Example 10.7. *The fillet welds of equal legs are used to fabricate a `T' as shown in Fig. 10.17 (a) and (b), where s is the leg size and l is the length of weld.*

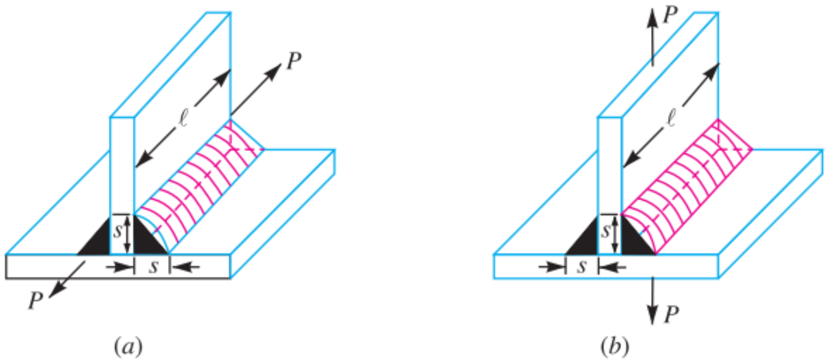


Fig. 10.17

Locate the plane of maximum shear stress in each of the following loading patterns:

1. Load parallel to the weld (neglect eccentricity), and

2. Load at right angles to the weld (transverse load).

Find the ratio of these limiting loads.

1. *Plane of maximum shear stress when load acts parallel to the weld (neglecting eccentricity)*

Let θ = Angle of plane of maximum shear stress, and

t = Throat thickness BD.

From the geometry of Fig. 10.18, we find that

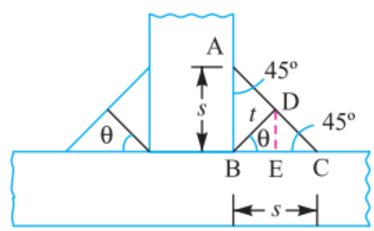


Fig. 10.18

$$BC = BE + EC$$

$$= BE + DE \quad ...(\because EC = DE)$$

or

$$s = BD\cos\theta + BD\sin\theta$$

$$= t\cos\theta + t\sin\theta$$

$$= t(\cos\theta + \sin\theta)$$

$$\therefore \quad t = \frac{s}{\cos\theta + \sin\theta}$$

We know that the minimum area of the weld or throat area,

$$A = 2t \times l = \frac{2s \times l}{(\cos\theta + \sin\theta)} \quad ...(\because \text{ of double fillet weld})$$

and shear stress, $$\tau = \frac{P}{A} = \frac{P(\cos\theta + \sin\theta)}{2s \times l} \qquad ...(i)$$

For maximum shear stress, differentiate the above expression with respect to θ and equate to zero.

$$\therefore \qquad \frac{d\tau}{d\theta} = \frac{P}{2s \times l}\,(-\sin\theta + \cos\theta) = 0$$

or $$\sin\theta = \cos\theta \quad \text{or} \quad \theta = 45°$$

Substituting the value of θ = 45° in equation *(i)*, we have maximum shear stress,

$$\tau_{max} = \frac{P\,(\cos 45° + \sin 45°)}{2s \times l} = \frac{1.414\,P}{2s \times l}$$

or $$P = \frac{2s \times l \times \tau_{max}}{1.414} = 1.414\,s \times l \times \tau_{max}$$ **Ans.**

2. *Plane of maximum shear stress when load acts at right angles to the weld*

When the load acts at right angles to the weld (transverse load), then the shear force and the normal force will act on each weld. Assuming that the two welds share the load equally, therefore summing up the vertical components, we have from Fig. 10.19,

$$P = \frac{P_s}{2}\sin\theta + \frac{P_n}{2}\cos\theta + \frac{P_s}{2}\sin\theta + \frac{P_n}{2}\cos\theta$$
$$= P_s \sin\theta + P_n \cos\theta \qquad ...(i)$$

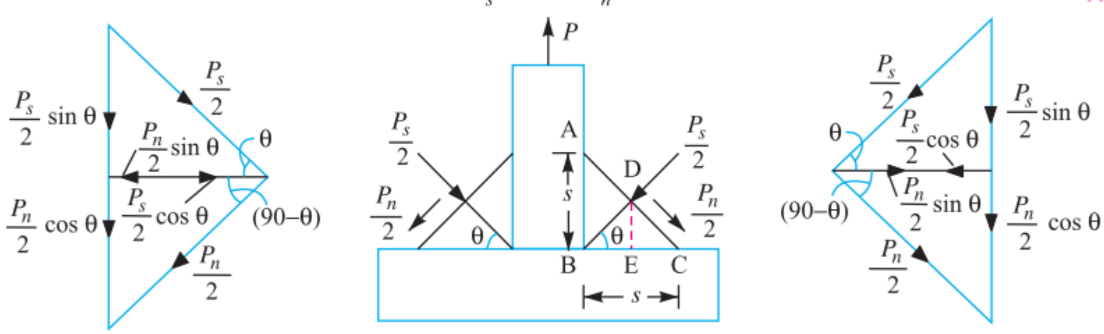


Fig. 10.19

Assuming that the resultant of $\frac{P_s}{2}$ and $\frac{P_n}{2}$ is vertical, then the horizontal components are equal and opposite. We know that

Horizontal component of $\frac{P_s}{2} = \frac{P_s}{2}\cos\theta$

and horizontal component of $\frac{P_n}{2} = \frac{P_n}{2}\sin\theta$

$$\therefore \qquad \frac{P_s}{2}\cos\theta = \frac{P_n}{2}\sin\theta \quad \text{or} \quad P_n = \frac{P_s\cos\theta}{\sin\theta}$$

Substituting the value of P_n in equation *(i)*, we have

$$P = P_s\sin\theta + \frac{P_s\cos\theta \times \cos\theta}{\sin\theta}$$

Multiplying throughout by sin θ, we have

$$P\sin\theta = P_s\sin^2\theta + P_s\cos^2\theta$$
$$= P_s(\sin^2\theta + \cos^2\theta) = P_s \qquad ...(ii)$$

From the geometry of Fig. 10.19, we have

$$BC = BE + EC = BE + DE \quad \text{...}(\because EC = DE)$$

or

$$s = t\cos\theta + t\sin\theta = t(\cos\theta + \sin\theta)$$

∴ Throat thickness, $t = \dfrac{s}{\cos\theta + \sin\theta}$

and minimum area of the weld or throat area,

$$A = 2t \times l \quad \text{... (}\because \text{ of double fillet weld)}$$

$$= 2 \times \frac{s}{\cos\theta + \sin\theta} \times l = \frac{2s \times l}{\cos\theta + \sin\theta}$$

∴ Shear stress, $\tau = \dfrac{P_s}{A} = \dfrac{P\sin\theta(\cos\theta + \sin\theta)}{2s \times l}$ ...[From equation (*ii*)] ...**(*iii*)**

For maximum shear stress, differentiate the above expression with respect to θ and equate to zero.

∴ $$\frac{d\tau}{d\theta} = \frac{P}{2\,s\,l}\left[\sin\theta(-\sin\theta + \cos\theta) + (\cos\theta + \sin\theta)\cos\theta\right] = 0$$

$$\text{...}\left(\because \frac{d(u.v)}{d\theta} = u\frac{dv}{d\theta} + v\frac{du}{d\theta}\right)$$

or

$$-\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta + \sin\theta\cos\theta = 0$$

$$\cos^2\theta - \sin^2\theta + 2\sin\theta\cos\theta = 0$$

Since $\cos^2\theta - \sin^2\theta = \cos 2\theta$ and $2\sin\theta\cos\theta = \sin 2\theta$, therefore,

$$\cos 2\theta + \sin 2\theta = 0$$

or

$$\sin 2\theta = -\cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -1 \quad \text{or} \quad \tan 2\theta = -1$$

∴ $2\theta = 135°$ or $\theta = 67.5°$ **Ans.**

Substituting the value of $\theta = 67.5°$ in equation **(*iii*)**, we have maximum shear stress,

$$\tau_{max} = \frac{P\sin 67.5°(\cos 67.5° + \sin 67.5°)}{2\,s \times l}$$

$$= \frac{P \times 0.9239\,(0.3827 + 0.9229)}{2\,s \times l} = \frac{1.21\,P}{2\,s \times l}$$

and

$$P = \frac{2s \times l \times \tau_{max}}{1.21} = 1.65\,s \times l \times \tau_{max}$$ **Ans.**

Ratio of the limiting loads

We know that the ratio of the limiting (or maximum) loads

$$= \frac{1.414\,s \times l \times \tau_{max}}{1.65\,s \times l \times \tau_{max}} = 0.857$$ **Ans.**

10.22 Axially Loaded Unsymmetrical Welded Sections

Sometimes unsymmetrical sections such as angles, channels, *T*-sections etc., welded on the flange edges are loaded axially as shown in Fig. 10.20. In such cases, the lengths of weld should be proportioned in such a way that the sum of resisting moments of the welds about the gravity axis is zero. Consider an angle section as shown in Fig. 10.20.

Let l_a = Length of weld at the top,

l_b = Length of weld at the bottom,

l = Total length of weld = $l_a + l_b$

P = Axial load,

a = Distance of top weld from gravity axis,

b = Distance of bottom weld from gravity axis, and

f = Resistance offered by the weld per unit length.

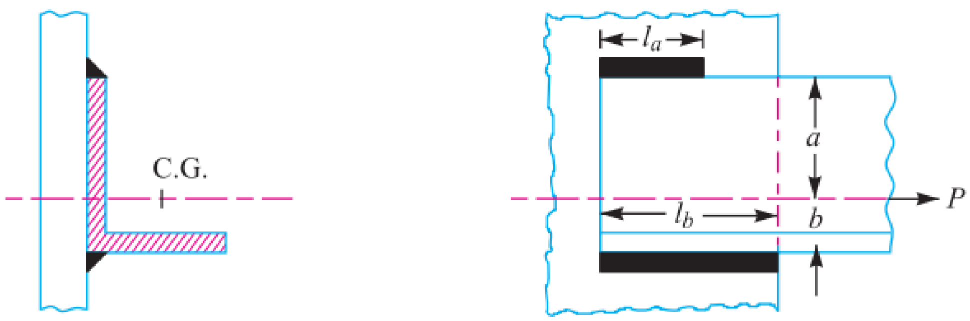


Fig. 10.20. Axially loaded unsymmetrical welded section.

∴ Moment of the top weld about gravity axis

∴ Moment of the top weld about gravity axis

$$= l_a \times f \times a$$

and moment of the bottom weld about gravity axis

$$= l_b \times f \times b$$

Since the sum of the moments of the weld about the gravity axis must be zero, therefore,

$$l_a \times f \times a - l_b \times f \times b = 0$$

or $$l_a \times a = l_b \times b \quad \text{...}(i)$$

We know that $$l = l_a + l_b \quad \text{...}(ii)$$

∴ From equations (*i*) and (*ii*), we have

$$l_a = \frac{l \times b}{a + b}, \quad \text{and} \quad l_b = \frac{l \times a}{a + b}$$

Example 10.8. *A 200 × 150 × 10 mm angle is to be welded to a steel plate by fillet welds as shown in Fig. 10.21. If the angle is subjected to a static load of 200 kN, find the length of weld at the top and bottom. The allowable shear stress for static loading may be taken as 75 MPa.*

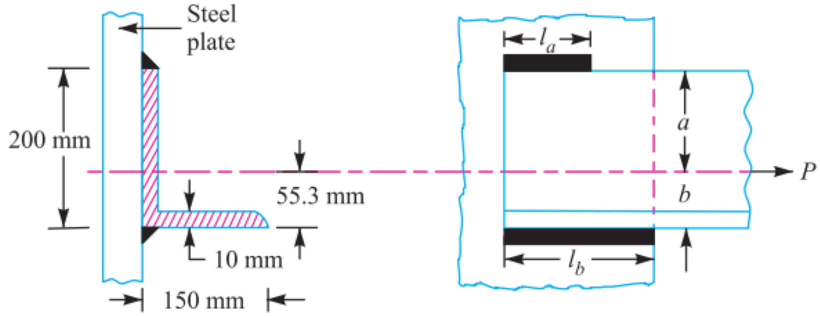


Fig. 10.21

Solution. Given : $a + b = 200$ mm ; $P = 200$ kN $= 200 \times 10^3$ N ; $\tau = 75$ MPa $= 75$ N/mm^2

Let l_a = Length of weld at the top,

l_b = Length of weld at the bottom, and

l = Total length of the weld $= l_a + l_b$

Since the thickness of the angle is 10 mm, therefore size of weld,

$$s = 10 \text{ mm}$$

We know that for a single parallel fillet weld, the maximum load (P),

$$200 \times 10^3 = 0.707\, s \times l \times \tau = 0.707 \times 10 \times l \times 75 = 530.25\, l$$

$$\therefore \quad l = 200 \times 10^3 / 530.25 = 377 \text{ mm}$$

or $$l_a + l_b = 377 \text{ mm}$$

Now let us find out the position of the centroidal axis.

Let b = Distance of centroidal axis from the bottom of the angle.

$$\therefore \quad b = \frac{(200 - 10)\, 10 \times 95 + 150 \times 10 \times 5}{190 \times 10 + 150 \times 10} = 55.3 \text{ mm}$$

and $$a = 200 - 55.3 = 144.7 \text{ mm}$$

We know that $$l_a = \frac{l \times b}{a + b} = \frac{377 \times 55.3}{200} = 104.2 \text{ mm}$$ **Ans.**

and $$l_b = l - l_a = 377 - 104.2 = 272.8 \text{ mm}$$ **Ans.**

10.23 Eccentrically Loaded Welded Joints

An eccentric load may be imposed on welded joints in many ways. The stresses induced on the joint may be of different nature or of the same nature. The induced stresses are combined depending upon the nature of stresses. When the shear and bending stresses are simultaneously present in a joint (see case 1), then maximum stresses are as follows:

Maximum normal stress,

$$\sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

where σ_b = Bending stress, and

τ = Shear stress.

When the stresses are of the same nature, these may be combined vectorially (see case 2).

We shall now discuss the two cases of eccentric loading as follows:

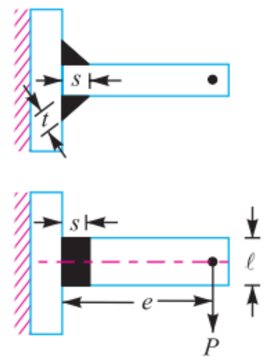


Fig. 10.22. Eccentrically loaded welded joint.

Case 1

Consider a *T*-joint fixed at one end and subjected to an eccentric load *P* at a distance *e* as shown in Fig. 10.22.

Let s = Size of weld,

l = Length of weld, and

t = Throat thickness.

The joint will be subjected to the following two types of stresses:

1. Direct shear stress due to the shear force *P* acting at the welds, and
2. Bending stress due to the bending moment $P \times e$.

We know that area at the throat,

$$A = \text{Throat thickness} \times \text{Length of weld}$$
$$= t \times l \times 2 = 2\,t \times l \quad \text{... (For double fillet weld)}$$
$$= 2 \times 0.707\,s \times l = 1.414\,s \times l \quad \text{... } (\because t = s\cos 45° = 0.707\,s)$$

∴ Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414\,s \times l}$$

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{6} \times 2 \quad \text{...(For both sides weld)}$$

$$= \frac{0.707\,s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$$

Bending moment, $M = P \times e$

∴ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2} = \frac{4.242\,P \times e}{s \times l^2}$

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Case 2

When a welded joint is loaded eccentrically as shown in Fig. 10.23, the following two types of the stresses are induced:

1. Direct or primary shear stress, and

2. Shear stress due to turning moment.

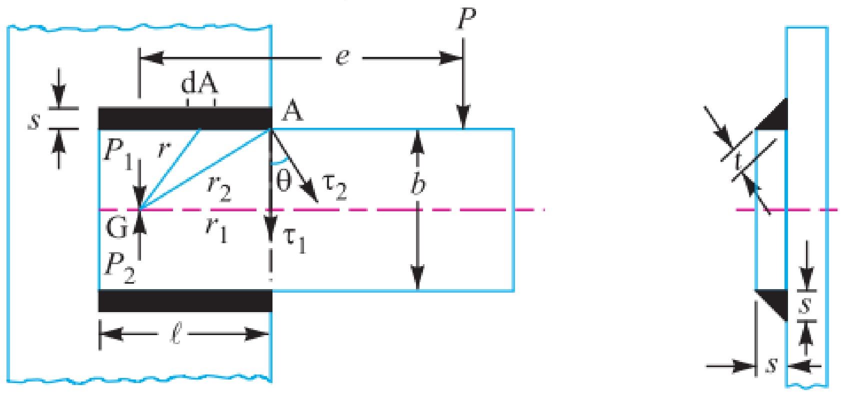


Fig. 10.23. Eccentrically loaded welded joint.

Let P = Eccentric load,

e = Eccentricity *i.e.* perpendicular distance between the line of action of load and centre of gravity (G) of the throat section or fillets,

l = Length of single weld,

s = Size or leg of weld, and

t = Throat thickness.

Let two loads P_1 and P_2 (each equal to P) are introduced at the centre of gravity 'G' of the weld system. The effect of load $P_1 = P$ is to produce direct shear stress which is assumed to be uniform over the entire weld length. The effect of load $P_2 = P$ is to produce a turning moment of magnitude $P \times e$ which tends of rotate the joint about the centre of gravity 'G' of the weld system. Due to the turning moment, secondary shear stress is induced.

We know that the direct or primary shear stress,

$$\tau_1 = \frac{\text{Load}}{\text{Throat area}} = \frac{P}{A} = \frac{P}{2\,t \times l}$$

$$= \frac{P}{2 \times 0.707\,s \times l} = \frac{P}{1.414\,s \times l}$$

... ($\because$ Throat area for single fillet weld = $t \times l = 0.707\,s \times l$)

Since the shear stress produced due to the turning moment ($T = P \times e$) at any section is proportional to its radial distance from G, therefore stress due to $P \times e$ at the point A is proportional to AG (r_2) and is in a direction at right angles to AG. In other words,

$$\frac{\tau_2}{r_2} = \frac{\tau}{r} = \text{Constant}$$

or

$$\tau = \frac{\tau_2}{r_2} \times r \qquad ...(i)$$

where τ_2 is the shear stress at the maximum distance (r_2) and τ is the shear stress at any distance r.

Consider a small section of the weld having area dA at a distance r from G.

$\therefore$ Shear force on this small section

$$= \tau \times dA$$

and turning moment of this shear force about G,

$$dT = \tau \times dA \times r = \frac{\tau_2}{r_2} \times dA \times r^2 \qquad \text{... [From equation } (i)]$$

$\therefore$ Total turning moment over the whole weld area,

$$T = P \times e = \int \frac{\tau_2}{r_2} \times dA \times r^2 = \frac{\tau_2}{r_2} \int dA \times r^2$$

$$= \frac{\tau_2}{r_2} \times J \qquad \left(\because J = \int dA \times r^2\right)$$

where J = Polar moment of inertia of the throat area about G.

$\therefore$ Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

In order to find the resultant stress, the primary and secondary shear stresses are combined vectorially.

$\therefore$ Resultant shear stress at A,

$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

where θ = Angle between τ_1 and τ_2, and

$$\cos\theta = r_1 / r_2$$

Note: The polar moment of inertia of the throat area (A) about the centre of gravity (G) is obtained by the parallel axis theorem, *i.e.*

$$J = 2\,[I_{xx} + A \times x^2] \qquad \text{... } (\because \text{ of double fillet weld})$$

$$= 2\left[\frac{A \times l^2}{12} + A \times x^2\right] = 2A\left(\frac{l_2}{12} + x^2\right)$$

where A = Throat area = $t \times l = 0.707\, s \times l$,

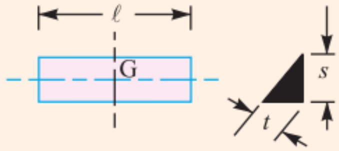
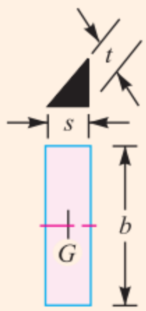
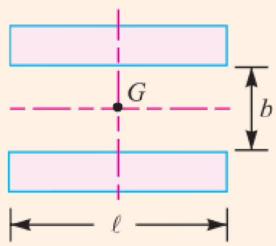
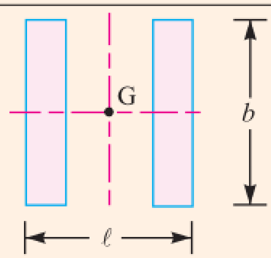
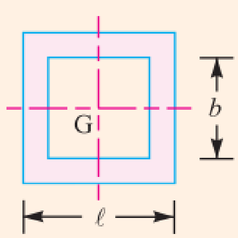
l = Length of weld, and

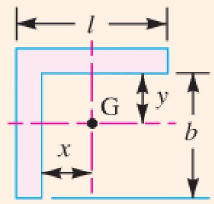
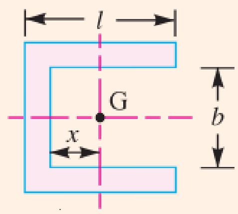
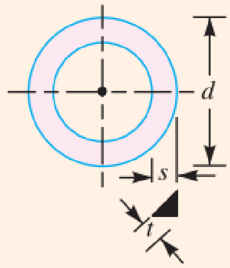
x = Perpendicular distance between the two parallel axes.

10.24 Polar Moment of Inertia and Section Modulus of Welds

The following table shows the values of polar moment of inertia of the throat area about the centre of gravity *'G'* and section modulus for some important types of welds which may be used for eccentric loading.

Table 10.7. Polar moment of inertia and section modulus of welds.

S.No	*Type of weld*	*Polar moment of inertia (J)*	*Section modulus (Z)*
1.		$\frac{t.l^3}{12}$	—
2.		$\frac{t.b^3}{12}$	$\frac{t.b^2}{6}$
3.		$\frac{t.l(3b^2 + l^2)}{6}$	$t.b.l$
4.		$\frac{t.b(b^2 + 3l^2)}{6}$	$\frac{t.b^2}{3}$
5.		$\frac{t(b + l)^3}{6}$	$t\left(b.l + \frac{b^2}{3}\right)$

S.No	Type of weld	Polar moment of inertia (J)	Section modulus (Z)
6.	 $x = \frac{l^2}{2\,(l+b)}, \; y = \frac{b^2}{2\,(l+b)}$	$t\left[\frac{(b+l)^4 - 6b^2l^2}{12\,(l+b)}\right]$	$t\left(\frac{4l.b + b^2}{6}\right)$ (Top) $t\left[\frac{b^2\,(4lb+b)}{6\,(2l+b)}\right]$ (Bottom)
7.	 $x = \frac{l^2}{2l+b}$	$t\left[\frac{(b+2l)^3}{12} - \frac{l^2\,(b+l)^2}{b+2l}\right]$	$t\left(l.b + \frac{b^2}{6}\right)$
8.		$\frac{\pi t d^3}{4}$	$\frac{\pi t d^2}{4}$

Note: In the above expressions, t is the throat thickness and s is the size of weld. It has already been discussed that $t = 0.707\,s$.

Example 10.9. *A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa.*

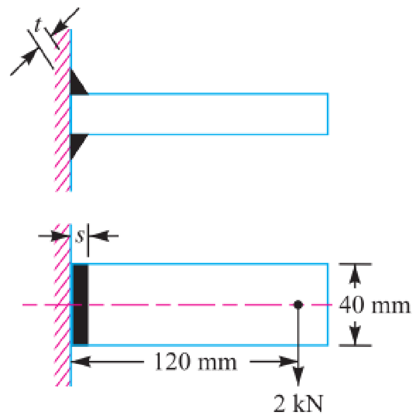


Fig. 10.24

Solution. Given: P = 2kN = 2000 N ; e = 120 mm ; l = 40 mm ; τ_{max} = 25 MPa = 25 N/mm^2

Let s = Size of weld in mm, and

t = Throat thickness.

The joint, as shown in Fig. 10.24, will be subjected to direct shear stress due to the shear force, P = 2000 N and bending stress due to the bending moment of $P \times e$.

We know that area at the throat,

$$A = 2t \times l = 2 \times 0.707\, s \times l$$
$$= 1.414\, s \times l$$
$$= 1.414\, s \times 40 = 56.56 \times s \text{ mm}^2$$

$\therefore$ Shear stress, $$\tau = \frac{P}{A} = \frac{2000}{56.56 \times s} = \frac{35.4}{s} \text{ N/mm}^2$$

Bending moment, $M = P \times e = 2000 \times 120 = 240 \times 10^3$ N-mm

Section modulus of the weld through the throat,

$$Z = \frac{s \times l^2}{4.242} = \frac{s\,(40)^2}{4.242} = 377 \times s \text{ mm}^3$$

$\therefore$ Bending stress, $$\sigma_b = \frac{M}{Z} = \frac{240 \times 10^3}{377 \times s} = \frac{636.6}{s} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

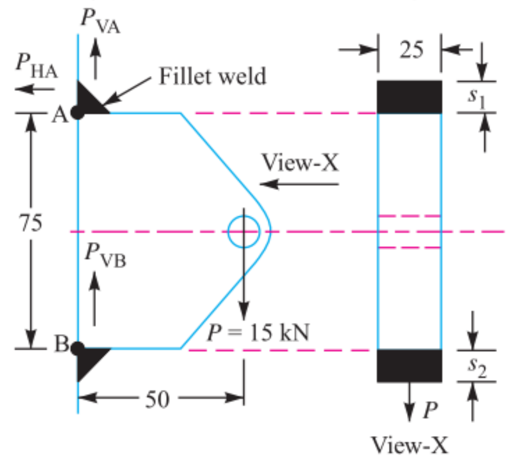
$$25 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4\left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$\therefore$ $s = 320.3 / 25 = 12.8$ mm **Ans.**

Example 10.16. *The bracket, as shown in Fig. 10.34, is designed to carry a dead weight of P = 15 kN.*

What sizes of the fillet welds are required at the top and bottom of the bracket? Assume the forces act through the points A and B. The welds are produced by shielded arc welding process with a permissible strength of 150 MPa.

Solution. Given : $P = 15$ kN ; $\tau = 150$ MPa $= 150$ N/mm^2 ; $l = 25$ mm



All dimensions in mm.

In the joint, as shown in Fig. 10.34, the weld at A is subjected to a vertical force P_{VA} and a horizontal force P_{HA}, whereas the weld at B is subjected only to a vertical force P_{VB}. We know that

$$P_{VA} + P_{VB} = P \quad \text{and} \quad P_{VA} = P_{VB}$$

∴ Vertical force at A and B,

$$P_{VA} = P_{VB} = P / 2 = 15 / 2 = 7.5 \text{ kN} = 7500 \text{ N}$$

The horizontal force at A may be obtained by taking moments about point B.

$$\therefore \quad P_{HA} \times 75 = 15 \times 50 = 750$$

or

$$P_{HA} = 750 / 75 = 10 \text{ kN}$$

Size of the fillet weld at the top of the bracket

Let s_1 = Size of the fillet weld at the top of the bracket in mm.

We know that the resultant force at A,

$$P_A = \sqrt{(P_{VA})^2 + (P_{HA})^2} = \sqrt{(7.5)^2 + (10)^2} = 12.5 \text{ kN} = 12\,500 \text{ N} \quad \text{...}(i)$$

We also know that the resultant force at A,

$$P_A = \text{Throat area} \times \text{Permissible stress}$$

$$= 0.707\, s_1 \times l \times \tau = 0.707\, s_1 \times 25 \times 150 = 2650\, s_1 \quad \text{...}(ii)$$

From equations ***(i)*** and ***(ii)***, we get

$$s_1 = 12\,500 / 2650 = 4.7 \text{ mm}$$ **Ans.**

Size of fillet weld at the bottom of the bracket

Let s_2 = Size of the fillet weld at the bottom of the bracket.

The fillet weld at the bottom of the bracket is designed for the vertical force (P_{VB}) only. We know that

$$P_{VB} = 0.707\, s_2 \times l \times \tau$$

$$7500 = 0.707\, s_2 \times 25 \times 150 = 2650\, s_2$$

$$\therefore \quad s_2 = 7500 / 2650 = 2.83 \text{ mm}$$ **Ans.**