Compression Members

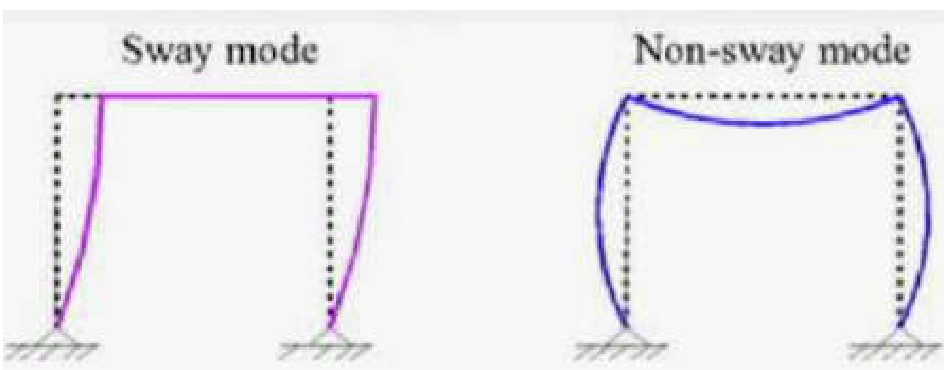
Design of the compression members (Column)

There are two methods for design the column: -

1. Direct Analysis Method (DM)
2. Effective Length Method (ELM) ← **This method will used**

Some suggested K factors are presented in Table 5.1 for columns which prevented the sidesway by bracing.

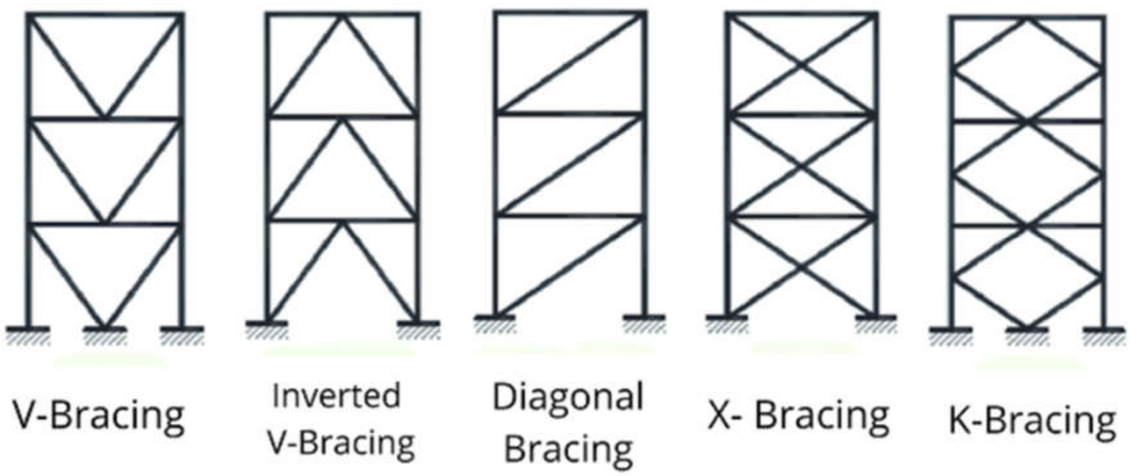
Sidesway refers to a type of buckling.



When the sidesway happen: -

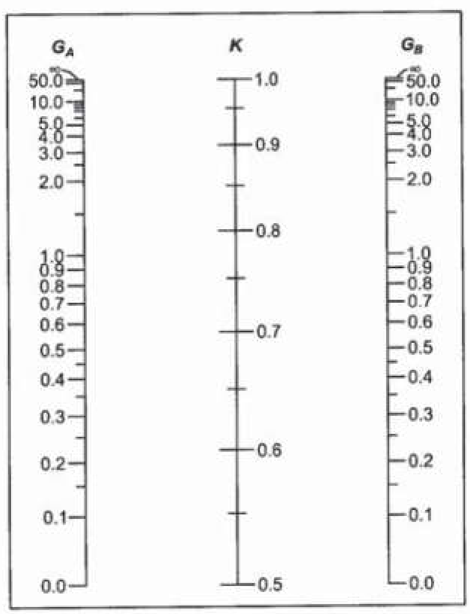
1. The frame defect laterally due to the presence of lateral loads.
2. Unsymmetrical vertical loads.
3. The frames are unsymmetrical.
4. Columns have ends can move transversely when they are loaded to the point that buckling occurs.

To prevent the sidesway in the column, some rotational restraint at their ends would be added.

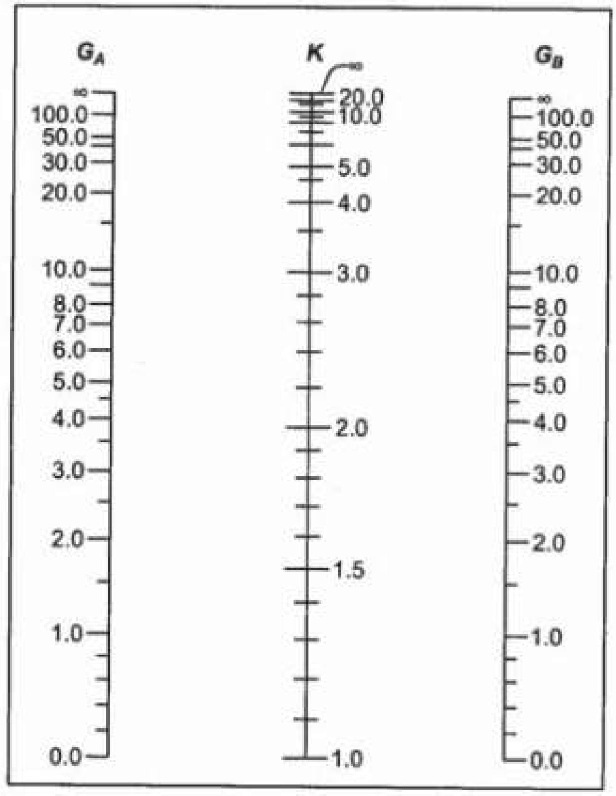


The AISC Specification states that K=1.0 should be used for columns in frames with sidesway inhibited, unless an analysis shows that a smaller value can be used.

The most common method for obtaining effective lengths by using the Jackson and Moreland charts.



a) Sidesway inhibited (Braced Frame)



b) Sidesway uninhibited (Moment Frame)

For pinned columns, the G value is equal to infinite, but it is recommended to be equal to 10 where nonrigid supports are used.

For rigid connections of column, the G value is equal to zero, but it is recommended to be equal to 1.0

$$G = \frac{\sum\left(\frac{E_c I_c}{L_c}\right)}{\sum\left(\frac{E_g I_g}{L_g}\right)} \quad \text{AISC Equation (C-A-7-2)}$$

E :- is the elastic modulus

I:- is the moment of inertia

L:- is the unsupported length.

C:- column

g:- girder

For pinned columns, the G value is equal to infinite, but it is recommended to be equal to 10 where nonrigid supports are used.

For rigid connections of column, the G value is equal to zero, but it is recommended to be equal to 1.0

To determine of K factors for the columns of a steel frame: -

1. Select the appropriate chart (sidesway inhibited of sidesway or uninhibited).
2. Compute G at each end of the column and label the values G_A and G_B.
3. Draw a straight line on the chart between the G_A and G_B values and read K where the line hits the center K scale.

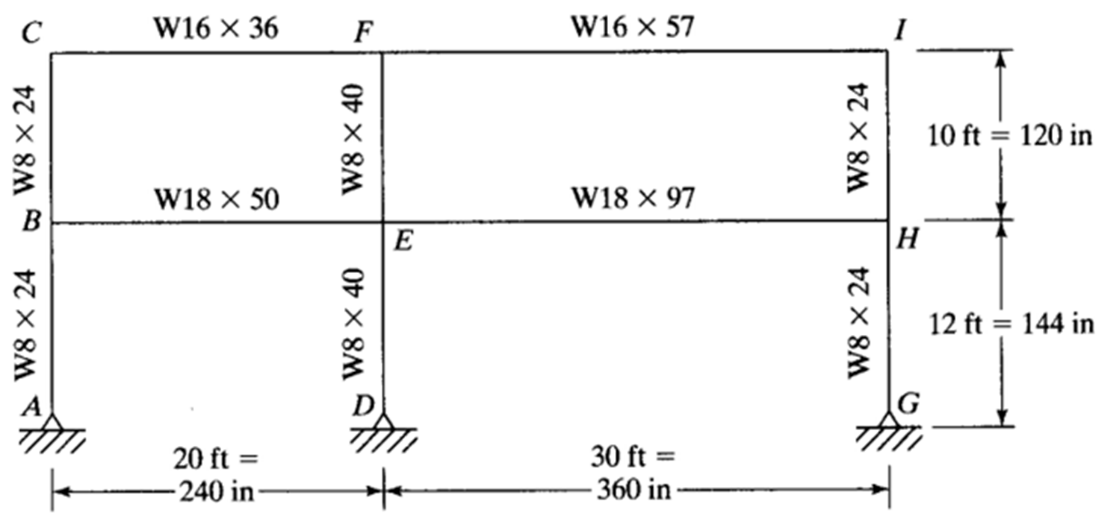
Frame meeting alignment chart assumptions: -

Members should have these criteria to use the charts

1. The members are elastic, have constant cross sections, and are connected with rigid joints.
2. All columns buckle simultaneously.
3. For braced frames, the rotations at opposite ends of each beam are equal in magnitude, and each beam bends in single curvature.
4. For unbraced frames, the rotations at opposite ends of each beam are equal in magnitude, but each beam bends in double curvature.
5. Axial compression forces in the girders are negligible.

Example:

Determine the effective length factor for each of the columns of the frame shown in the figure. If the frame is not braced against sidesway. Use the alignment charts.



Solution. Stiffness factors: E is assumed to be 29,000 ksi for all members and is therefore neglected in the equation to calculate G.

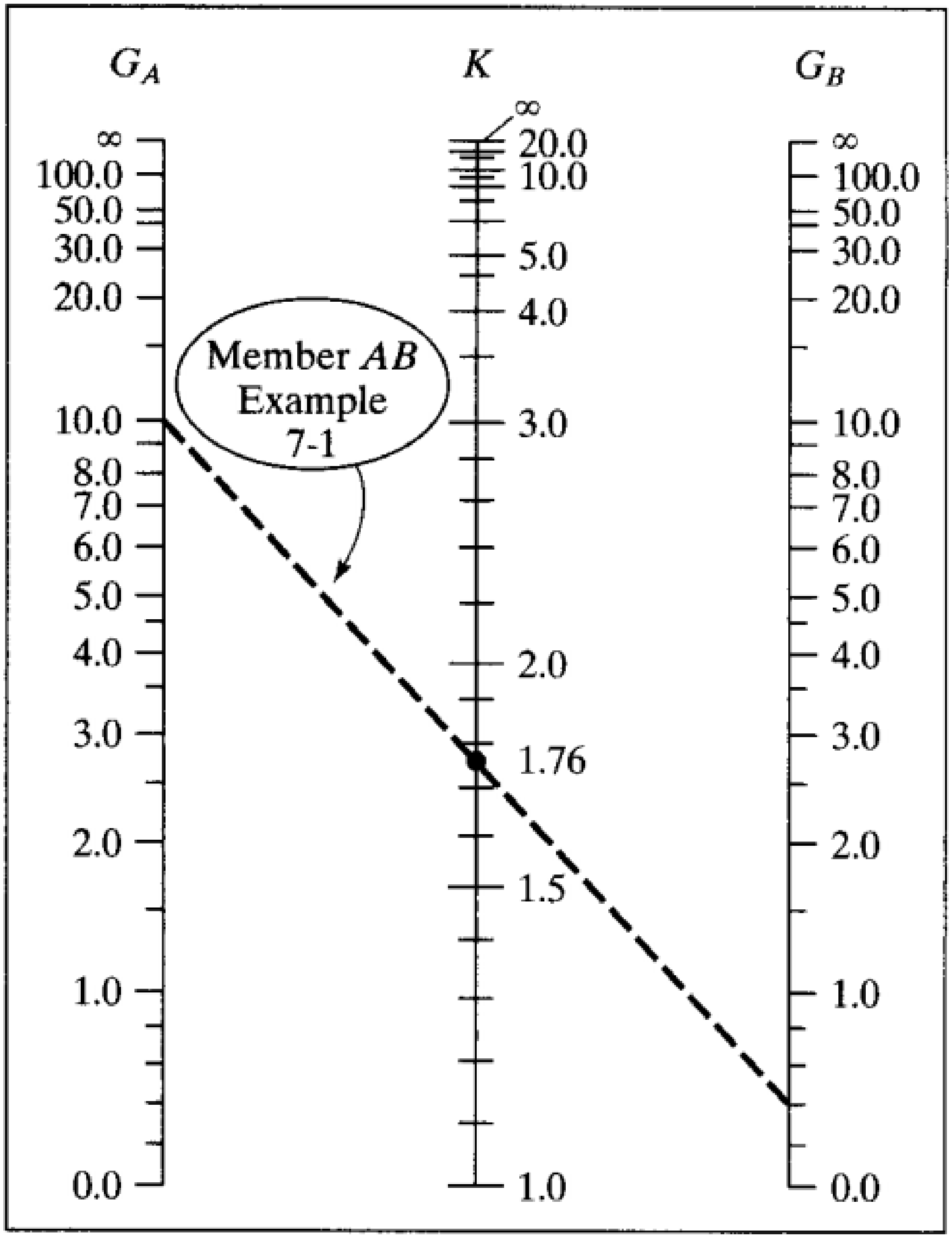
	Member	Shape	I	L	I/L
Columns	*AB*	W8 × 24	82.7	144	0.574
	BC	W8 × 24	82.7	120	0.689
	DE	W8 × 40	146	144	1.014
	EF	W8 × 40	146	120	1.217
	GH	W8 × 24	82.7	144	0.574
	HI	W8 × 24	82.7	120	0.689
Girders	*BE*	W18 × 50	800	240	3.333
	CF	W16 × 36	448	240	1.867
	EH	W18 × 97	1750	360	4.861
	FI	W16 × 57	758	360	2.106

G factors for each joint:

Joint	$\Sigma(I_c/L_c)/\Sigma(I_g/L_g)$	G
A	Pinned Column, $G = 10$	10.0
B	$\frac{0.574 + 0.689}{3.333}$	0.379
C	$\frac{0.689}{1.867}$	0.369
D	Pinned Column, $G = 10$	10.0
E	$\frac{1.014 + 1.217}{(3.333 + 4.861)}$	0.272
F	$\frac{1.217}{(1.867 + 2.106)}$	0.306
G	Pinned Column, $G = 10$	10.0
H	$\frac{0.574 + 0.689}{4.861}$	0.260
I	$\frac{0.689}{2.106}$	0.327

Column K factors from chart [Fig. 7.2(b)]:

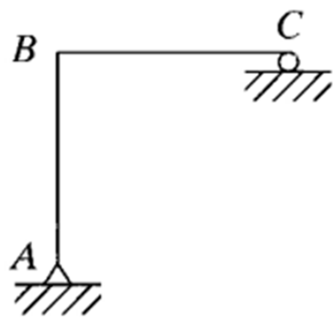
Column	G_A	G_B	K*
AB	10.0	0.379	1.76
BC	0.379	0.369	1.12
DE	10.0	0.272	1.74
EF	0.272	0.306	1.10
GH	10.0	0.260	1.73
HI	0.260	0.327	1.10



(b) Sidesway uninhibited (Moment Frame)

For most buildings, the values of Kx and Ky should be examined separately, because of different possible framing conditions.

FRAMES NOT MEETING ALIGNMENT CHART ASSUMPTIONS AS TO JOINT ROTATIONS



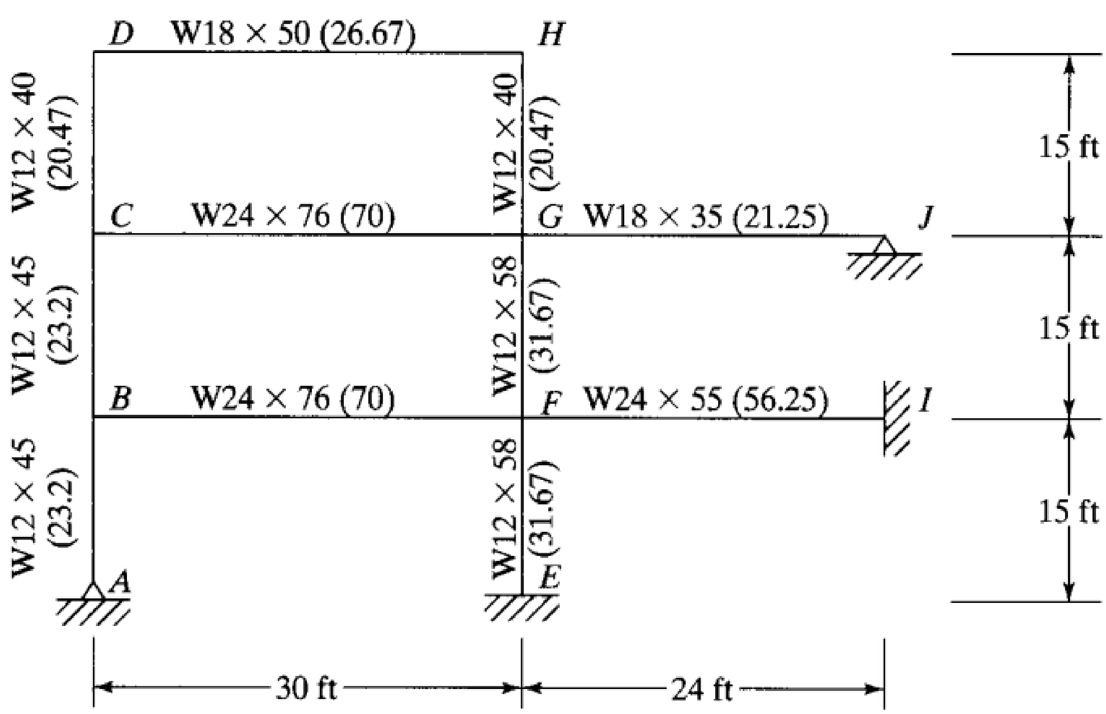
Frames not Meeting Alignment Chart Assumptions as to Joint Rotations

TABLE 7.1 Multipliers for Rigidly Attached Members

Condition at Far End of Girder	Sidesway Prevented, Multiply by:	Sidesway Uninhibited, Multiply by:
Pinned	1.5	0.5
Fixed against rotation	2.0	0.67

Example:

Determine K factors for each of the columns of the frame shown in the fig. Here, W sections have been tentatively selected for each of the members of the frame and their I/L values determined and shown in the figure.



Solution. First, the G factors are computed for each joint in the frame. In this calculation, the I/L values for members FI and GJ are multiplied by the appropriate factors from Table 7.1.

1. For member *FI*, the *I*/*L* value is multiplied by 2.0, because its far end is fixed and there is no sidesway on that level.
2. For member, *GJ*, *I*/*L* is multiplied by 1.5, because its far end is pinned and there is no sidesway on that level.

$$G_A = 10 \text{ as described in Section 7.2, Pinned Column}$$

$$G_B = \frac{23.2 + 23.2}{70} = 0.663$$

$$G_C = \frac{23.2 + 20.47}{70} = 0.624$$

$$G_D = \frac{20.47}{26.67} = 0.768$$

$$G_E = 1.0 \text{ as described in Section 7.2, Fixed Column}$$

$$G_F = \frac{31.67 + 31.67}{70 + (2.0)(56.25)} = 0.347$$

$$G_G = \frac{31.67 + 20.47}{70 + (1.5)(21.25)} = 0.512$$

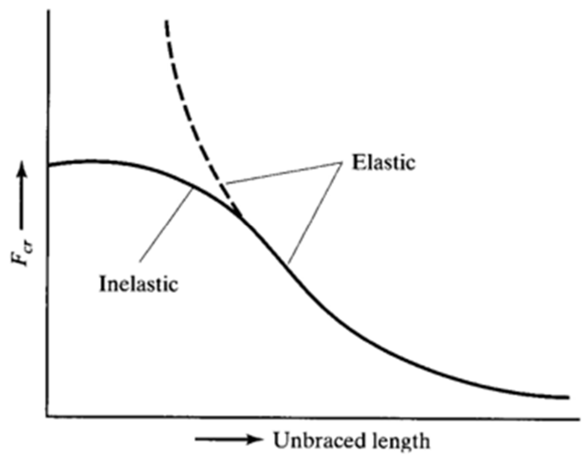
$$G_H = \frac{20.47}{26.67} = 0.768$$

Finally, the *K* factors are selected from the appropriate alignment chart of Fig. 7.2.

Column	*G* Factors	Chart used	*K* Factors
AB	10 and 0.663	7.2 (a) no sidesway	0.83
BC	0.663 and 0.624	7.2 (a) no sidesway	0.72
CD	0.624 and 0.768	7.2 (b) has sidesway	1.23
EF	1.0 and 0.347	7.2 (a) no sidesway	0.71
FG	0.347 and 0.512	7.2 (a) no sidesway	0.67
GH	0.512 and 0.768	7.2 (b) has sidesway	1.21

STIFFNESS-REDUCTION FACTORS

When the real conditions are not idealized, unrealistically high K factors may be obtained from the charts and overconservative designs may result.



If the column behavior is inelastic, the G factor used to enter the alignment chart will be smaller, and the K factor selected from the chart will be smaller.

Though the alignment charts were developed for elastic column action, they may be used for an inelastic column situation if the G value is multiplied by a correction factor, τ_b.

TABLE 7.2 Stiffness Reduction Factor, τ_b

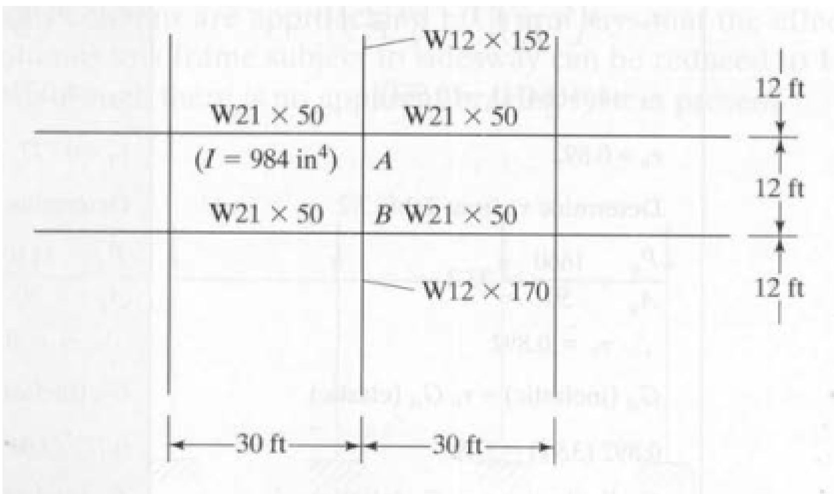
ASD $\frac{P_a}{A_g}$ / LRFD $\frac{P_u}{A_g}$	F_y, ksi: 35 ASD	35 LRFD	36 ASD	36 LRFD	42 ASD	42 LRFD	46 ASD	46 LRFD	50 ASD	50 LRFD
45	–	–	–	–	–	–	–	0.0851	–	0.360
44	–	–	–	–	–	–	–	0.166	–	0.422
43	–	–	–	–	–	–	–	0.244	–	0.482
42	–	–	–	–	–	–	–	0.318	–	0.538
41	–	–	–	–	–	0.0930	–	0.388	–	0.590
40	–	–	–	–	–	0.181	–	0.454	–	0.640
39	–	–	–	–	–	0.265	–	0.516	–	0.686
38	–	–	–	–	–	0.345	–	0.575	–	0.730
37	–	–	–	–	–	0.420	–	0.629	–	0.770
36	–	–	–	–	–	0.490	–	0.681	–	0.806
35	–	–	–	0.108	–	0.556	–	0.728	–	0.840
34	–	0.111	–	0.210	–	0.617	–	0.771	–	0.870
33	–	0.216	–	0.306	–	0.673	–	0.811	–	0.898
32	–	0.313	–	0.395	–	0.726	–	0.847	–	0.922
31	–	0.405	–	0.478	–	0.773	–	0.879	0.0317	0.942
30	–	0.490	–	0.556	–	0.816	–	0.907	0.154	0.960
29	–	0.568	–	0.627	–	0.855	–	0.932	0.267	0.974
28	–	0.640	–	0.691	–	0.889	0.102	0.953	0.373	0.986
27	–	0.705	–	0.750	–	0.918	0.229	0.970	0.470	0.994
26	–	0.764	–	0.802	0.0377	0.943	0.346	0.983	0.559	0.998
25	–	0.816	–	0.849	0.181	0.964	0.454	0.992	0.640	1.00
24	–	0.862	–	0.889	0.313	0.980	0.552	0.998	0.713	
23	–	0.901	–	0.923	0.434	0.991	0.640	1.00	0.777	
22	–	0.934	0.0869	0.951	0.543	0.998	0.719		0.834	
21	0.154	0.960	0.249	0.972	0.640	1.00	0.788		0.882	
20	0.313	0.980	0.395	0.988	0.726		0.847		0.922	
19	0.457	0.993	0.525	0.997	0.800		0.896		0.953	
18	0.583	0.999	0.640	1.00	0.862		0.936		0.977	
17	0.693	1.00	0.739		0.913		0.967		0.992	
16	0.786		0.822		0.952		0.987		0.999	
15	0.862		0.889		0.980		0.998		1.00	
14	0.922		0.940		0.996		1.00			
13	0.964		0.976		1.00					
12	0.991		0.996							
11	1.00		1.00							
10										
9										
8										
7										
6										
5										

– Indicates the stiffness reduction parameter is not applicable because the required strength exceeds the available strength for $KL/r = 0$.

Example:

(a) Determine the effective length factor for column AB of the unbraced frame shown in Fig. 7.8, assuming that we have elastic behavior and that all of the other assumptions on which the alignment charts were developed are met. $P_D = 450$ k, $P_L = 700$ k, $F_y = 50$ ksi. Assume that column AB is a W12 × 170 and the columns above and below are as indicated on the figure.

(b) Repeat part (a) if inelastic column behavior is considered.



(a) Assuming that the column is in the elastic range.
Using W12 × 170 ($A = 50$ in², $I_x = 1650$ in⁴) for column AB and the column below.
Using W12 × 152 ($A = 44.7$ in², $I_x = 1430$ in⁴) for column above.

$$G_A = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)} = \frac{\frac{1430}{12} + \frac{1650}{12}}{2\left(\frac{984}{30}\right)} = 3.91$$

$$G_B = \frac{\sum(I_c/L_c)}{\sum(I_g/L_g)} = \frac{2\left(\frac{1650}{12}\right)}{2\left(\frac{984}{30}\right)} = 4.19$$

From Fig. 7.2(b) alignment chart

$$K = 2.05$$

(b) Inelastic solution

LRFD

$\alpha = 1.0$

$P_r = P_u = 1660$ k

$P_y = F_y A_g = 50$ ksi $(50 \text{ in}^2) = 2500$ k

$$\alpha \frac{P_r}{P_y} = \frac{1.0(1660)}{2500} = 0.664 > 0.5$$

Use AISC Equation C2-2b

$$\tau_b = 4\left(\alpha \frac{P_r}{P_y}\right)\left[1 - \left(\alpha \frac{P_r}{P_y}\right)\right]$$

$\tau_b = 4(0.664)\,[1 - (0.664)]$

$\tau_b = 0.892$

Determine τ_b from Table 7.2

$$\frac{P_u}{A_g} = \frac{1660}{50} = 33.2$$

$\therefore\ \tau_b = 0.892$

G_A (inelastic) $= \tau_b\, G_A$ (elastic)

$0.892\,(3.91) = 3.49$

G_B (inelastic) $= \tau_b\, G_B$ (elastic)

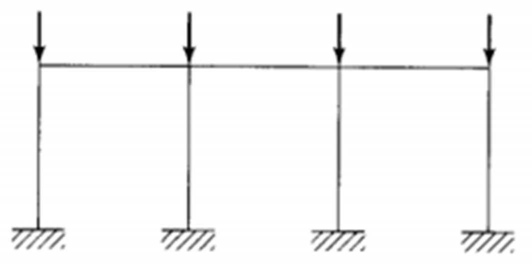
$0.892\,(4.19) = 3.74$

From Fig. 7.2(b) alignment chart

$K = 1.96$

Columns leaning on each other for In-Plane design

A pin-ended column that does not help provide lateral stability to a structure is referred to as a **leaning column**. Such a column depends on the other parts of the structure to provide lateral stability.



If the exterior columns are bracing the interior ones against sidesway, the K factores for those interior columns are approaching 1.0.

Example:

For the frame of Fig. 7.11, which consists of 50 ksi steel, beams are rigidly connected to the exterior columns, while all other connections are simple. The columns are braced top and bottom against sidesway, out of the plane of the frame, so that $K_y = 1.0$ in that direction. Sidesway is possible in the plane of the frame. Using the LRFD method, design the interior columns assuming that $K_x = K_y = 1.0$, and design the exterior columns with K_x as determined from the alignment chart and $P_u = 1100$ k. (With this approach to column buckling, the interior columns could carry no load at all, since they appear to be unstable under sidesway conditions.) The end columns are assumed to have no bending moment at the top of the member.

Solution. *Design of interior columns:*

Assume $K_x = K_y = 1.0$, $KL = (1.0)(15) = 15$ ft, $P_u = 660$ k.

Use W14 × 74; $\phi P_n = 667\text{ k} > P_u = 660\text{ k}$

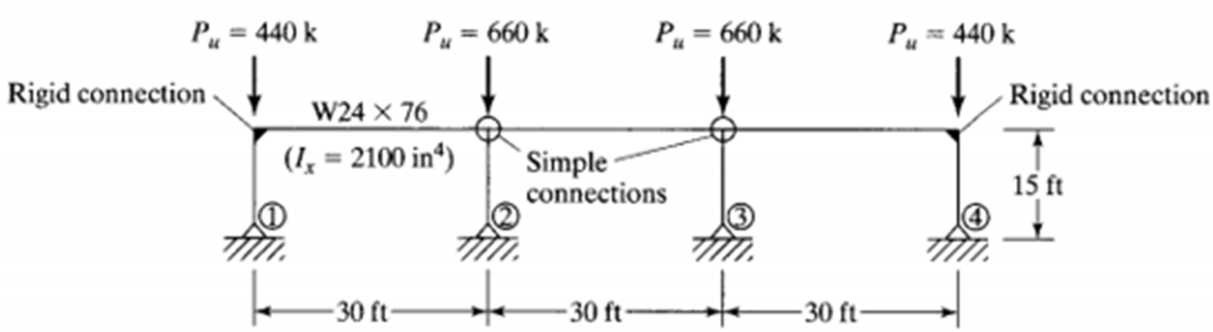
Design of exterior columns:

In plane $P_u = 440 + 660 = 1100$ k, K_x to be determined from alignment chart. Estimating a column size a little larger than would be required for $P_u = 1100$ k. Try W14 × 120 ($A = 35.3\text{ in}^2$, $I_x = 1380\text{ in}^4$, $r_x = 6.24$ in, $r_y = 3.74$ in).

$$G_{top} = \frac{1380/15}{2100/30 \times 0.5} = 2.63$$

(noting that girder stiffness is multiplied by 0.5, since sidesway is permitted and far end of girder is hinged).

$$G_{bottom} = 10$$
$$K_x = 2.22 \text{ from Fig. 7.2(b)}$$



$$\frac{K_x L_x}{r_x} = \frac{(2.22)(12 \times 15)}{6.24} = 64.04$$

$$\phi_c F_{cr} = 33.38 \text{ ksi}$$

$$\phi_c P_n = (33.38)(35.3) = 1178 \text{ k} > P_u = 1100 \text{ k}$$

Out of plane: $K_y = 1.0$, $P_u = 440$ k

$$\frac{K_y L_y}{r_y} = \frac{1.0\,(12 \times 15)}{3.74} = 48.13$$

$$\phi F_{cr} = 37.96 \text{ ksi}$$

$$\phi_c P_n = (37.96)\,(35.3) = 1340 \text{ k} > P_u = 440 \text{ k}$$

Use W14 × 120.