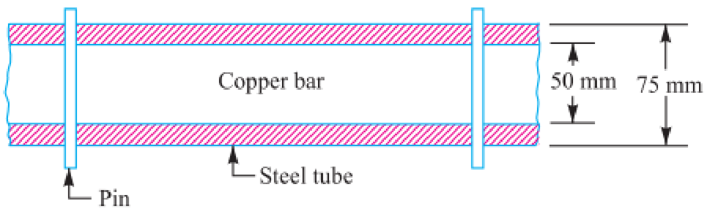
Example 4.15. *A copper bar 50 mm in diameter is placed within a steel tube 75 mm external diameter and 50 mm internal diameter of exactly the same length. The two pieces are rigidly fixed together by two pins 18 mm in diameter, one at each end passing through the bar and tube. Calculate the stress induced in the copper bar, steel tube and pins if the temperature of the combination is raised by 50°C. Take* $E_s = 210$ *GN/m²* ; $E_c = 105$ *GN/m²* ; $\alpha_s = 11.5 \times 10^{-6}/°C$ *and* $\alpha_c = 17 \times 10^{-6}/°C$.

Solution. Given: d_c = 50 mm ; d_{se} = 75 mm ; d_{si} = 50 mm ; d_p = 18 mm = 0.018 m ; t = 50°C; E_s = 210 GN/m² = 210 × 10⁹ N/m² ; E_c = 105 GN/m² = 105 × 10⁹ N/m² ; $\alpha_s = 11.5 \times 10^{-6}/°C$; $\alpha_c = 17 \times 10^{-6}/°C$

The copper bar in a steel tube is shown in Fig. 4.18.



We know that cross-sectional area of the copper bar,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(50)^2 = 1964 \text{ mm}^2 = 1964 \times 10^{-6} \text{ m}^2$$

and cross-sectional area of the steel tube,

$$A_s = \frac{\pi}{4}\left[(d_{se})^2 - (d_{si})^2\right] = \frac{\pi}{4}\left[(75)^2 - (50)^2\right] = 2455 \text{ mm}^2$$

$$= 2455 \times 10^{-6} \text{ m}^2$$

Let l = Length of the copper bar and steel tube.

We know that free expansion of copper bar

$$= \alpha_c \, . \, l \, . \, t = 17 \times 10^{-6} \times l \times 50 = 850 \times 10^{-6}\, l$$

and free expansion of steel tube

$$= \alpha_s \, . \, l \, . \, t = 11.5 \times 10^{-6} \times l \times 50 = 575 \times 10^{-6}\, l$$

∴ Difference in free expansion

$$= 850 \times 10^{-6}\, l - 575 \times 10^{-6}\, l = 275 \times 10^{-6}\, l \quad ...(i)$$

Since the free expansion of the copper bar is more than the free expansion of the steel tube, therefore the copper bar is subjected to a *compressive stress, while the steel tube is subjected to a tensile stress.

Let a compressive force P newton on the copper bar opposes the extra expansion of the copper bar and an equal tensile force P on the steel tube pulls the steel tube so that the net effect of reduction in length of copper bar and the increase in length of steel tube equalises the difference in free expansion of the two.

∴ Reduction in length of copper bar due to force P

$$= \frac{P.l}{A_c.E_c}$$

$$= \frac{P.l}{1964 \times 10^{-6} \times 105 \times 10^{9}} = \frac{P.l}{206.22 \times 10^{6}} \text{ m}$$

and increase in length of steel bar due to force P

$$= \frac{P.l}{A_s.E_s} = \frac{P.l}{2455 \times 10^{-6} \times 210 \times 10^{9}} = \frac{P.l}{515.55 \times 10^{6}} \text{ m}$$

$$\therefore \text{Net effect in length} = \frac{P.l}{206.22 \times 10^{6}} + \frac{P.l}{515.55 \times 10^{6}}$$

$$= 4.85 \times 10^{-9}\, P.l + 1.94 \times 10^{-9}\, P.l = 6.79 \times 10^{-9}\, P.l$$

Equating this net effect in length to the difference in free expansion, we have

$$6.79 \times 10^{-9}\, P.l = 275 \times 10^{-6}\, l \quad \text{or} \quad P = 40\,500 \text{ N}$$

Stress induced in the copper bar, steel tube and pins

We know that stress induced in the copper bar,

$$\sigma_c = P / A_c = 40\,500 / (1964 \times 10^{-6}) = 20.62 \times 10^{6} \text{ N/m}^2 = 20.62 \text{ MPa}$$ **Ans.**

Stress induced in the steel tube,

$$\sigma_s = P / A_s = 40\,500 / (2455 \times 10^{-6}) = 16.5 \times 10^{6} \text{ N / m}^2 = 16.5 \text{ MPa}$$ **Ans.**

* In other words, we can also say that since the coefficient of thermal expansion for copper (α_c) is more than the coefficient of thermal expansion for steel (α_s), therefore the copper bar will be subjected to compressive stress and the steel tube will be subjected to tensile stress.

and shear stress induced in the pins,

$$\tau_p = \frac{P}{2\, A_p} = \frac{40\,500}{2 \times \frac{\pi}{4} (0.018)^2} = 79.57 \times 10^{6} \text{ N/m}^2 = 79.57 \text{ MPa}$$ **Ans.**

...(∵ The pin is in double shear)

4.17 Linear and Lateral Strain

Consider a circular bar of diameter d and length l, subjected to a tensile force P as shown in Fig. 4.19 (a).

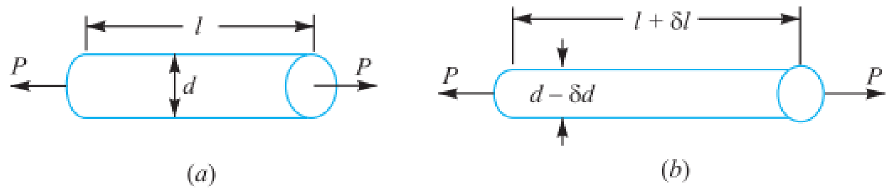


Fig. 4.19. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in Fig. 4.19 (b). Similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

4.18 Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

4.19 Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

where δV = Change in volume, and V = Original volume.

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains in the directions x-axis, y-axis and z-axis respectively.

4.20 Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by K. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

4.21 Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = \frac{m.E}{3\,(m-2)} = \frac{E}{3\,(1-2\mu)}$$

4.22 Relation Between Young's Modulus and Modulus of Rigidity

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m.E}{2\,(m+1)} = \frac{E}{2\,(1+\mu)}$$

Example 4.16. *A mild steel rod supports a tensile load of 50 kN. If the stress in the rod is limited to 100 MPa, find the size of the rod when the cross-section is 1. circular, 2. square, and 3. rectangular with width = 3 × thickness.*

Solution. Given : $P = 50$ kN $= 50 \times 10^3$ N ; $\sigma_t = 100$ MPa $= 100$ N/mm^2

1. ***Size of the rod when it is circular***

Let d = Diameter of the rod in mm.

$\therefore$ Area, $$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times 0.7854\, d^2 = 78.54\, d^2$$

$\therefore$ $$d^2 = 50 \times 10^3 / 78.54 = 636.6 \quad \text{or} \quad d = 25.23 \text{ mm}$$ **Ans.**

Example 4.17. *A steel bar 2.4 m long and 30 mm square is elongated by a load of 500 kN. If poisson's ratio is 0.25, find the increase in volume. Take $E = 0.2 \times 10^6$ N/mm^2.*

Solution. Given : $l = 2.4$ m $= 2400$ mm ; $A = 30 \times 30 = 900$ mm^2 ; $P = 500$ kN $= 500 \times 10^3$ N ; $1/m = 0.25$; $E = 0.2 \times 10^6$ N/mm^2

Let δV = Increase in volume.

We know that volume of the rod,

$$V = \text{Area} \times \text{length} = 900 \times 2400 = 2160 \times 10^3 \text{ mm}^3$$

and Young's modulus, $$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\varepsilon}$$

$\therefore$ $$\varepsilon = \frac{P}{A.E} = \frac{500 \times 10^3}{900 \times 0.2 \times 10^6} = 2.8 \times 10^{-3}$$

We know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right) = 2.8 \times 10^{-3}\ (1 - 2 \times 0.25) = 1.4 \times 10^{3}$$

$\therefore$ $$\delta V = V \times 1.4 \times 10^{-3} = 2160 \times 10^3 \times 1.4 \times 10^{-3} = 3024 \text{ mm}^3$$ **Ans.**

4.23 Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as ***impact stress.***

Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig. 4.20.

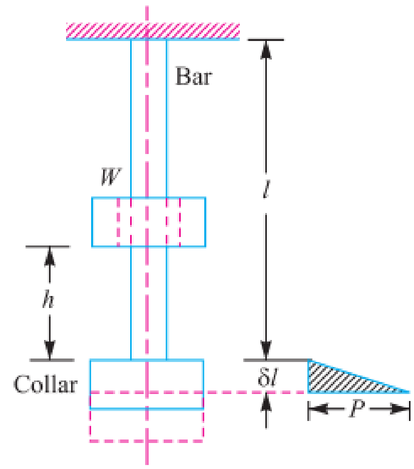


Fig. 4.20. Impact stress.

Let A = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

nd potential energy lost by the weight

$$= W(h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W\ (h + \delta l)$$

$$\frac{1}{2}\ \sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i \times l}{E}\right) \qquad \ldots\left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E}\right]$$

$$\therefore \qquad \frac{Al}{2E}\ (\sigma_i)^2 - \frac{Wl}{E}\ (\sigma_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2\,h\,A\,E}{W\,l}}\right) \qquad \ldots \text{[Taking +ve sign for maximum value]}$$

Note : When $h = 0$, then $\sigma_i = 2W/A$. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

Example 4.18. *An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm² in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm².*

Solution. Given : h = 10 mm ; l = 3 m = 3000 mm ; A = 600 mm² ; δl = 2 mm ; E = 200 kN/mm² = 200 × 10³ N/mm²

Stress in the bar

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

$$\therefore \qquad \sigma = \frac{E . \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3 \text{ N/mm}^2$$ **Ans.**

Value of the unknown weight

Let W = Value of the unknown weight.

We know that
$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3\,W} = 1 + \sqrt{1 + \frac{800\,000}{W}}$$

$$\frac{80\,000}{W} - 1 = \sqrt{1 + \frac{800\,000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160\,000}{W} = 1 + \frac{800\,000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \quad \text{or} \quad \frac{6400 \times 10^2}{W} = 96$$

$\therefore$ $W = 6400 \times 10^2 / 96 = 6666.7$ N **Ans.**

4.24 Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as ***strain energy.*** The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as ***resilience*** and the maximum energy which can be stored in a body up to the elastic limit is called ***proof resilience***. The proof resilience per unit volume of a material is known as ***modulus of resilience.*** It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy stored in a body due to tensile or compressive load or resilience,

$$U = \frac{\sigma^2 \times V}{2E}$$

and
$$\text{Modulus of resilience} = \frac{\sigma^2}{2E}$$

where σ = Tensile or compressive stress,

V = Volume of the body, and

E = Young's modulus of the material of the body.

Notes : 1. When a body is subjected to a shear load, then modulus of resilience (shear)

$$= \frac{\tau^2}{2C}$$

where τ = Shear stress, and

C = Modulus of rigidity.

2. When the body is subjected to torsion, then modulus of resilience

$$= \frac{\tau^2}{4C}$$

Example 4.19. *A wrought iron bar 50 mm in diameter and 2.5 m long transmits a shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m².*

Solution. Given : $d = 50$ mm ; $l = 2.5$ m $= 2500$ mm ; $U = 100$ N-m $= 100 \times 10^3$ N-mm ; $E = 200$ GN/m^2 $= 200 \times 10^3$ N/mm^2

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} (50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\ \sigma^2$$

$$\therefore \quad \sigma^2 = 100 \times 10^3 / 12.25 = 8163 \quad \text{or} \quad \sigma = 90.3 \text{ N/mm}^2$$ **Ans.**

Elongation produced

Let δl = Elongation produced.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\delta l / l}$$

$$\therefore \quad \delta l = \frac{\sigma \times l}{E} = \frac{90.3 \times 2500}{200 \times 10^3} = 1.13 \text{ mm}$$ **Ans.**