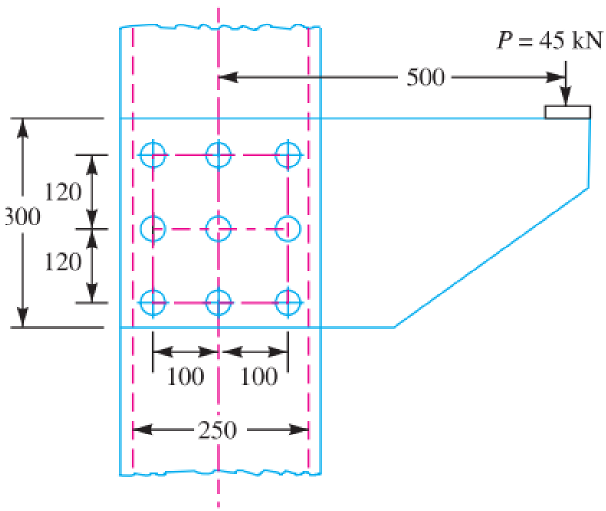
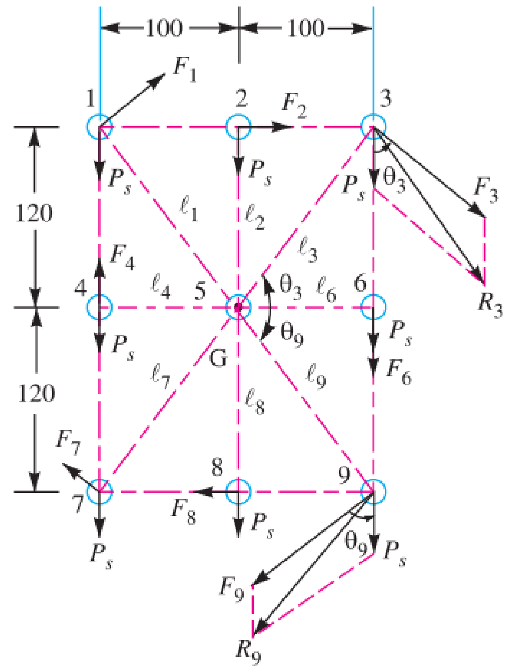
Example 9.15. *The bracket as shown in Fig. 9.27, is to carry a load of 45 kN. Determine the size of the rivet if the shear stress is not to exceed 40 MPa. Assume all rivets of the same size.*

Solution. Given : $P = 45$ kN $= 45 \times 10^3$ N ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $e = 500$ mm; $n = 9$





All dimensions in mm.

Fig. 9.27 **Fig. 9.28**

First of all, let us find the centre of gravity of the rivet system.

Since all the rivets are of same size and placed symmetrically, therefore the centre of gravity of the rivet system lies at G (rivet 5) as shown in Fig. 9.28.

We know that direct shear load on each rivet,

$$P_s = P / n = 45 \times 10^3 / 9 = 5000 \text{ N}$$

The direct shear load acts parallel to the direction of load P, *i.e.* vertically downward as shown in the figure.

Turning moment produced by the load P due to eccentricity e

$$= P.e = 45 \times 10^3 \times 500 = 22.5 \times 10^6 \text{ N-mm}$$

This turning moment tends to rotate the joint about the centre of gravity (G) of the rivet system in a clockwise direction. Due to this turning moment, secondary shear load on each rivet is produced. It may be noted that rivet 5 does not resist any moment.

Let $F_1, F_2, F_3, F_4, F_6, F_7, F_8$ and F_9 be the secondary shear load on rivets 1, 2, 3, 4, 6, 7, 8 and 9 at distances $l_1, l_2, l_3, l_4, l_6, l_7, l_8$ and l_9 from the centre of gravity (G) of the rivet system as shown in Fig. 9.28. From the symmetry of the figure, we find that

$$l_1 = l_3 = l_7 = l_9 = \sqrt{(100)^2 + (120)^2} = 156.2 \text{ mm}$$

Now equating the turning moment due to eccentricity of the load to the resisting moments of the rivets, we have

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_6)^2 + (l_7)^2 + (l_8)^2 + (l_9)^2\right]$$

$$= \frac{F_1}{l_1}\left[4(l_1)^2 + 2(l_2)^2 + 2(l_4)^2\right] \quad ....(\because l_1 = l_3 = l_7 = l_9;\ l_2 = l_8 \text{ and } l_4 = l_6)$$

$$\therefore \quad 45 \times 10^3 \times 500 = \frac{F_1}{156.2}\left[4(156.2)^2 + 2(120)^2 + 2(100)^2\right] = 973.2\ F_1$$

or $$F_1 = 45 \times 10^3 \times 500 / 973.2 = 23\ 120 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity (G), therefore

$$F_2 = F_1 \times \frac{l_2}{l_1} = F_8 = 23\ 120 \times \frac{120}{156.2} = 17\ 762 \text{ N} \quad ...(\because l_2 = l_8)$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = F_7 = F_9 = 23\ 120 \text{ N} \quad ...(\because l_3 = l_7 = l_9 = l_1)$$

and $$F_4 = F_1 \times \frac{l_4}{l_1} = F_6 = 23\ 120 \times \frac{100}{156.2} = 14\ 800 \text{ N} \quad ...(\because l_4 = l_6)$$

The secondary shear loads acts perpendicular to the line joining the centre of rivet and the centre of gravity of the rivet system, as shown in Fig. 9.28 and their direction is clockwise.

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 6 and 9 are heavily loaded. Let us now find the angle between the direct and secondary shear loads for these rivets. From the geometry of the figure, we find that

$$\cos\theta_3 = \cos\theta_9 = \frac{100}{l_3} = \frac{100}{156.2} = 0.64$$

$\therefore$ Resultant shear load on rivets 3 and 9,

$$R_3 = R_9 = \sqrt{(P_s)^2 + (F_3)^3 + 2\ P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(5000)^2 + (23\ 120)^2 + 2 \times 5000 \times 23\ 120 \times 0.64} = 26\ 600 \text{ N}$$

$$...(\because F_3 = F_9 \text{ and } \cos\theta_3 = \cos\theta_9)$$

and resultant shear load on rivet 6,

$$R_6 = P_s + F_6 = 5000 + 14\ 800 = 19\ 800 \text{ N}$$

The resultant shear load (R_3 or R_9) may be determined graphically as shown in Fig. 9.28.

From above we see that the maximum resultant shear load is on rivets 3 and 9.

If d is the diameter of the rivet hole, then maximum resultant shear load (R_3),

$$26\ 600 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 40 = 31.42\ d^2$$

$$\therefore \quad d^2 = 26\ 600 / 31.42 = 846 \quad \text{or} \quad d = 29 \text{ mm}$$

From Table 9.7, we see that according to IS : 1929 – 1982 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 29 mm and the corresponding diameter of the rivet is 27 mm. **Ans.**

Example 9.16. *Find the value of P for the joint shown in Fig. 9.29 based on a working shear stress of 100 MPa for the rivets. The four rivets are equal, each of 20 mm diameter.*

Solution. Given : τ = 100 MPa = 100 N/mm^2 ; n = 4 ; d = 20 mm

We know that the direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{P}{4} = 0.25\ P$$

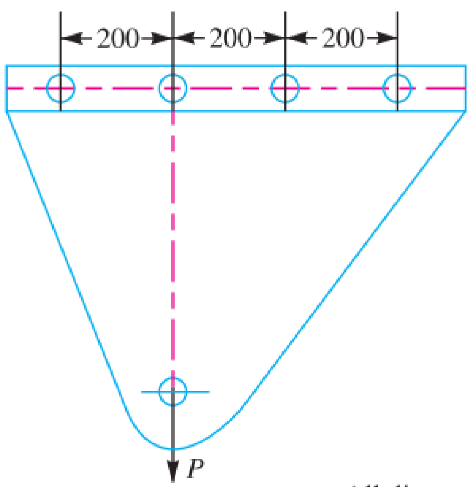
The direct shear load on each rivet acts in the direction of the load P, as shown in Fig. 9.30. The centre of gravity of the rivet group will lie at E (because of symmetry). From Fig. 9.30, we find that the perpendicular distance from the centre of gravity E to the line of action of the load (or eccentricity),

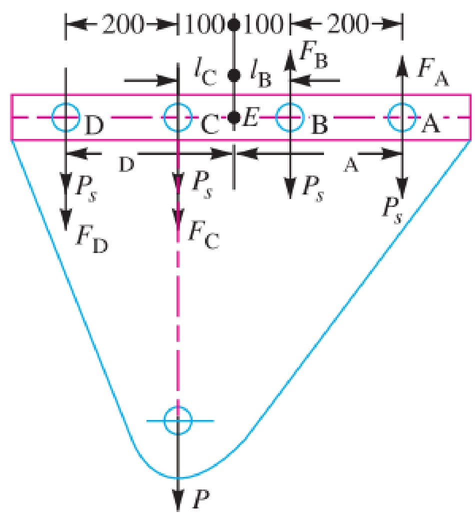
$$EC = e = 100 \text{ mm}$$

$\therefore$ Turning moment produced by the load at the centre of gravity (E) of the rivet system due to eccentricity

$$= P.e = P \times 100 \text{ N-mm (anticlockwise)}$$

This turning moment is resisted by four rivets as shown in Fig. 9.30. Let F_A, F_B, F_C and F_D be the secondary shear load on the rivets, A, B, C, and D placed at distances l_A, l_B, l_C and l_D respectively from the centre of gravity of the rivet system.





All dimensions in mm.

Fig. 9.29 **Fig. 9.30**

From Fig. 9.30, we find that

$$l_A = l_D = 200 + 100 = 300 \text{ mm} \text{ ; and } l_B = l_C = 100 \text{ mm}$$

We know that

$$P \times e = \frac{F_A}{l_A}\left[(l_A)^2 + (l_B)^2 + (l_C)^2 + (l_D)^2\right] = \frac{F_A}{l_A}\left[2(l_A)^2 + 2(l_B)^2\right]$$

...($\because l_A = l_D$ and $l_B = l_C$)

$$P \times 100 = \frac{F_A}{300}\left[2\ (300)^2 + 2\ (100)^2\right] = \frac{2000}{3} \times F_A$$

$$\therefore \quad F_A = P \times 100 \times 3 / 2000 = 3\ P/20 = 0.15\ P \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

$$F_B = F_A \times \frac{l_B}{l_A} = \frac{3\,P}{20} \times \frac{100}{300} = 0.05\,P\,\text{N}$$

$$F_C = F_A \times \frac{l_C}{l_A} = \frac{3\,P}{20} \times \frac{100}{300} = 0.05\,P\,\text{N}$$

and

$$F_D = F_A \times \frac{l_D}{l_A} = \frac{3\,P}{20} \times \frac{300}{300} = 0.15\,P\,\text{N}$$

The secondary shear loads on each rivet act at right angles to the lines joining the centre of the rivet to the centre of gravity of the rivet system as shown in Fig. 9.30.

Now let us find out the resultant shear load on each rivet. From Fig. 9.30, we find that

Resultant load on rivet *A*,

$$R_A = P_s - F_A = 0.25\,P - 0.15\,P = 0.10\,P$$

Resultant load on rivet *B*,

$$R_B = P_s - F_B = 0.25\,P - 0.05\,P = 0.20\,P$$

Resultant load on rivet *C*,

$$R_C = P_s + F_C = 0.25\,P + 0.05\,P = 0.30\,P$$

and resultant load on rivet *D*,

$$R_D = P_s + F_D = 0.25\,P + 0.15\,P = 0.40\,P$$

From above we see that the maximum shear load is on rivet *D*. We know that the maximum shear load (R_D),

$$0.40\,P = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4}\,(20)^2\,100 = 31\,420$$

$$\therefore \quad P = 31\,420 / 0.40 = 78\,550\text{ N} = 78.55\text{ kN}$$ **Ans.**