Variable Stresses in Machine Parts

6.1 Introduction

We have discussed, in the previous chapter, the stresses due to static loading only. But only a few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses.

6.2 Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load *W*, as shown in Fig. 6.1. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (*i.e.* at point *A*) are under compressive stress and the lower fibres (*i.e.* at point *B*) are under tensile stress. After half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.

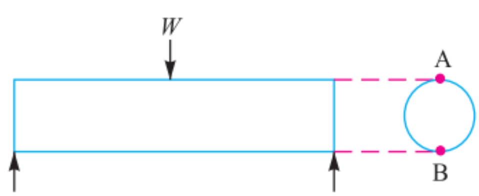


Fig. 6.1. Reversed or cyclic stresses.

From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or ***vice versa***, are known as ***completely reversed*** or ***cyclic stresses.***

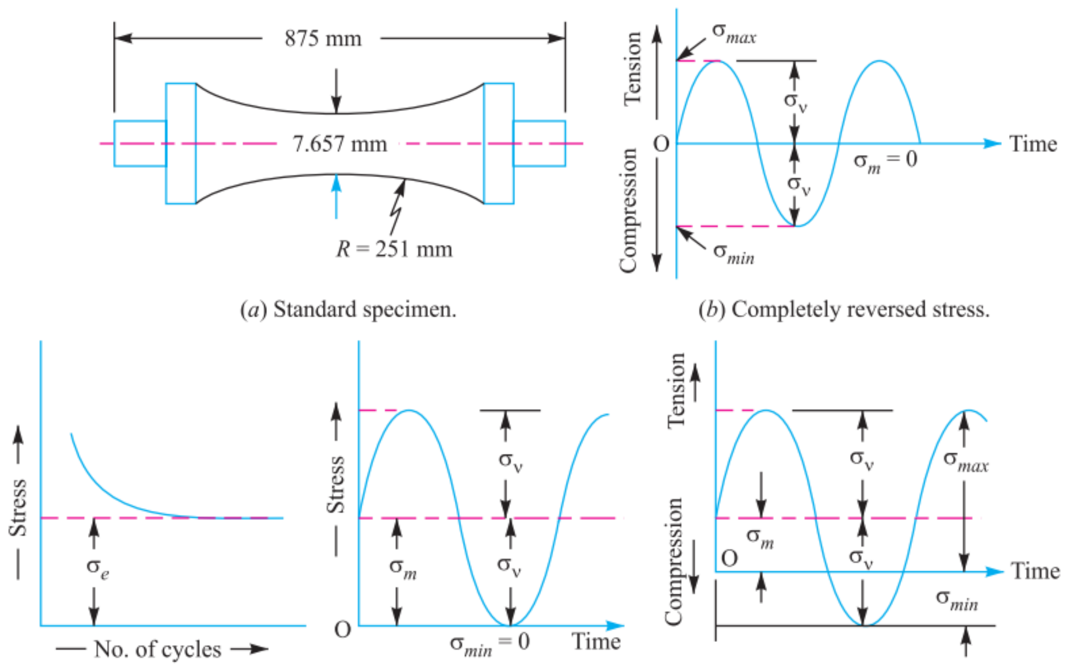
Notes: 1. The stresses which vary from a minimum value to a maximum value of the same nature, (*i.e.* tensile or compressive) are called ***fluctuating stresses.***

2. The stresses which vary from zero to a certain maximum value are called ***repeated stresses.***

3. The stresses which vary from a minimum value to a maximum value of the opposite nature (*i.e.* from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called ***alternating stresses.***

6.3 Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(*a*) Standard specimen. (*b*) Completely reversed stress.

(*c*) Endurance or fatigue limit. (*d*) Repeated stress. (*e*) Fluctuating stress.

Fig. 6.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 6.2 (*a*), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 6.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig. 6.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown stress, as represented by dotted line, is known as ***endurance*** or ***fatigue limit*** (σ_e). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 6.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress.

The stress ***verses*** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. 6.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 6.2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Note: For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma_{min} = 0$) in each cycle as shown in Fig. 6.2 (*d*).

$$\therefore \qquad \sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \dfrac{\sigma_{max}}{\sigma_{min}}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

where σ'_e = Endurance limit for any stress range represented by R.

σ_e = Endurance limit for completely reversed stresses, and

R = Stress ratio.

6.4 Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e.K_b = \sigma_e$ ...(∵ $K_b = 1$)

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e.K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e.K_s$

6.5 Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 6.3 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

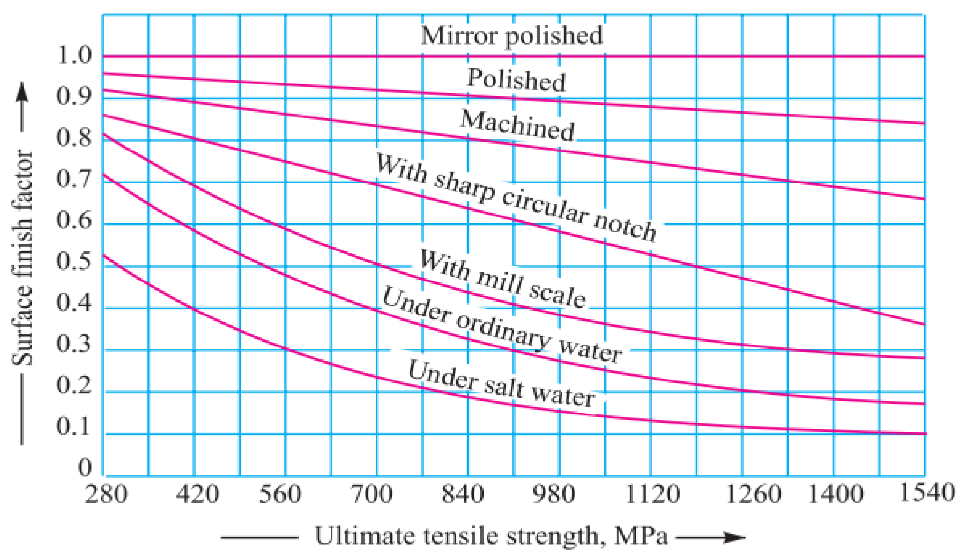


Fig. 6.3. Surface finish factor for various surface conditions.

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur} \quad \text{...}(\because K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad \text{...(For reversed torsional or shear load)}$$

Note : The surface finish factor for non-ferrous metals may be taken as unity.

6.6 Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig. 6.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

∴ Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$

$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$

$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

Notes: 1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

6.8 Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u). Fig. 6.4 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice :

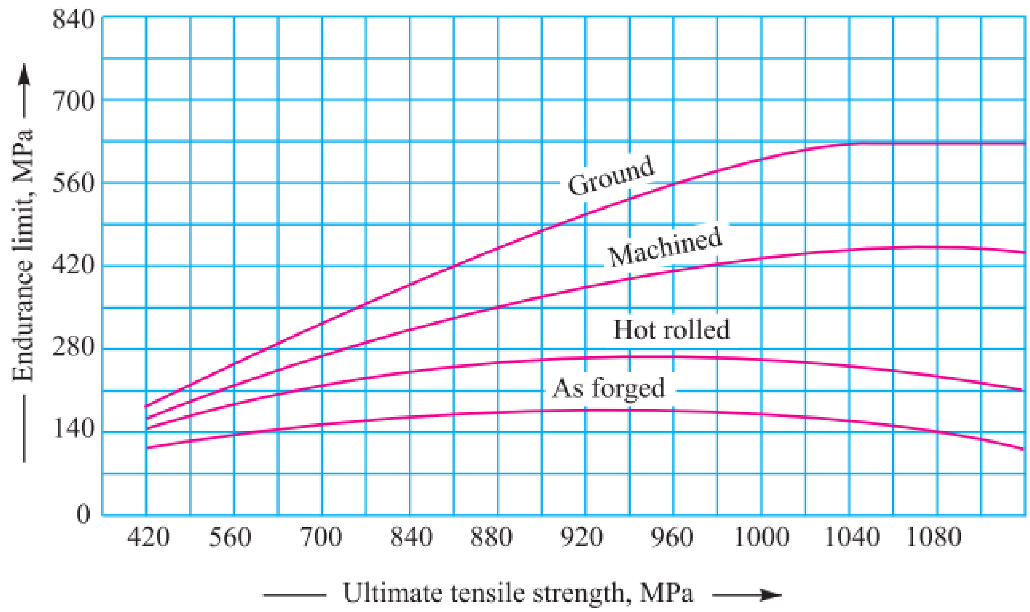


Fig. 6.4. Endurance limit of steel corresponding to ultimate tensile strength.

For steel, $\sigma_e = 0.5\ \sigma_u$;

For cast steel, $\sigma_e = 0.4\ \sigma_u$;

For cast iron, $\sigma_e = 0.35\ \sigma_u$;

For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

6.9 Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for faliure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

Note: For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$

where σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Example 6.1. *Determine the design stress for a piston rod where the load is completely reversed. The surface of the rod is ground and the surface finish factor is 0.9. There is no stress concentration. The load is predictable and the factor of safety is 2.*

Piston rod

Solution. Given : $K_{sur} = 0.9$; $F.S. = 2$

The piston rod is subjected to reversed axial loading. We know that for reversed axial loading, the load correction factor (K_a) is 0.8.

If σ_e is the endurance limit for reversed bending load, then endurance limit for reversed axial load,

$$\sigma_{ea} = \sigma_e \times K_a \times K_{sur} = \sigma_e \times 0.8 \times 0.9 = 0.72\ \sigma_e$$

We know that design stress,

$$\sigma_d = \frac{\sigma_{ea}}{F.S.} = \frac{0.72\ \sigma_e}{2} = 0.36\ \sigma_e$$ **Ans.**

6.16 Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

6.17 Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig. 6.14, may be used for determining the values of q for two steels.

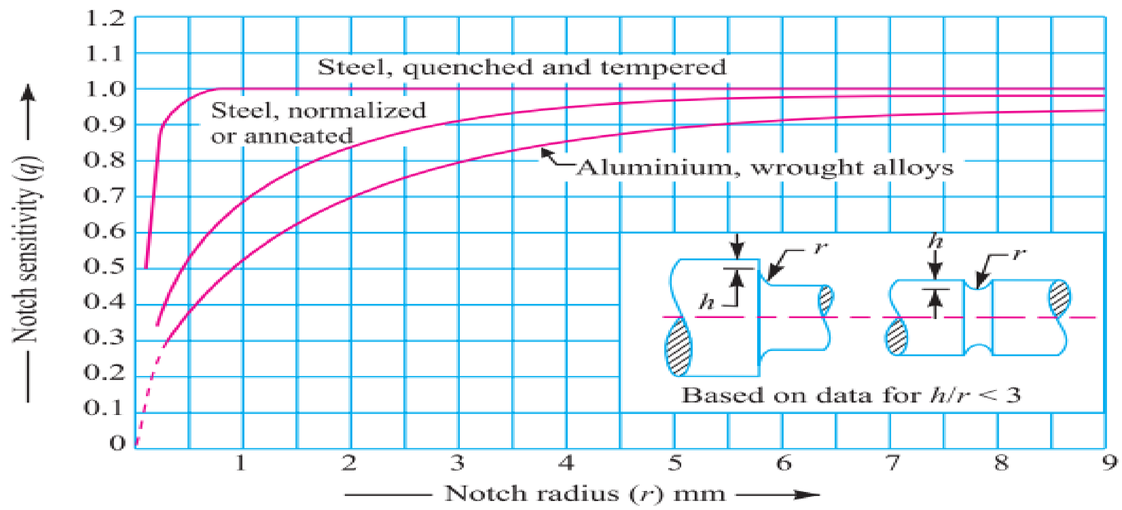


Fig. 6.14. Notch sensitivity.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

or $K_f = 1 + q\,(K_t - 1)$ ...[For tensile or bending stress]

and $K_{fs} = 1 + q\,(K_{ts} - 1)$ ...[For shear stress]

where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

6.18 Combined Steady and Variable Stress

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. 6.15 as functions of variable stress (σ_v) and mean stress (σ_m). The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

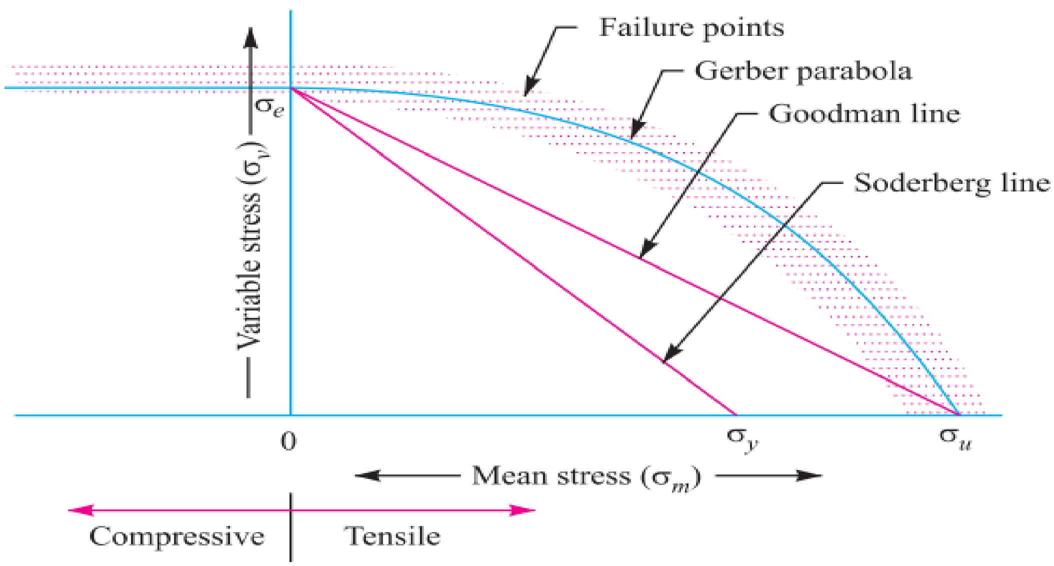


Fig. 6.15. Combined mean and variable stress.

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

1. Gerber method, **2.** Goodman method, and **3.** Soderberg method.

We shall now discuss these methods, in detail, in the following pages.

6.19 Gerber Method for Combination of Stresses

The relationship between variable stress (σ_v) and mean stress (σ_m) for axial and bending loading for ductile materials are shown in Fig. 6.15. The point σ_e represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point σ_u represents the static ultimate strength corresponding to $\sigma_v = 0$.

A parabolic curve drawn between the endurance limit (σ_e) and ultimate tensile strength (σ_u) was proposed by Gerber in 1874. Generally, the test data for ductile material fall closer to Gerber parabola as shown in Fig. 6.15, but because of scatter in the test points, a straight line relationship (*i.e.* Goodman line and Soderberg line) is usually preferred in designing machine parts.

According to Gerber, variable stress,

$$\sigma_v = \sigma_e \left[\frac{1}{F.S.} - \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. \right]$$

or

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v}{\sigma_e} \qquad \ldots(i)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_u = Ultimate stress (tensile or compressive), and

σ_e = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor (K_f), the equation (*i*) may be written as

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}$$

6.20 Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line *AB* in Fig. 6.16, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

In Fig. 6.16, line *AB* connecting σ_e and

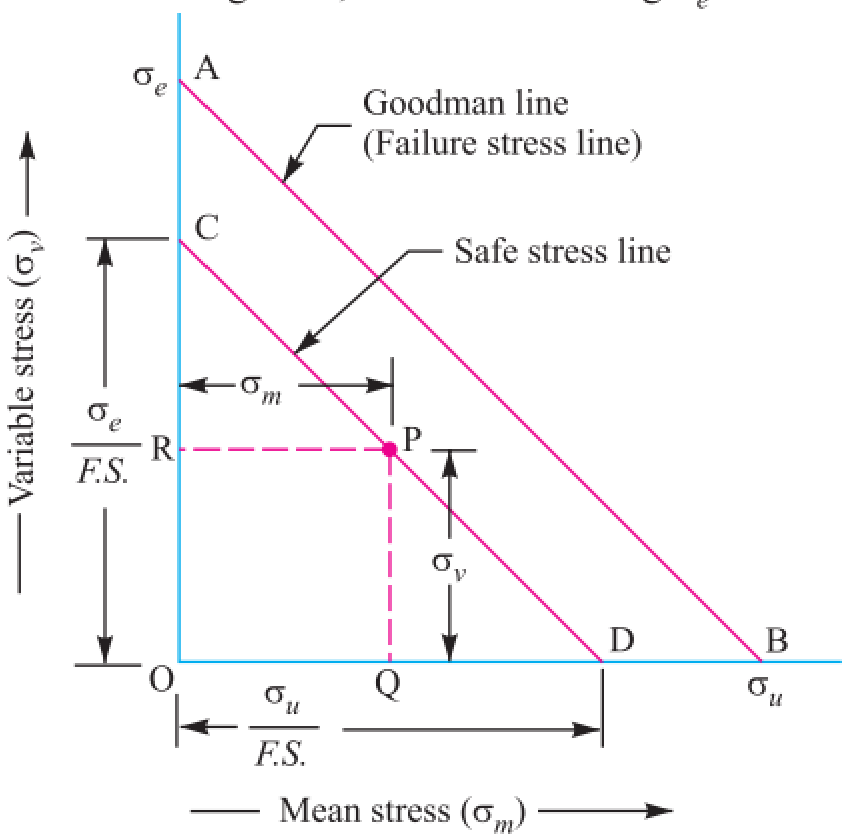


Fig. 6.16. Goodman method.

σ_u is called ***Goodman's failure stress line.*** If a suitable factor of safety (*F.S.*) is applied to endurance limit and ultimate strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*.

Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad \ldots(\because\ QD = OD - OQ)$$

$\therefore$

$$\frac{^*\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad \ldots(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (*i*) must be altered to include this effect. In such cases, the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v). The equation **(*i*)** may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad \ldots(ii)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation **(*ii*)** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad \ldots(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad \ldots(\because\ \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

where K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

---

* Here we have assumed the same factor of safety ($F.S.$) for the ultimate tensile strength (σ_u) and endurance limit (σ_e). In case the factor of safety relating to both these stresses is different, then the following relation may be used :

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m}{\sigma_u / (F.S.)_u}$$

where $(F.S.)_e$ = Factor of safety relating to endurance limit, and

$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Notes : 1. The equation ***(iii)*** is applicable to ductile materials subjected to reversed bending loads (tensile or compressive). For brittle materials, the theoretical stress concentration factor (K_t) should be applied to the mean stress and fatigue stress concentration factor (K_f) to the variable stress. Thus for brittle materials, the equation ***(iii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...}(iv)$$

2. When a machine component is subjected to a load other than reversed bending, then the endurance limit for that type of loading should be taken into consideration. Thus for reversed axial loading (tensile or compressive), the equations ***(iii)*** and ***(iv)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...(For ductile materials)}$$

and

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...(For brittle materials)}$$

Similarly, for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...(For ductile materials)}$$

and

$$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...(For brittle materials)}$$

where suffix '*s*' denotes for shear.

For reversed torsional or shear loading, the values of ultimate shear strength (τ_u) and endurance shear strength (τ_e) may be taken as follows:

$$\tau_u = 0.8\ \sigma_u; \text{ and } \tau_e = 0.8\ \sigma_e$$

6.21 Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line *AB* in Fig. 6.17, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.

Proceeding in the same way as discussed in Art 6.20, the line *AB* connecting σ_e and σ_y, as shown in Fig. 6.17, is called ***Soderberg's failure stress line***. If a suitable factor of safety (*F.S.*) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*. Now from similar triangles *COD* and *PQD*,

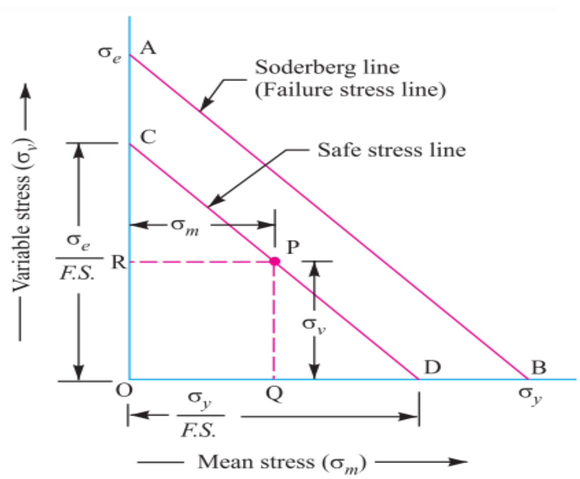


Fig. 6.17. Soderberg method.

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD} \quad \text{...}(\because QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (K_f) should be applied to only variable stress (σ_v). Thus the equations **(*i*)** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

Considering the load factor, surface finish factor and size factor, the equation **(*ii*)** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

Notes: 1. The Soderberg method is particularly used for ductile materials. The equation **(*iii*)** is applicable to ductile materials subjected to reversed bending load (tensile or compressive).

2. When a machine component is subjected to reversed axial loading, then the equation **(*iii*)** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

3. When a machine component is subjected to reversed shear loading, then equation **(*iii*)** may be written as

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

where K_{fs} is the fatigue stress concentration factor for reversed shear loading. The yield strength in shear (τ_y) may be taken as one-half the yield strength in reversed bending (σ_y).

Example 6.7. *A steel rod is subjected to a reversed axial load of 180 kN. Find the diameter of the rod for a factor of safety of 2. Neglect column action. The material has an ultimate tensile strength of 1070 MPa and yield strength of 910 MPa. The endurance limit in reversed bending may be assumed to be one-half of the ultimate tensile strength. Other correction factors may be taken as follows:*

For axial loading = 0.7; For machined surface = 0.8 ; For size = 0.85 ; For stress concentration = 1.0.

Solution. Given : W_{max} = 180 kN ; W_{min} = – 180 kN ; $F.S.$ = 2 ; σ_u = 1070 MPa = 1070 N/mm²; σ_y = 910 MPa = 910 N/mm² ; $\sigma_e = 0.5\,\sigma_u$; K_a = 0.7 ; K_{sur} = 0.8 ; K_{sz} = 0.85 ; K_f = 1

Let $\qquad d$ = Diameter of the rod in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{180 + (-180)}{2} = 0$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = 0$$

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{180 - (-180)}{2} = 180 \text{ kN} = 180 \times 10^3 \text{ N}$

$\therefore$ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{180 \times 10^3}{0.7854\, d^2} = \frac{229 \times 10^3}{d^2} \text{ N/mm}^2$

Endurance limit in reversed axial loading,

$$\sigma_{ea} = \sigma_e \times K_a = 0.5\, \sigma_u \times 0.7 = 0.35\, \sigma_u \qquad ...(\because\ \sigma_e = 0.5\, \sigma_u)$$

$$= 0.35 \times 1070 = 374.5 \text{ N/mm}^2$$

We know that according to Soderberg's formula for reversed axial loading,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = 0 + \frac{229 \times 10^3 \times 1}{d^2 \times 374.5 \times 0.8 \times 0.85} = \frac{900}{d^2}$$

$\therefore$ $d^2 = 900 \times 2 = 1800$ or $d = 42.4$ mm **Ans.**

Example 6.8. *A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by : ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.*

Solution. Given : l = 500 mm ; W_{min} = 20 kN = 20 × 10^3 N ; W_{max} = 50 kN = 50 × 10^3 N ; $F.S.$ = 1.5 ; K_{sz} = 0.85 ; K_{sur} = 0.9 ; σ_u = 650 MPa = 650 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 350 MPa = 350 N/mm^2

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N-mm}$$

Section modulus of the bar,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$\therefore$ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.0982\, d^3} = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\, d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore \quad d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$\therefore \quad d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3$ or $d = 62.1$ mm

Taking larger of the two values, we have $d = 62.1$ mm **Ans.**

Example 6.10. *A cantilever beam made of cold drawn carbon steel of circular cross-section as shown in Fig. 6.18, is subjected to a load which varies from – F to 3 F. Determine the maximum load that this member can withstand for an indefinite life using a factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9. Assume the following values :*

Ultimate stress = 550 MPa
Yield stress = 470 MPa
Endurance limit = 275 MPa
Size factor = 0.85
Surface finish factor = 0.89

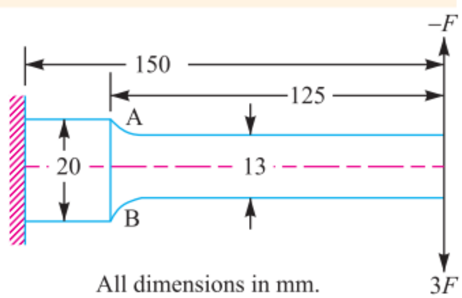


Fig. 6.18

Solution. Given : $W_{min} = -F$; $W_{max} = 3F$; $F.S. = 2$; $K_t = 1.42$; $q = 0.9$; $\sigma_u = 550$ MPa $= 550$ N/mm² ; $\sigma_y = 470$ MPa $= 470$ N/mm² ; $\sigma_e = 275$ MPa $= 275$ N/mm² ; $K_{sz} = 0.85$; $K_{sur} = 0.89$

The beam as shown in Fig. 6.18 is subjected to a reversed bending load only. Since the point *A* at the change of cross section is critical, therefore we shall find the bending moment at point *A*.

We know that maximum bending moment at point *A*,

$$M_{max} = W_{max} \times 125 = 3F \times 125 = 375\, F \text{ N-mm}$$

and minimum bending moment at point *A*,

$$M_{min} = W_{min} \times 125 = -F \times 125 = -125\, F \text{ N-mm}$$

$\therefore$ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{375\ F + (-125\ F)}{2} = 125\,F \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{375\ F - (-125\ F)}{2} = 250\,F \text{ N-mm}$$

Section modulus, $Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (13)^3 = 215.7 \text{ mm}^3$ ...($\because$ d = 13 mm)

$\therefore$ Mean bending stress, $\sigma_m = \frac{M_m}{Z} = \frac{125\ F}{215.7} = 0.58\ F \text{ N/mm}^2$

and variable bending stress, $\sigma_v = \frac{M_v}{Z} = \frac{250\ F}{215.7} = 1.16\ F \text{ N/mm}^2$

Fatigue stress concentration factor, $K_f = 1 + q\,(K_t - 1) = 1 + 0.9\,(1.42 - 1) = 1.378$

We know that according to Goodman's formula

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{550} + \frac{1.16\,F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 05\ F + 0.007\ 68\ F = 0.008\ 73\ F$$

$$\therefore \quad F = \frac{1}{2 \times 0.008\,73} = 57.3 \text{ N}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{470} + \frac{1.16\,F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 23\ F + 0.007\ 68\ F = 0.008\ 91\ F$$

$$\therefore \quad F = \frac{1}{2 \times 0.008\ 91} = 56 \text{ N}$$

Taking larger of the two values, we have F = 57.3 N **Ans.**

Example 6.11. *A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.*

Solution. Given : $W_{min} = P$; $W_{max} = 4P$; L = 500 mm; d = 60 mm ; σ_u = 700 MPa = 700 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 330 MPa = 330 N/mm^2 ; $F.S.$ = 1.3 ; K_{sz} = 0.85 ; K_{sur} = 0.9

We know that maximum bending moment,

$$M_{max} = \frac{W_{max} \times L}{4} = \frac{4\,P \times 500}{4} = 500\,P \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125\,P \text{ N-mm}$$

$\therefore$ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500\,P + 125\,P}{2} = 312.5\,P \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500\,P - 125\,P}{2} = 187.5\,P \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}\,(60)^3 = 21.21 \times 10^3 \text{ mm}^3$$

$\therefore$ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5\,P}{21.21 \times 10^3} = 0.0147\,P \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5\,P}{21.21 \times 10^3} = 0.0088\,P \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{700} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1)$$

$$= \frac{21\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{55.8\,P}{10^6}$$

$$\therefore \quad P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13\ 785 \text{ N} = 13.785 \text{ kN}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{500} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} = \frac{29.4\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{64.2P}{10^6}$$

$$\therefore \quad P = \frac{1}{1.3} \times \frac{10^6}{64.2} = 11\ 982 \text{ N} = 11.982 \text{ kN}$$

From the above, we find that maximum value of P = 13.785 kN **Ans.**

6.22 Combined Variable Normal Stress and Variable Shear Stress

When a machine part is subjected to both variable normal stress and a variable shear stress; then it is designed by using the following two theories of combined stresses :

1. Maximum shear stress theory, and **2.** Maximum normal stress theory.

We have discussed in Art. 6.21, that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...(For reversed bending load)}$$

Multiplying throughout by σ_y, we get

$$\frac{\sigma_y}{F.S.} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as ***equivalent normal stress*** due to reversed bending.

∴ Equivalent normal stress due to reversed bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...}(i)$$

Similarly, equivalent normal stress due to reversed axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...}(ii)$$

and total equivalent normal stress,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{\sigma_y}{F.S.} \quad \text{...}(iii)$$

We have also discussed in Art. 6.21, that for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

Multiplying throughout by τ_y, we get

$$\frac{\tau_y}{F.S.} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as ***equivalent shear stress.***

∴ Equivalent shear stress due to reversed torsional or shear loading,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...}(iv)$$

The maximum shear stress theory is used in designing machine parts of ductile materials. According to this theory, maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\tau_y}{F.S.}$$

The maximum normal stress theory is used in designing machine parts of brittle materials. According to this theory, maximum equivalent normal stress,

$$\sigma_{ne(max)} = \frac{1}{2}(\sigma_{ne}) + \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\sigma_y}{F.S.}$$

Example 6.12. *A steel cantilever is 200 mm long. It is subjected to an axial load which varies from 150 N (compression) to 450 N (tension) and also a transverse load at its free end which varies from 80 N up to 120 N down. The cantilever is of circular cross-section. It is of diameter 2d for the first 50 mm and of diameter d for the remaining length. Determine its diameter taking a factor of safety of 2. Assume the following values :*

Yield stress	= *330 MPa*
Endurance limit in reversed loading	= *300 MPa*
Correction factors	= *0.7 in reversed axial loading*
	= *1.0 in reversed bending*
Stress concentration factor	= *1.44 for bending*
	= *1.64 for axial loading*
Size effect factor	= *0.85*
Surface effect factor	= *0.90*
Notch sensitivity index	= *0.90*

Solution. Given : l = 200 mm; $W_{a(max)}$ = 450 N; $W_{a(min)}$ = – 150 N ; $W_{t(max)}$ = 120 N ; $W_{t(min)}$ = – 80 N; $F.S.$ = 2 ; σ_y = 330 MPa = 330 N/mm^2 ; σ_e = 300 MPa = 300 N/mm^2 ; K_a = 0.7; K_b = 1; K_{tb} = 1.44 ; K_{ta} = 1.64; K_{sz} = 0.85 ; K_{sur} = 0.90 ; q = 0.90

First of all, let us find the equivalent normal stress for point A which is critical as shown in Fig. 6.19. It is assumed that the equivalent normal stress at this point will be the algebraic sum of the equivalent normal stress due to axial loading and equivalent normal stress due to bending (*i.e.* due to transverse load acting at the free end).

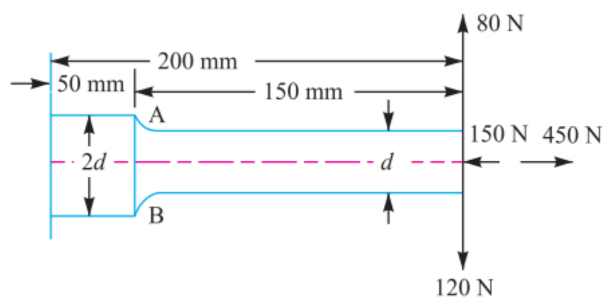


Fig. 6.19

Let us first consider the reversed axial loading. We know that mean or average axial load,

$$W_m = \frac{W_{a(max)} + W_{a(min)}}{2} = \frac{450 + (-150)}{2} = 150 \text{ N}$$

and variable axial load,

$$W_v = \frac{W_{a(max)} - W_{a(min)}}{2} = \frac{450 - (-150)}{2} = 300 \text{ N}$$

∴ Mean or average axial stress,

$$\sigma_m = \frac{W_m}{A} = \frac{150 \times 4}{\pi d^2} = \frac{191}{d^2} \text{ N/mm}^2 \qquad \ldots \left(\because A = \frac{\pi}{4} \times d^2\right)$$

and variable axial stress,

$$\sigma_v = \frac{W_v}{A} = \frac{300 \times 4}{\pi d^2} = \frac{382}{d^2} \text{ N/mm}^2$$

We know that fatigue stress concentration factor for reversed axial loading,

$$K_{fa} = 1 + q\,(K_{ta} - 1) = 1 + 0.9\,(1.64 - 1) = 1.576$$

and endurance limit stress for reversed axial loading,

$$\sigma_{ea} = \sigma_e \times K_a = 300 \times 0.7 = 210 \text{ N/mm}^2$$

We know that equivalent normal stress at point A due to axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}} = \frac{191}{d^2} + \frac{382 \times 330 \times 1.576}{d^2 \times 210 \times 0.9 \times 0.85}$$

$$= \frac{191}{d^2} + \frac{1237}{d^2} = \frac{1428}{d^2} \text{ N/mm}^2$$

Now let us consider the reversed bending due to transverse load. We know that mean or average bending load,

$$W_m = \frac{W_{t(max)} + W_{t(min)}}{2}$$

$$= \frac{120 + (-80)}{2} = 20 \text{ N}$$

and variable bending load,

$$W_v = \frac{W_{t(max)} - W_{t(min)}}{2}$$

$$= \frac{120 - (-80)}{2} = 100 \text{ N}$$

∴ Mean bending moment at point A,

$$M_m = W_m (l - 50) = 20 (200 - 50) = 3000 \text{ N-mm}$$

and variable bending moment at point A,

$$M_v = W_v (l - 50) = 100 (200 - 50) = 15\,000 \text{ N-mm}$$

We know that section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{3000}{0.0982\, d^3} = \frac{30\,550}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{15\,000}{0.0982\, d^3} = \frac{152\,750}{d^3} \text{ N/mm}^2$$

We know that fatigue stress concentration factor for reversed bending,

$$K_{fb} = 1 + q\,(K_{tb} - 1) = 1 + 0.9\,(1.44 - 1) = 1.396$$

Since the correction factor for reversed bending load is 1 (*i.e.* $K_b = 1$), therefore the endurance limit for reversed bending load,

$$\sigma_{eb} = \sigma_e \,.\, K_b = \sigma_e = 300 \text{ N/mm}^2$$

We know that the equivalent normal stress at point *A* due to bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{30\ 550}{d^3} + \frac{152\ 750 \times 330 \times 1.396}{d^3 \times 300 \times 0.9 \times 0.85}$$

$$= \frac{30\ 550}{d^3} + \frac{306\ 618}{d^3} = \frac{337\ 168}{d^3} \text{ N/mm}^2$$

∴ Total equivalent normal stress at point *A*,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{337\ 168}{d^3} + \frac{1428}{d^2} \text{ N/mm}^2 \qquad ...(i)$$

We know that equivalent normal stress at point *A*,

$$\sigma_{ne} = \frac{\sigma_y}{F.S.} = \frac{330}{2} = 165 \text{ N/mm}^2 \qquad ...(ii)$$

Equating equations (*i*) and (*ii*), we have

$$\frac{337\ 168}{d^3} + \frac{1428}{d^2} = 165 \quad \text{or} \quad 337\ 168 + 1428\, d = 165\, d^3$$

∴ $236.1 + d = 0.116\, d^3$ or $d = 12.9$ mm **Ans.** ...(By hit and trial)