23.14 Stress and Deflection in Helical Springs of Non-circular Wire

The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space. However these springs have the following main disadvantages :

1. The quality of material used for springs is not so good.
2. The shape of the wire does not remain square or rectangular while forming helix, resulting in trapezoidal cross-sections. It reduces the energy absorbing capacity of the spring.
3. The stress distribution is not as favourable as for circular wires. But this effect is negligible where loading is of static nature.

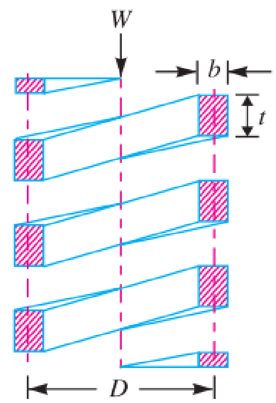


Fig. 23.18. Spring of rectangular wire.

For springs made of rectangular wire, as shown in Fig. 23.18, the maximum shear stress is given by

$$\tau = K \times \frac{W.D\,(1.5\,t + 0.9\,b)}{b^2.t^2}$$

This expression is applicable when the longer side (*i.e.* $t > b$) is parallel to the axis of the spring. But when the shorter side (*i.e.* $t < b$) is parallel to the axis of the spring, then maximum shear stress,

$$\tau = K \times \frac{W.D\,(1.5\,b + 0.9\,t)}{b^2.t^2}$$

and deflection of the spring,

$$\delta = \frac{2.45\,W.D^3.n}{G.b^3\,(t - 0.56\,b)}$$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\,W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568\,W.D^3.n}{G.b^4} = \frac{5.568\,W.C^3.n}{G.b} \qquad \ldots\left(\because C = \frac{D}{b}\right)$$

where b = Side of the square.

Note : In the above expressions,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}, \text{ and } C = \frac{D}{b}$$

Example 23.16. *A loaded narrow-gauge car of mass 1800 kg and moving at a velocity 72 m/min., is brought to rest by a bumper consisting of two helical steel springs of square section. The mean diameter of the coil is six times the side of the square section. In bringing the car to rest, the springs are to be compressed 200 mm. Assuming the allowable shear stress as 365 MPa and spring index of 6, find :*

1. Maximum load on each spring, 2. Side of the square section of the wire, 3. Mean diameter of coils, and 4. Number of active coils.

Take modulus of rigidity as 80 kN/mm².

Solution. Given : m = 1800 kg ; v = 72 m/min = 1.2 m/s ; δ = 200 mm ; τ = 365 MPa = 365 N/mm^2 ; C = 6 ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. *Maximum load on each spring,*

Let W = Maximum load on each spring.

We know that kinetic energy of the car

$$= \frac{1}{2} m.v^2 = \frac{1}{2} \times 1800 \, (1.2)^2 = 1296 \text{ N-m} = 1296 \times 10^3 \text{ N-mm}$$

This energy is absorbed in the two springs when compressed to 200 mm. If the springs are loaded gradually from 0 to W, then

$$\left(\frac{0 + W}{2}\right) 2 \times 200 = 1296 \times 10^3$$

$\therefore$ $W = 1296 \times 10^3 / 200 = 6480$ N **Ans.**

2. *Side of the square section of the wire*

Let b = Side of the square section of the wire, and

D = Mean diameter of the coil = 6 b ... ($\because$ $C = D/b = 6$)

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stres (τ),

$$365 = K \times \frac{2.4 \, W.D}{b^3} = 1.2525 \times \frac{2.4 \times 6480 \times 6 \, b}{b^3} = \frac{116\,870}{b^2}$$

$\therefore$ $b^2 = 116\,870 / 365 = 320$ or $b = 17.89$ say 18 mm **Ans.**

3. *Mean diameter of the coil*

We know that mean diameter of the coil,

$$D = 6\,b = 6 \times 18 = 108 \text{ mm}$$ **Ans.**

4. *Number of active coils*

Let n = Number of active coils.

We know that the deflection of the spring (δ),

$$200 = \frac{5.568 \, W.C^3.n}{G.b} = \frac{5.568 \times 6480 \times 6^3 \times n}{80 \times 10^3 \times 18} = 5.4\,n$$

$\therefore$ $n = 200 / 5.4 = 37$ **Ans.**

23.15 Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the *Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig. 23.19.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to $\tau_e / 2$ and the variable shear stress is also equal to $\tau_e / 2$. A line drawn from A to B (the yield point in shear, τ_y) gives the Soderberg's failure stress line. If a suitable factor of safety ($F.S.$) is applied to the yield strength (τ_y), a safe stress line CD may be drawn parallel to the line AB, as shown in Fig. 23.19. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below :

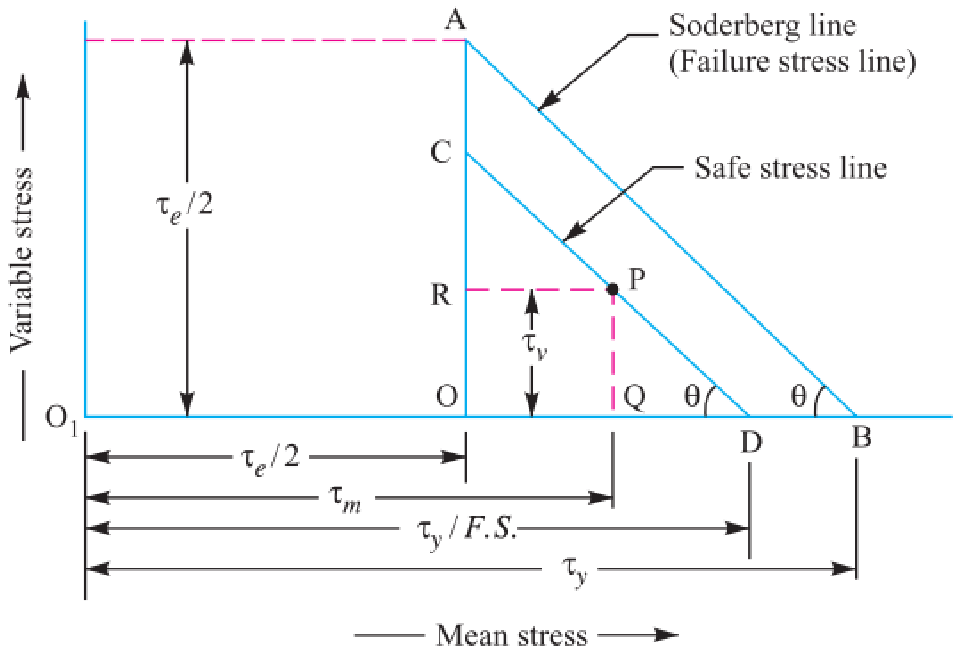


Fig. 23.19. Modified Soderberg method for helical springs.

From similar triangles *PQD* and *AOB*, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\dfrac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e/2}{\tau_y - \dfrac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or
$$2\,\tau_v\,.\,\tau_y - \tau_v.\,\tau_e = \frac{\tau_e\,.\tau_y}{F.S.} - \tau_m.\tau_e$$

$\therefore$
$$\frac{\tau_e\,.\tau_y}{F.S.} = 2\,\tau_v.\tau_y - \tau_v.\tau_e + \tau_m.\tau_e$$

Dividing both sides by $\tau_e.\tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e} \quad \text{...}(i)$$

Notes : 1. From equation ***(i)***, the expression for the factor of safety (*F.S.*) may be written as

$$F.S. = \frac{\tau_y}{\tau_m - \tau_v + \dfrac{2\,\tau_v.\tau_y}{\tau_e}}$$

2. The value of mean shear stress (τ_m) is calculated by using the shear stress factor (K_S), while the variable shear stress is calculated by using the full value of the Wahl's factor (*K*). Thus

Mean shear stress,

$$\tau_m = K_s \times \frac{8\,W_m \times D}{\pi d^3}$$

where
$$K_S = 1 + \frac{1}{2C}; \text{ and } W_m = \frac{W_{max} + W_{min}}{2}$$

and variable shear stress, $\tau_v = K \times \dfrac{8 W_v \times D}{\pi d^3}$

where $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$; and $W_v = \dfrac{W_{max} - W_{min}}{2}$

Example 23.17. *A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and the design factor of safety is 1.25. If the yield stress in shear is 770 MPa and endurance stress in shear is 350 MPa, find : 1. Size of the spring wire, 2. Diameters of the spring, 3. Number of turns of the spring, and 4. Free length of the spring.*

The compression of the spring at the maximum load is 30 mm. The modulus of rigidity for the spring material may be taken as 80 kN/mm².

Solution. Given : W_{min} = 400 N ; W_{max} = 1000 N ; C = 6 ; $F.S.$ = 1.25 ; τ_y = 770 MPa = 770 N/mm² ; τ_e = 350 MPa = 350 N/mm² ; δ = 30 mm ; G = 80 kN/mm² = 80 × 10³ N/mm²

1. ***Size of the spring wire***

Let d = Diameter of the spring wire, and

D = Mean diameter of the spring = $C.d = 6\,d$ ... ($\because D/d = C = 6$)

We know that the mean load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{1000 + 400}{2} = 700 \text{ N}$$

and variable load,

$$W_v = \frac{W_{max} - W_{min}}{2} = \frac{1000 - 400}{2} = 300 \text{ N}$$

Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 6} = 1.083$$

Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We know that mean shear stress,

$$\tau_m = K_S \times \frac{8 W_m \times D}{\pi d^3} = 1.083 \times \frac{8 \times 700 \times 6\,d}{\pi d^3} = \frac{11\,582}{d^2} \text{ N/mm}^2$$

and variable shear stress,

$$\tau_v = K \times \frac{8 W_v \times D}{\pi d^3} = 1.2525 \times \frac{8 \times 300 \times 6\,d}{\pi d^3} = \frac{5740}{d^2} \text{ N/mm}^2$$

We know that

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\,\tau_v}{\tau_e}$$

$$\frac{1}{1.25} = \frac{\dfrac{11\,582}{d^2} - \dfrac{5740}{d^2}}{770} + \frac{2 \times \dfrac{5740}{d^2}}{350} = \frac{7.6}{d^2} + \frac{32.8}{d^2} = \frac{40.4}{d^2}$$

$\therefore$ $d^2 = 1.25 \times 40.4 = 50.5$ or $d = 7.1$ mm **Ans.**

2. *Diameters of the spring*

We know that mean diameter of the spring,

$$D = C.d = 6 \times 7.1 = 42.6 \text{ mm } \textbf{Ans.}$$

Outer diameter of the spring,

$$D_o = D + d = 42.6 + 7.1 = 49.7 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring,

$$D_i = D - d = 42.6 - 7.1 = 35.5 \text{ mm } \textbf{Ans.}$$

3. *Number of turns of the spring*

Let n = Number of active turns of the spring.

We know that deflection of the spring (δ),

$$30 = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times 1000\,(42.6)^3\,n}{80 \times 10^3\,(7.1)^4} = 3.04\,n$$

$\therefore$ $n = 30 / 3.04 = 9.87$ say 10 **Ans.**

Assuming the ends of the spring to be squared and ground, the total number of turns of the spring,

$$n' = n + 2 = 10 + 2 = 12 \textbf{ Ans.}$$

4. *Free length of the spring*

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 12 \times 7.1 + 30 + 0.15 \times 30 \text{ mm}$$
$$= 119.7 \text{ say } 120 \text{ mm } \textbf{Ans.}$$

23.16 Springs in Series

Consider two springs connected in series as shown in Fig. 23.20.

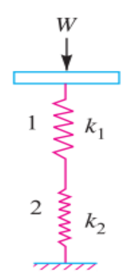
Let W = Load carried by the springs,

δ_1 = Deflection of spring 1,

δ_2 = Deflection of spring 2,

k_1 = Stiffness of spring 1 = W / δ_1, and

k_2 = Stiffness of spring 2 = W / δ_2



Springs in series.
Fig. 23.20

A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

$\therefore$ Total deflection of the springs,

$$\delta = \delta_1 + \delta_2$$

or
$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$\therefore$
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

where k = Combined stiffness of the springs.

23.17 Springs in Parallel

Consider two springs connected in parallel as shown in Fig 23.21.

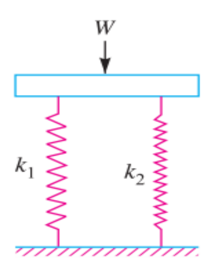
Let W = Load carried by the springs,

W_1 = Load shared by spring 1,

W_2 = Load shared by spring 2,

k_1 = Stiffness of spring 1, and

k_2 = Stiffness of spring 2.



Springs in parallel.
Fig. 23.21

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

We know that $W = W_1 + W_2$

or $\delta.k = \delta.k_1 + \delta.k_2$

$\therefore$ $k = k_1 + k_2$

where k = Combined stiffness of the springs, and

δ = Deflection produced.

Example 23.18. *A close coiled helical compression spring of 12 active coils has a spring stiffness of k. It is cut into two springs having 5 and 7 turns. Determine the spring stiffnesses of resulting springs.*

Solution. Given : $n = 12$; $n_1 = 5$; $n_2 = 7$

We know that the deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} \quad \text{or} \quad \frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n}$$

Since G, D and d are constant, therefore substituting

$$\frac{G.d^4}{8\,D^3} = X, \text{ a constant, we have } \frac{W}{\delta} = k = \frac{X}{n}$$

or $X = k.n = 12\,k$

The spring is cut into two springs with $n_1 = 5$ and $n_2 = 7$.

Let k_1 = Stiffness of spring having 5 turns, and

k_2 = Stiffness of spring having 7 turns.

$$\therefore \quad k_1 = \frac{X}{n_1} = \frac{12\,k}{5} = 2.4\,k \quad \textbf{Ans.}$$

and

$$k_2 = \frac{X}{n_2} = \frac{12\,k}{7} = 1.7\,k \quad \textbf{Ans.}$$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig. 23.12.

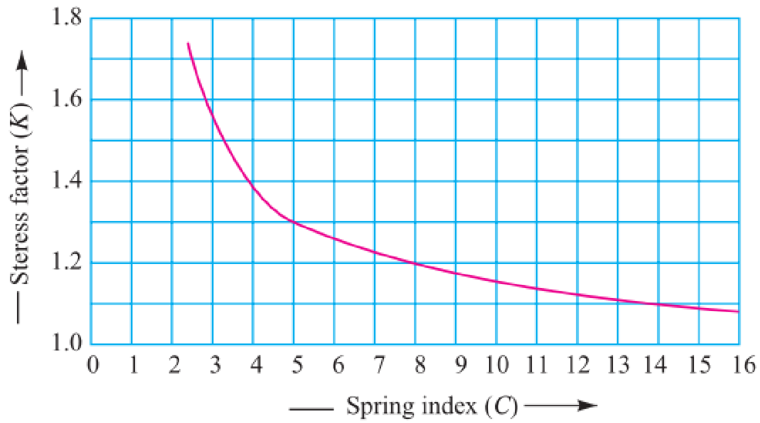


Fig. 23.12. Wahl's stress factor for helical springs.

Table 23.4. Values of buckling factor (K_B).

L_F/D	*Hinged end spring*	*Built-in end spring*	L_F / D	*Hinged end spring*	*Built-in end spring*
1	0.72	0.72	5	0.11	0.53
2	0.63	0.71	6	0.07	0.38
3	0.38	0.68	7	0.05	0.26
4	0.20	0.63	8	0.04	0.19