_1}{J_3} = \frac{8.32 \times 10^6}{5.75 \times 10^6} = 1.447 \qquad ...(ii)$$

We know that $l_1 + l_2 + l_3 = L = 3.5$ m = 3500 mm

$$l_1\left(1+\frac{l_2}{l_1}+\frac{l_3}{l_1}\right) = 3500$$

$$l_1\left(1+\frac{1}{0.847}+\frac{1}{1.447}\right) = 3500$$

$l_1 \times 2.8717 = 3500$ or $l_1 = 3500 / 2.8717 = 1218.8$ mm **Ans.**

From equation **(*i*),**

$l_2 = l_1 / 0.847 = 1218.8 / 0.847 = 1439$ mm **Ans.**

and from equation **(*ii*),** $l_3 = l_1 / 1.447 = 1218.8 / 1.447 = 842.2$ mm **Ans.**

Value of the applied torque

We know that the maximum shear stress in the hollow portion,

$$\tau = 47.5 \text{ MPa} = 47.5 \text{ N/mm}^2$$

For a hollow shaft, the applied torque,

$$T = \frac{\pi}{16}\times\tau\left[\frac{(d_o)^4-(d_i)^4}{d_o}\right] = \frac{\pi}{16}\times 47.5\left[\frac{(100)^4-(62.5)^4}{100}\right]$$

$$= 7.9\times 10^6 \text{ N-mm} = 7900 \text{ N-m}$$ **Ans.**

Total angle of twist

When the shafts are connected in series, the total angle of twist is equal to the sum of angle of twists of the individual shafts. Mathematically, the total angle of twist,

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$= \frac{T\,.\,l_1}{C\,.\,J_1}+\frac{T\,.\,l_2}{C\,.\,J_2}+\frac{T\,.\,l_3}{C\,.\,J_3} = \frac{T}{C}\left[\frac{l_1}{J_1}+\frac{l_2}{J_2}+\frac{l_3}{J_3}\right]$$

$$= \frac{7.9\times10^6}{82.5\times10^3}\left[\frac{1218.8}{8.32\times10^6}+\frac{1439}{9.82\times10^6}+\frac{842.2}{5.75\times10^6}\right]$$

$$= \frac{7.9\times10^6}{82.5\times10^3\times10^6}\,[146.5+146.5+146.5] = 0.042 \text{ rad}$$

$= 0.042 \times 180 / \pi = 2.406°$ **Ans.**

5.4 Bending Stress in Straight Beams

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses.

Consider a straight beam subjected to a bending moment *M* as shown in Fig. 5.4. The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).
2. The material of the beam obeys Hooke's law.
3. The transverse sections (*i.e.* *BC* or *GH*) which were plane before bending, remain plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.
5. The Young's modulus (*E*) is the same in tension and compression.
6. The loads are applied in the plane of bending.

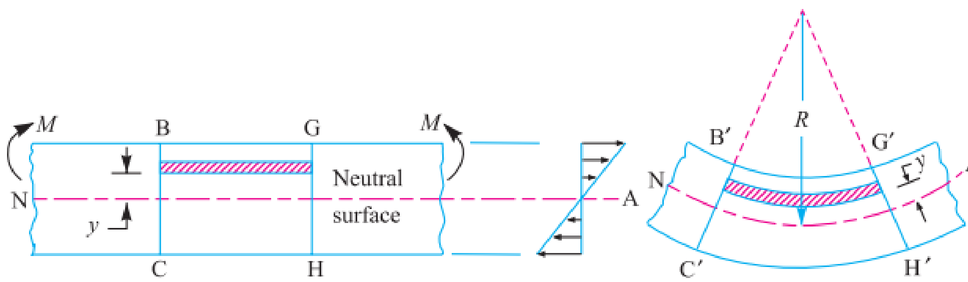


Fig. 5.4. Bending stress in straight beams.

A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called ***neutral surface.*** The intersection of the neutral surface with any normal cross-section of the beam is known *as* ***neutral axis.*** The stress distribution of a beam is shown in Fig. 5.4. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I / y} = \frac{M}{Z}$$

The ratio I/y is known as ***section modulus*** and is denoted by Z.

Notes : 1. The neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis is $y = d / 2$, where d is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Example 5.6. *A pump lever rocking shaft is shown in Fig. 5.5. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.*

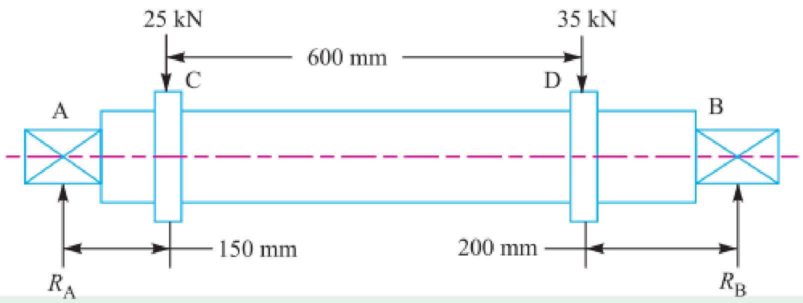


Fig. 5.5

Solution. Given : σ_b = 100 MPa = 100 N/mm^2

Let R_A and R_B = Reactions at *A* and *B* respectively.

Taking moments about *A*, we have

$$R_B \times 950 = 35 \times 750 + 25 \times 150 = 30\ 000$$

$$\therefore \quad R_B = 30\ 000 / 950 = 31.58 \text{ kN} = 31.58 \times 10^3 \text{ N}$$

and $$R_A = (25 + 35) - 31.58 = 28.42 \text{ kN} = 28.42 \times 10^3 \text{ N}$$

$\therefore$ Bending moment at *C*

$$= R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N-mm}$$

and bending moment at $D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6$ N-mm

We see that the maximum bending moment is at *D*, therefore maximum bending moment, $M = 6.316 \times 10^6$ N-mm.

Let d = Diameter of the shaft.

$\therefore$ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

We know that bending stress (σ_b),

$$100 = \frac{M}{Z} = \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

$$\therefore \quad d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3 \text{ or } d = 86.3 \text{ say } 90 \text{ mm}$$ **Ans.**

Example 5.7. *An axle 1 metre long supported in bearings at its ends carries a fly wheel weighing 30 kN at the centre. If the stress (bending) is not to exceed 60 MPa, find the diameter of the axle.*

Solution. Given : $L = 1$ m = 1000 mm ; $W = 30$ kN = 30×10^3 N ; $\sigma_b = 60$ MPa = 60 N/mm^2

The axle with a flywheel is shown in Fig. 5.6.

Let d = Diameter of the axle in mm.

∴ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

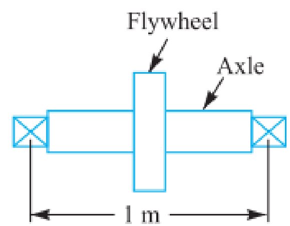


Fig. 5.6

Maximum bending moment at the centre of the axle,

$$M = \frac{W.L}{4} = \frac{30 \times 10^3 \times 1000}{4} = 7.5 \times 10^6 \text{ N-mm}$$

We know that bending stress (σ_b),

$$60 = \frac{M}{Z} = \frac{7.5 \times 10^6}{0.0982\, d^3} = \frac{76.4 \times 10^6}{d^3}$$

∴ $d^3 = 76.4 \times 10^6/60 = 1.27 \times 10^6$ or $d = 108.3$ say 110 mm **Ans.**

Example 5.9. *A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.*

Solution. Given : $P = 10$ kW = 10×10^3 W ; $N = 400$ r.p.m ; $D = 1.2$ m = 1200 mm or $R = 600$ mm ; $\sigma_b = 15$ MPa = 15 N/mm^2

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\,\pi\, N\,.\,T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\, T$$

∴ $T = 10 \times 10^3/42 = 238$ N-m $= 238 \times 10^3$ N-mm

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm $= 2 \times 2b = 4b$ ...(Given)

∴ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4} (2b)^2 \times b = \pi\, b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\ 520}{\pi\, b^3} = \frac{18\ 943}{b^3}$$

or $b^3 = 18\ 943/15 = 1263$ or $b = 10.8$ mm

∴ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

5.5 Bending Stress in Curved Beams

We have seen in the previous article that for the straight beams, the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in case of curved beams, the neutral axis of the cross-section is shifted towards the centre of curvature of the beam causing a non-linear (hyperbolic) distribution of stress, as shown in Fig. 5.8. It may be noted that the neutral axis lies between the centroidal axis and the centre of curvature and always occurs within the curved beams. The application of curved beam principle is used in crane hooks, chain links and frames of punches, presses, planers etc.

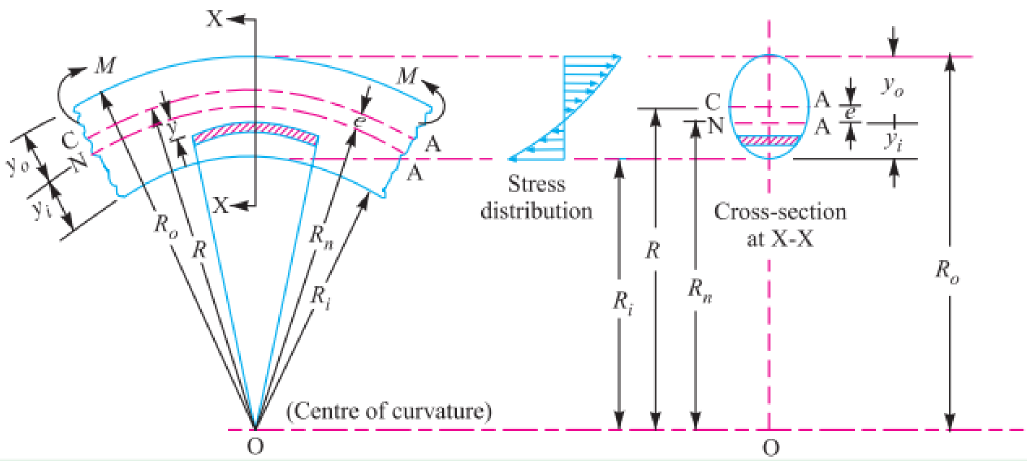


Fig. 5.8. Bending stress in a curved beam.

Consider a curved beam subjected to a bending moment M, as shown in Fig. 5.8. In finding the bending stress in curved beams, the same assumptions are used as for straight beams. The general expression for the bending stress (σ_b) in a curved beam at any fibre at a distance y from the neutral axis, is given by

$$\sigma_b = \frac{M}{A\,.\,e}\left(\frac{y}{R_n - y}\right)$$

where M = Bending moment acting at the given section about the centroidal axis,

A = Area of cross-section,

e = Distance from the centroidal axis to the neutral axis = $R - R_n$,

R = Radius of curvature of the centroidal axis,

R_n = Radius of curvature of the neutral axis, and

y = Distance from the neutral axis to the fibre under consideration. It is positive for the distances towards the centre of curvature and negative for the distances away from the centre of curvature.

Notes : 1. The bending stress in the curved beam is zero at a point other than at the centroidal axis.

2. If the section is symmetrical such as a circle, rectangle, I-beam with equal flanges, then the maximum bending stress will always occur at the inside fibre.

3. If the section is unsymmetrical, then the maximum bending stress may occur at either the inside fibre or the outside fibre. The maximum bending stress at the inside fibre is given by

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i}$$

where y_i = Distance from the neutral axis to the inside fibre = $R_n - R_i$, and

R_i = Radius of curvature of the inside fibre.

The maximum bending stress at the outside fibre is given by

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o}$$

where y_o = Distance from the neutral axis to the outside fibre = $R_o - R_n$, and

R_o = Radius of curvature of the outside fibre.

It may be noted that the bending stress at the inside fibre is ***tensile*** while the bending stress at the outside fibre is ***compressive.***

4. If the section has an axial load in addition to bending, then the axial or direct stress (σ_d) must be added algebraically to the bending stress, in order to obtain the resultant stress on the section. In other words,

Resultant stress, $\sigma = \sigma_d \pm \sigma_b$

Example 5.10. *The frame of a punch press is shown in Fig. 5.9. Find the stresses at the inner and outer surface at section X-X of the frame, if W = 5000 N.*

Solution. Given : W = 5000 N ; b_i = 18 mm ; b_o = 6 mm ; h = 40 mm ; R_i = 25 mm ; R_o = 25 + 40 = 65 mm

We know that area of section at X-X,

$$A = \frac{1}{2}(18 + 6)\,40 = 480 \text{ mm}^2$$

The various distances are shown in Fig. 5.10.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{\left(\dfrac{b_i + b_o}{2}\right) h}{\left(\dfrac{b_i R_o - b_o R_i}{h}\right) \log_e \left(\dfrac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$= \frac{\left(\dfrac{18 + 6}{2}\right) \times 40}{\left(\dfrac{18 \times 65 - 6 \times 25}{40}\right) \log_e \left(\dfrac{65}{25}\right) - (18 - 6)}$$

$$= \frac{480}{(25.5 \times 0.9555) - 12} = 38.83 \text{ mm}$$

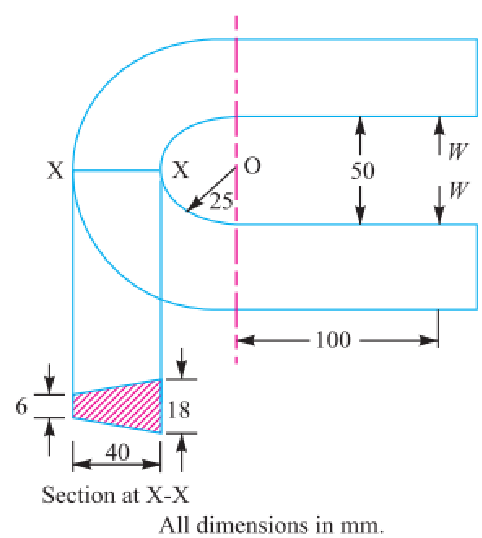


Fig. 5.9

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)} = 25 + \frac{40\,(18 + 2 \times 6)}{3\,(18 + 6)} \text{ mm}$$

$$= 25 + 16.67 = 41.67 \text{ mm}$$

Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 41.67 - 38.83 = 2.84 \text{ mm}$$

and the distance between the load and centroidal axis,

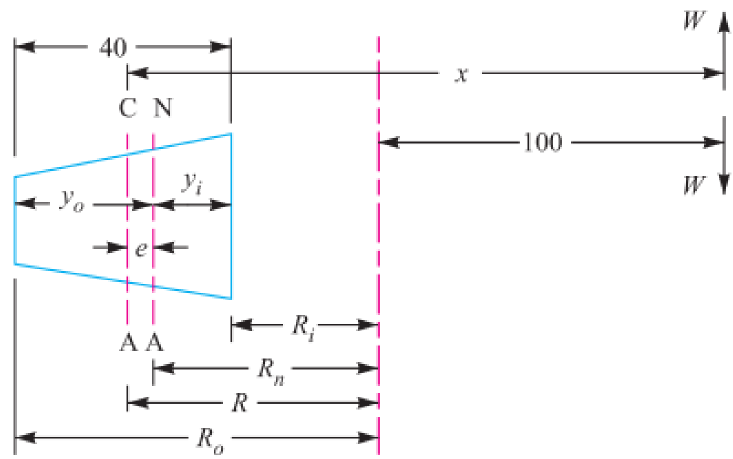
$$x = 100 + R = 100 + 41.67 = 141.67 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W.x = 5000 \times 141.67 = 708\ 350 \text{ N-mm}$$

The section at *X-X* is subjected to a direct tensile load of $W = 5000$ N and a bending moment of $M = 708\ 350$ N-mm. We know that direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{5000}{480} = 10.42 \text{ N/mm}^2 = 10.42 \text{ MPa}$$



All dimensions in mm.

Fig. 5.10

Distance from the neutral axis to the inner surface,

$$y_i = R_n - R_i = 38.83 - 25 = 13.83 \text{ mm}$$

Distance from the neutral axis to the outer surface,

$$y_o = R_o - R_n = 65 - 38.83 = 26.17 \text{ mm}$$

We know that maximum bending stress at the inner surface,

$$\sigma_{bi} = \frac{M\,.\,y_i}{A\,.\,e\,.\,R_i} = \frac{708\ 350 \times 13.83}{480 \times 2.84 \times 25} = 287.4 \text{ N/mm}^2$$

$$= 287.4 \text{ MPa (tensile)}$$

and maximum bending stress at the outer surface,

$$\sigma_{b0} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{708\ 350 \times 26.17}{480 \times 2.84 \times 65} = 209.2 \text{ N/mm}^2$$

$$= 209.2 \text{ MPa (compressive)}$$

∴ Resultant stress on the inner surface

$$= \sigma_t + \sigma_{bi} = 10.42 + 287.4 = 297.82 \text{ MPa (tensile)}$$ **Ans.**

and resultant stress on the outer surface,

$$= \sigma_t - \sigma_{bo} = 10.42 - 209.2 = -198.78 \text{ MPa}$$

$$= 198.78 \text{ MPa (compressive)}$$ **Ans.**

Example 5.11. *The crane hook carries a load of 20 kN as shown in Fig. 5.11. The section at X-X is rectangular whose horizontal side is 100 mm. Find the stresses in the inner and outer fibres at the given section.*

Solution. Given : $W = 20$ kN $= 20 \times 10^3$ N ; $R_i = 50$ mm ; $R_o = 150$ mm ; $h = 100$ mm ; $b = 20$ mm

We know that area of section at *X-X*,

$$A = b.h = 20 \times 100 = 2000 \text{ mm}^2$$

The various distances are shown in Fig. 5.12.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{h}{\log_e \left(\frac{R_o}{R_i}\right)} = \frac{100}{\log_e \left(\frac{150}{50}\right)} = \frac{100}{1.098} = 91.07 \text{ mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = 100 \text{ mm}$$

∴ Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 100 - 91.07 = 8.93 \text{ mm}$$

and distance between the load and the centroidal axis,

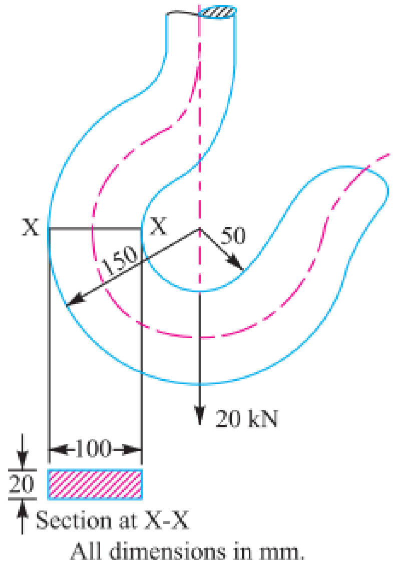
$$x = R = 100 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W \times x = 20 \times 10^3 \times 100 = 2 \times 10^6 \text{ N-mm}$$

The section at *X-X* is subjected to a direct tensile load of $W = 20 \times 10^3$ N and a bending moment of $M = 2 \times 10^6$ N-mm. We know that direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{20 \times 10^3}{2000} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$



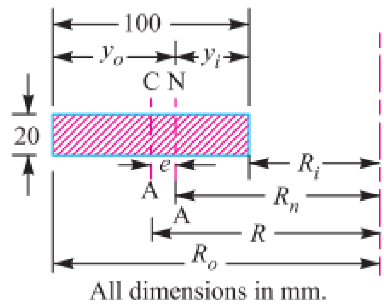


Fig. 5.11 **Fig. 5.12**

We know that the distance from the neutral axis to the inside fibre,

$$y_i = R_n - R_i = 91.07 - 50 = 41.07 \text{ mm}$$

and distance from the neutral axis to outside fibre,

$$y_o = R_o - R_n = 150 - 91.07 = 58.93 \text{ mm}$$

∴ Maximum bending stress at the inside fibre,

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{2 \times 10^6 \times 41.07}{2000 \times 8.93 \times 50} = 92 \text{ N/mm}^2 = 92 \text{ MPa (tensile)}$$

and maximum bending stress at the outside fibre,

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{2 \times 10^6 \times 58.93}{2000 \times 8.93 \times 150} = 44 \text{ N/mm}^2$$

$$= 44 \text{ MPa (compressive)}$$

∴ Resultant stress at the inside fibre

$= \sigma_t + \sigma_{bi} = 10 + 92 = 102$ MPa (tensile) **Ans.**

and resultant stress at the outside fibre

$= \sigma_t - \sigma_{bo} = 10 - 44 = -34$ MPa $= 34$ MPa (compressive) **Ans.**

5.6 Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as ***principal planes*** and the direct stresses along these planes are known as ***principal stresses***. The planes on which the maximum shear stress act are known as planes of maximum shear.

5.7 Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (*i.e.* direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body *ABCD* of uniform cross-sectional area and unit thickness subjected to normal stresses σ_1 and σ_2 as shown in Fig. 5.15 (*a*). In addition to these normal stresses, a shear stress τ also acts.

It has been shown in books on '***Strength of Materials***' that the normal stress across any oblique section such as *EF* inclined at an angle θ with the direction of σ_2, as shown in Fig. 5.15 (*a*), is given by

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \qquad \ldots(i)$$

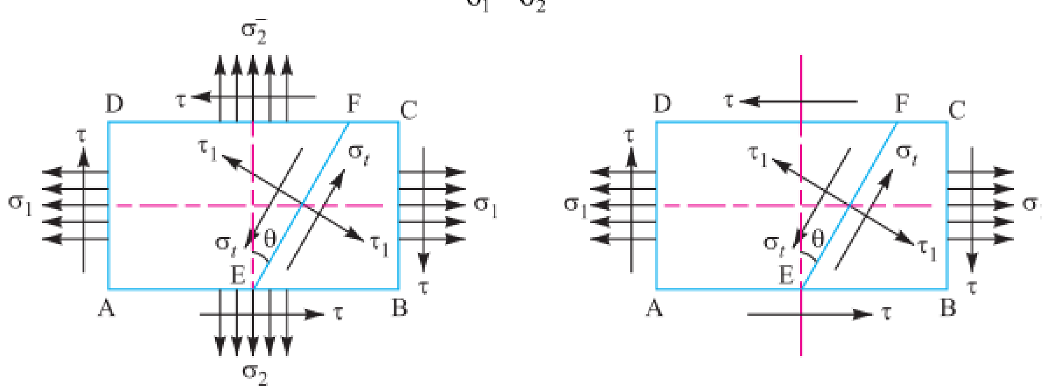
and tangential stress (*i.e.* shear stress) across the section *EF*,

$$\tau_1 = \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta \qquad \ldots(ii)$$

Since the planes of maximum and minimum normal stress (*i.e.* principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating $\tau_1 = 0$ in the above equation **(*ii*)**, *i.e.*

$$\frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta = 0$$

$$\therefore \qquad \tan 2\theta = \frac{2\,\tau}{\sigma_1 - \sigma_2} \qquad \ldots(iii)$$



(*a*) Direct stress in two mutually prependicular planes accompanied by a simple shear stress.

(*b*) Direct stress in one plane accompanied by a simple shear stress.

Fig. 5.15. Principal stresses for a member subjected to bi-axial stress.

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section.

From Fig. 5.16, we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

$$\therefore \qquad \sin 2\theta_1 = +\frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2}}$$

and

$$\sin 2\theta_2 = -\frac{2\tau}{\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2}}$$

Also

$$\cos 2\theta = \pm\frac{\sigma_1-\sigma_2}{\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2}}$$

$$\therefore \qquad \cos 2\theta_1 = +\frac{\sigma_1-\sigma_2}{\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2}}$$

and

$$\cos 2\theta_2 = -\frac{\sigma_1-\sigma_2}{\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2}}$$

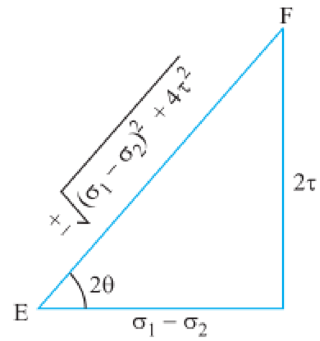


Fig. 5.16

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation ***(i)***.

∴ Maximum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1+\sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2} \qquad ...(iv)$$

and minimum principal (or normal) stress,

$$\sigma_{t2} = \frac{\sigma_1+\sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2} \qquad ...(v)$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by ***one-half the algebraic difference between the principal stresses,*** *i.e.*

$$\tau_{max} = \frac{\sigma_{t1}-\sigma_{t2}}{2} = \frac{1}{2}\sqrt{(\sigma_1-\sigma_2)^2+4\,\tau^2} \qquad ...(vi)$$

Notes: 1. When a member is subjected to direct stress in one plane accompanied by a simple shear stress as shown in Fig. 5.15 (*b*), then the principal stresses are obtained by substituting $\sigma_2 = 0$ in equation ***(iv)***, ***(v)*** and ***(vi)***.

$$\therefore \qquad \sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\,\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\,\tau^2}\right]$$

and

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\,\tau^2}\right]$$

2. In the above expression of σ_{t2}, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2+4\,\tau^2}\right]$ is more than $\frac{\sigma_1}{2}$. Therefore the nature of σ_{t2} will be opposite to that of σ_{t1}, *i.e.* if σ_{t1} is tensile then σ_{t2} will be compressive and *vice-versa*.

5.8 Application of Principal Stresses in Designing Machine Members

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained.

The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

2. Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

where σ_t = Tensile stress due to direct load and bending,

σ_c = Compressive stress, and

τ = Shear stress due to torsion.

Notes : 1. When $\tau = 0$ as in the case of thin cylindrical shell subjected in internal fluid pressure, then

$$\sigma_{t\,(max)} = \sigma_t$$

2. When the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). This will give the resultant tensile stress or compressive stress (σ_t or σ_c) depending upon the type of axial load (*i.e.* pull or push).

Example 5.13. *A hollow shaft of 40 mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously, it is subjected to an axial thrust of 10 kN and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses.*

Solution. Given: d_o = 40 mm ; d_i = 25 mm ; T = 120 N-m = 120 × 10³ N-mm ; P = 10 kN = 10 × 10³ N ; M = 80 N-m = 80 × 10³ N-mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] = \frac{\pi}{4}\left[(40)^2 - (25)^2\right] = 766 \text{ mm}^2$$

∴ Direct compressive stress due to axial thrust,

$$\sigma_o = \frac{P}{A} = \frac{10 \times 10^3}{766} = 13.05 \text{ N/mm}^2 = 13.05 \text{ MPa}$$

Section modulus of the shaft,

$$Z = \frac{\pi}{32}\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{32}\left[\frac{(40)^4 - (25)^4}{40}\right] = 5325 \text{ mm}^3$$

∴ Bending stress due to bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{80 \times 10^3}{5325} = 15.02 \text{ N/mm}^2 = 15.02 \text{ MPa (compressive)}$$

and resultant compressive stress,

$$\sigma_c = \sigma_b + \sigma_o = 15.02 + 13.05 = 28.07 \text{ N/mm}^2 = 28.07 \text{ MPa}$$

We know that twisting moment (T),

$$120 \times 10^3 = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \left[\frac{(40)^4 - (25)^4}{40} \right] = 10\ 650\ \tau$$

$$\therefore \quad \tau = 120 \times 10^3 / 10\ 650 = 11.27 \text{ N/mm}^2 = 11.27 \text{ MPa}$$

Maximum compressive stress

We know that maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

$$= \frac{28.07}{2} + \frac{1}{2}\left[\sqrt{(28.07)^2 + 4\,(11.27)^2}\right]$$

$$= 14.035 + 18 = 32.035 \text{ MPa} \quad \textbf{Ans.}$$

Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(28.07)^2 + 4(11.27)^2}\right] = 18 \text{ MPa} \quad \textbf{Ans.}$$

Example 5.14. *A shaft, as shown in Fig. 5.17, is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN.*

Calculate the stresses at A and B.

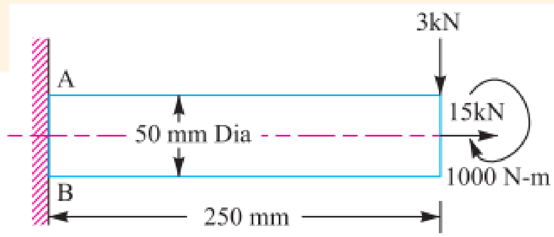


Fig. 5.17

Solution. Given : W = 3 kN = 3000 N ; T = 1000 N-m = 1×10^6 N-mm ; P = 15 kN = 15×10^3 N ; d = 50 mm; x = 250 mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4}\,(50)^2 = 1964 \text{ mm}^2$$

$\therefore$ Tensile stress due to axial pulling at points A and B,

$$\sigma_o = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64 \text{ N/mm}^2 = 7.64 \text{ MPa}$$

Bending moment at points A and B,

$$M = W.x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}\,(50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

$\therefore$ Bending stress at points *A* and *B*,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point *A* and compressive at point *B*.

$\therefore$ Resultant tensile stress at point *A*,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$
$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point *B*,

$$\sigma_B = \sigma_b - \sigma_o = 61.1 - 7.64 = 53.46 \text{ MPa}$$

We know that the shear stress at points *A* and *B* due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 1 \times 10^6}{\pi\,(50)^3} = 40.74 \text{ N/mm}^2 = 40.74 \text{ MPa} \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Stresses at point A

We know that maximum principal (or normal) stress at point *A*,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right]$$

$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 34.37 + 53.3 = 87.67 \text{ MPa (tensile)}$$ **Ans.**

Minimum principal (or normal) stress at point *A*,

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = 34.37 - 53.3 = -\,18.93 \text{ MPa}$$

$$= 18.93 \text{ MPa (compressive)}$$ **Ans.**

and maximum shear stress at point *A*,

$$\tau_{A(max)} = \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 53.3 \text{ MPa}$$ **Ans.**

Stresses at point B

We know that maximum principal (or normal) stress at point *B*,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right]$$

$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive)}$$ **Ans.**

Minimum principal (or normal) stress at point B,

$$\sigma_{B(min)} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$

$$= 26.73 - 48.73 = -22 \text{ MPa}$$

$$= 22 \text{ MPa (tensile)}$$ **Ans.**

and maximum shear stress at point B,

$$\tau_{B(max)} = \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 48.73 \text{ MPa}$$ **Ans.**

Example 5.15. *An overhang crank with pin and shaft is shown in Fig. 5.18. A tangential load of 15 kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crankshaft bearing.*

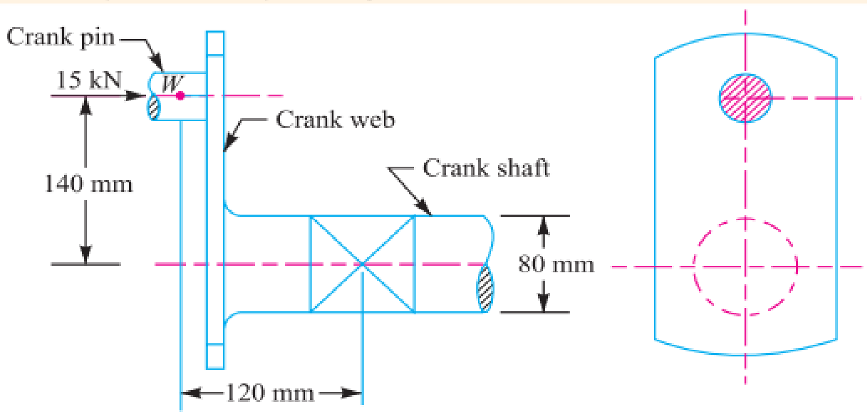


Fig. 5.18

Solution. Given : $W = 15$ kN $= 15 \times 10^3$ N ; $d = 80$ mm ; $y = 140$ mm ; $x = 120$ mm

Bending moment at the centre of the crankshaft bearing,

$$M = W \times x = 15 \times 10^3 \times 120 = 1.8 \times 10^6 \text{ N-mm}$$

and torque transmitted at the axis of the shaft,

$$T = W \times y = 15 \times 10^3 \times 140 = 2.1 \times 10^6 \text{ N-mm}$$

We know that bending stress due to the bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{32\,M}{\pi\,d^3} \qquad \ldots\left(\because Z = \frac{\pi}{32} \times d^3\right)$$

$$= \frac{32 \times 1.8 \times 10^6}{\pi\,(80)^3} = 35.8 \text{ N/mm}^2 = 35.8 \text{ MPa}$$

and shear stress due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 2.1 \times 10^6}{\pi\,(80)^3} = 20.9 \text{ N/mm}^2 = 20.9 \text{ MPa}$$

Maximum principal stress

We know that maximum principal stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

$$= \frac{35.8}{2} + \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right] \qquad \text{... (Substituting } \sigma_t = \sigma_b)$$

$$= 17.9 + 27.5 = 45.4 \text{ MPa}$$ **Ans.**

Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right]$$

$$= 27.5 \text{ MPa}$$ **Ans.**