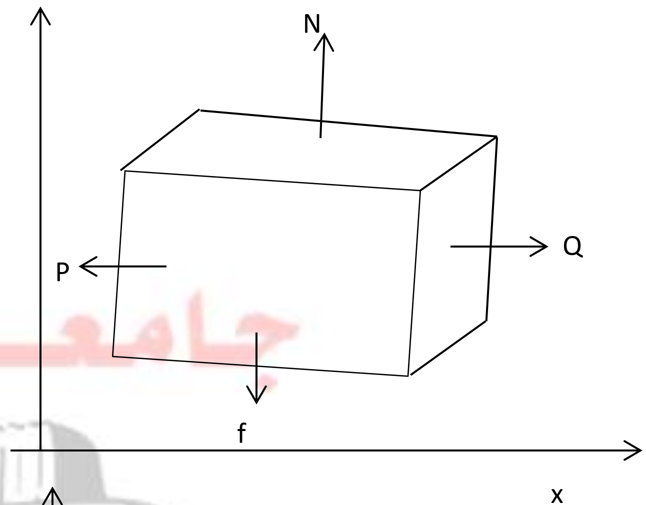
Plane stress analysis :

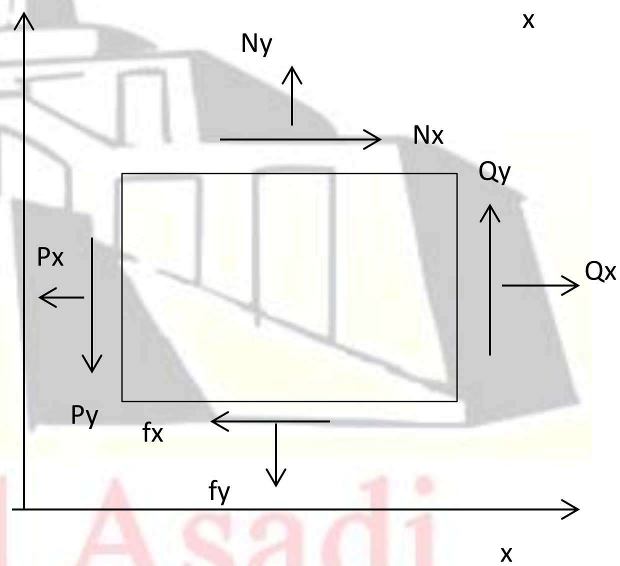
A two –dimensional stress system is one in which the stresses at any point in a body in the same plane



N , Q , P , F lies in the X- y plane

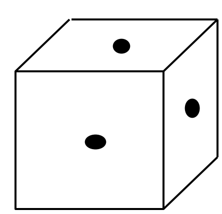
The perpendicular compounds introduce (direct stress σ) , and the tangential components introduce (shearing stresses τ).

Stress at a point :



The stress acting at a point is represented by the stresses acting on the faces of a differential element including the point . The element is usually represented by its front view .

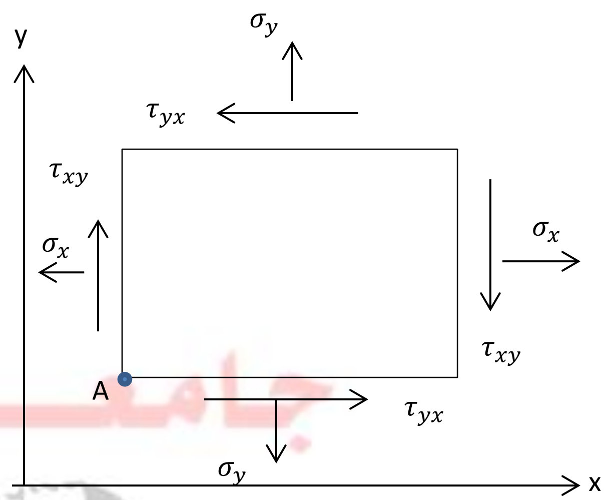
σ_x: normal stress (direct stress) acting on a plane Perpendicular to the X-axis



σ_y: normal stress (direct stress) acting on a plane Perpendicular y-axis

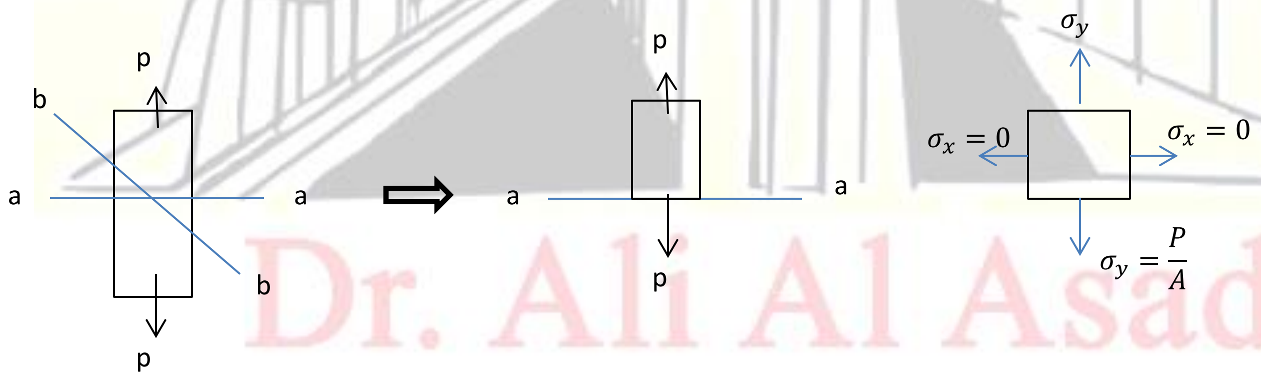
τ_{xy}: shearing stress acting on a plane perpendicular to the x-axis and directed parallel to the y-axis .

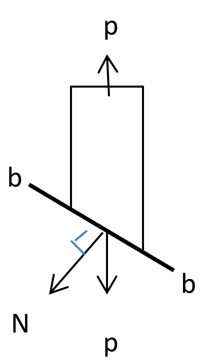
$$\sum MA = 0 : \ \tau_{xy} = \tau_{yx}$$

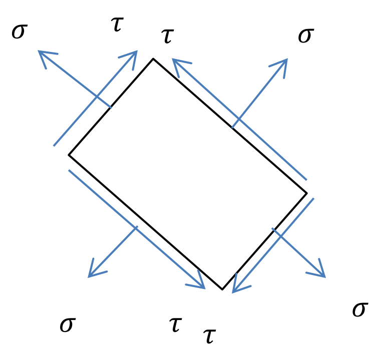


- ***Variation of stress at a point :***

The stress on an element (point) vary with the orientation of the element .







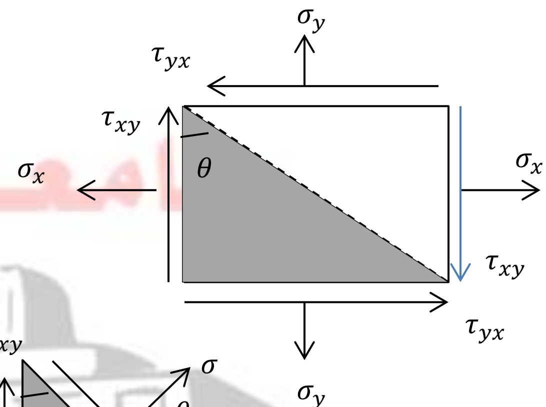
- ***Stress on an inclined plane :***

To find the stresses acting on an inclined plane tow method may be used :

1- Analytical method

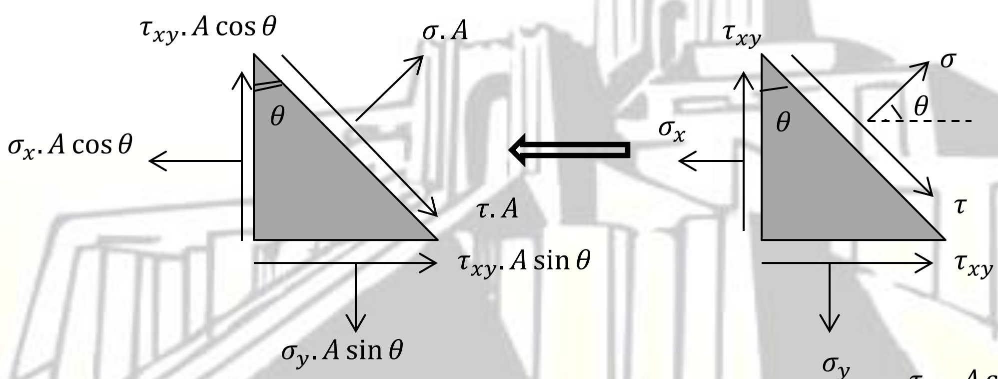
2- Graphical method

Analytical method :

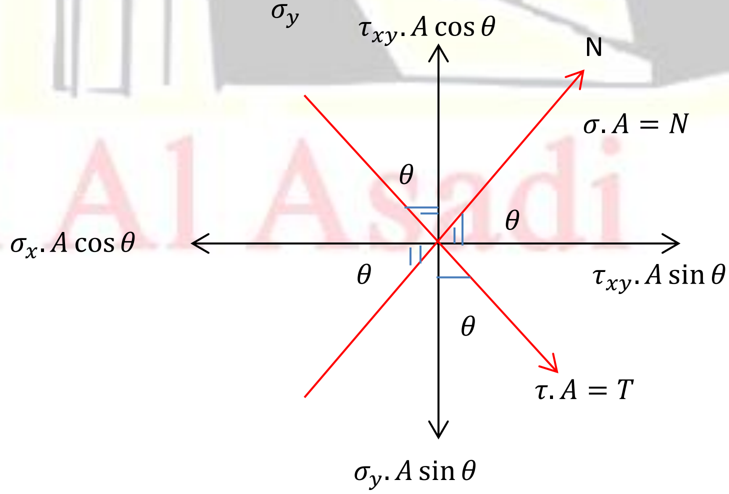


σ : direct stress acting on the inclined plane .

τ : shearing stress acting on the inclined plane .



If A is the area of the inclined plane



$$\therefore \sum FN = 0$$

$$\sigma.A + \tau_{xy}.Acos\theta\ sin\theta + \tau_{yx}Asin\theta cos\theta = \sigma_x.Acos\theta\ cos\theta + \sigma_y Asin\theta\ sin\theta$$

We have :

$$\tau_{xy} = \tau_{yx}$$

Then :

$$\sigma = \sigma_x cos^2\theta + \sigma_y sin^2\theta - 2\tau_{xy} cos\theta sin\theta$$

This equation may be written more convenient

$$\sigma = \sigma_x \left(\frac{1 + cos2\theta}{2}\right) + \sigma_y \left(\frac{1 - cos2\theta}{2}\right) - \tau_{xy} sin2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_x}{2} cos2\theta + \frac{\sigma_y}{2} - \frac{\sigma_y}{2} cos2\theta - \tau_{xy} sin2\theta$$

$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right) cos2\theta - \tau_{xy} sin2\theta$$

$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right) cos2\theta - \tau_{xy} sin2\theta \text{ -------- (1)}$$

$$\sum F_t = 0 : \ \tau.A + \tau_{yx} sin\theta.Asin\theta + \sigma_y.Asin\theta cos\theta$$
$$= \tau_{xy}.Acos\theta cos\theta + \sigma_x.Acos\theta sin\theta$$

$$\tau = (\sigma_x - \sigma_y) sin\theta cos\theta + \tau_{xy}(cos^2\theta - sin^2\theta)$$

$$\therefore \tau = \left(\frac{\sigma_x-\sigma_y}{2}\right) sin2\theta + \tau_{xy} cos2\theta \text{ ------------ (2)}$$

Principal Stresses :

Are the maximum and minimum normal stress the planes defining max. or min, normal stresses are found by differentiating eq. (1) with respect to θ and :

$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right) cos2\theta - \tau_{xy} sin2\theta \text{ ------------- (1)}$$

$$\frac{d\sigma}{d\theta} = 0 + \left(\frac{\sigma_x - \sigma_y}{2}\right)(-2sin2\theta) - 2\tau_{xy}cos2\theta = 0$$

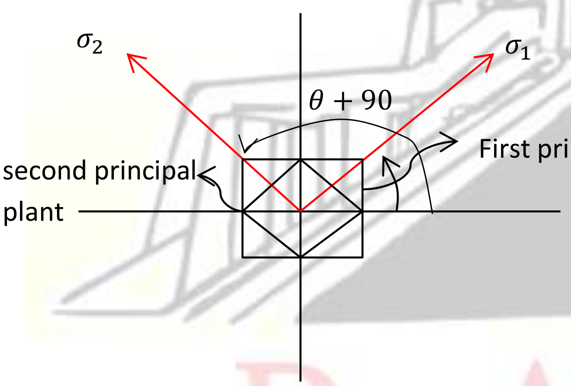
$$\therefore (\sigma_x - \sigma_y)sin2\theta = -2\tau_{xy}cos2\theta$$

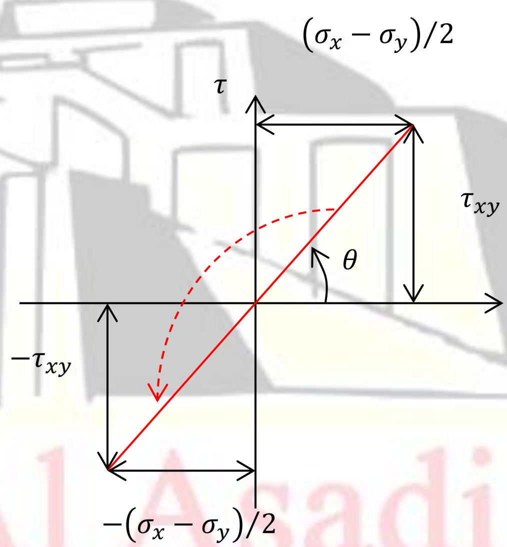
$$\therefore \frac{sin2\theta}{cos2\theta} = \frac{-2\tau_{xy}}{(\sigma_x-\sigma_y)}$$

$$\therefore tan2\theta = \frac{-2\tau_{xy}}{(\sigma_x-\sigma_y)} \rightarrow tan2\theta = -\frac{\tau_{xy}}{(\sigma_x-\sigma_y)/2} \quad \text{------(3)}$$

$$\therefore 2\theta = \tan^{-1}\left(-\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right) + n * 180 \quad \text{Where n = 0,1,2,3,........}$$

$$\text{Or } \theta = \frac{1}{2}\tan^{-1}\left(-\frac{\tau_{xy}}{\sigma_x-\sigma_y}\right) + n * 90$$



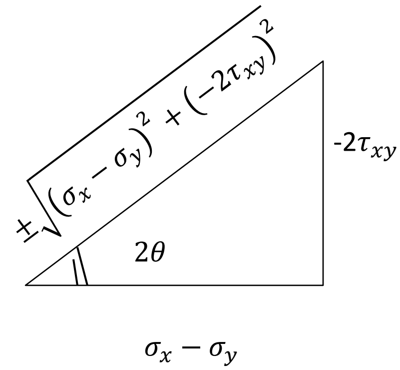


$$\therefore \sigma = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\therefore \sigma = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right) * \frac{(\sigma_x-\sigma_y)}{\pm\sqrt{(\sigma_x-\sigma_y)^2+(2\tau_{xy})^2}} - \tau_{xy}\frac{-2\tau_{xy}}{\pm\sqrt{(\sigma_x-\sigma_y)^2+(2\tau_{xy})^2}}$$

$$\therefore \sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{(\sigma_x-\sigma_y)^2}{\pm2\sqrt{(\sigma_x-\sigma_y)^2+(2\tau_{xy})^2}} + \frac{2\tau_{xy}^2}{\pm\sqrt{(\sigma_x-\sigma_y)^2+(2\tau_{xy})^2}}$$

$$\therefore \sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}{\pm 2\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$



$$\therefore \sigma_{\substack{1,2\\ max\\ min}} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} \text{ -------------- (4)}$$

Where the positive sign front of the radical must be used to obtain σ_1 , and the negative sign to be obtain σ_2.

Maximum shearing stresses :

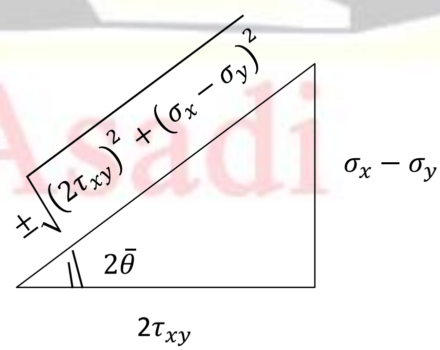
If σ_x , σ_y, τ_{xy} , known for an element , the shear stress on any plane defined by an angle θ is given by :

$$\tau = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \text{ from eqs. (2)}$$

$\therefore$ Eq (2) must be differentiated with respect to θ and the derivative set equal to Zero .

$$\therefore \frac{d\tau}{d\theta} = \left(\frac{\sigma_x-\sigma_y}{2}\right) 2.\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

$$\therefore \frac{\sin 2\theta}{\cos 2\theta} = \frac{\sigma_x-\sigma_y}{2\tau_{xy}} \rightarrow \tan 2\bar{\theta} = \frac{\sigma_x-\sigma_y}{2\tau_{xy}} \text{ ------------(5)}$$



direction of

$$\bar{\theta} = \frac{1}{2}\tan^{-1}\frac{\sigma_x-\sigma_y}{2\tau_{xy}} + n.90 \qquad \{n=0 , 1 , 2 , \}$$

Sub ($\bar{\theta}$) in eq (2)

$$\therefore \tau_{max} = \frac{\sigma_x-\sigma_y}{2} * \frac{\sigma_x-\sigma_y}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} + \frac{2\tau_{xy}^2}{\pm\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$\therefore\ \tau_{max} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$ ------------ (6)

Note : Thus , the max. shearing stress differs from the min. shearing stress only in sign , therefore shearing regardless sign will be called (Max. shear stress) .

Note : when shearing stress equal to zero , then

$$\tau = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta = 0$$

$\therefore \left\{\tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x-\sigma_y}\right\}$ this is same for the angle of max. and min. normal stress

Therefore no shearing stress on planes of max. , min. normal stress .

$\therefore\ \tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x-\sigma_y}$ and $\tan 2\bar{\theta} = \frac{\sigma_x-\sigma_y}{2\tau_{xy}}$

$\therefore\ \tan 2\theta = -\cot 2\bar{\theta}$

$\therefore\ \tan(2\bar{\theta}+90) = -\cot 2\bar{\theta}$

$\tan 2\theta = \tan(2\bar{\theta}+90) \rightarrow 2\theta = 2\bar{\theta}+90$

$\therefore\ \theta = \bar{\theta} + 45$ ---------- (7)

$\therefore$ The max . shearing stress locate with angles is 45^o with the planes of the principle stresses .

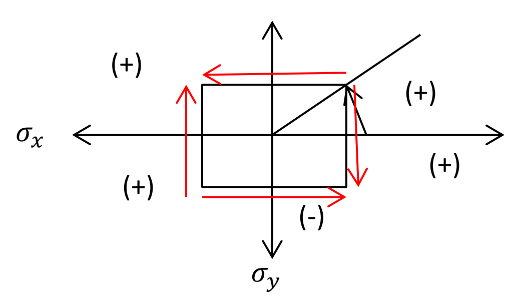
Note : for sign :

1- τ : +ve if its moment about the center of the element is clockwise .

2- θ : +ve anti-clockwise

3- Tension : +ve

Compression –ve



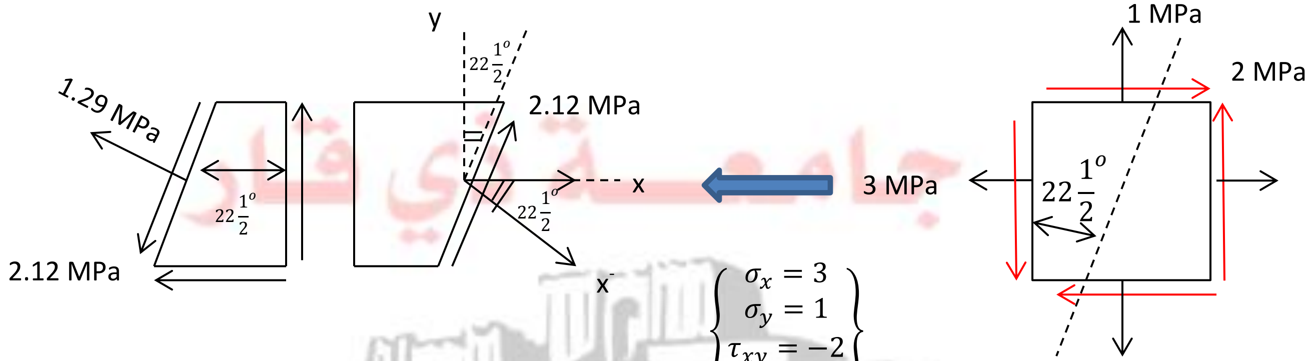
Example 1 : For the state of stress as shown below ,

(a) find normal stresses and shearing stress on the plane at ($-22\frac{1}{2}^{o}$)with y-axis.

(b) find principal stresses

(c) Max. shearing stresses . Show the result on a properly oriented element .

Sol.



$$\begin{Bmatrix} \sigma_x = 3 \\ \sigma_y = 1 \\ \tau_{xy} = -2 \end{Bmatrix}$$

$\bar{\sigma}_x = \frac{3+1}{2} + \frac{3-1}{2}\cos(-45) + 2\sin(-45)$

$\bar{\sigma}_x = 2 + 1(0.707) - 2(0.707) = +1.29$ MPa

$$\tau_{\bar{x}\bar{y}} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{\bar{x}\bar{y}} = +\frac{3-1}{2}\sin(-45) - 2\cos(-45)$$

=1(0.707)+2(0.707)=2.12 MPa

(b) $\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

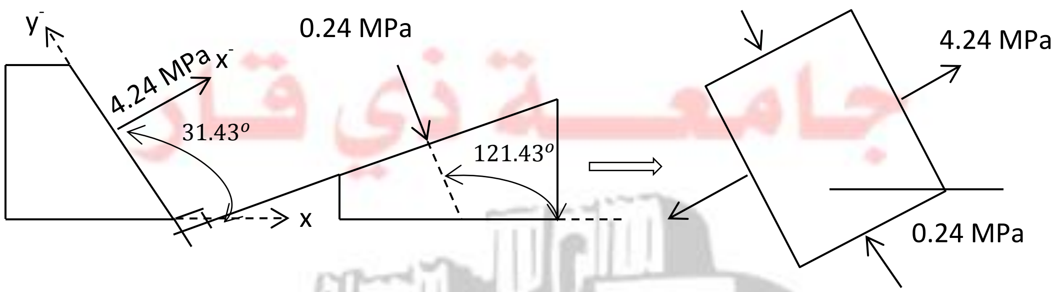
$$\therefore\ \sigma_{1,2} = \frac{3+1}{2} \pm \sqrt{\left(\frac{3-1}{2}\right)^2 + (-2)^2} = 2 \pm 2.24$$

$\therefore\ \sigma_1 = +4.24$ MPa (tension)

$\sigma_2 = -0.24$ MPa (compression)

$$\tan 2\theta = -\frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-(-2)}{(3-1)/2} = 2$$

$$\therefore 2\theta = 63.26^o \rightarrow \theta = 31.43^o \text{ or } 63.26^o + 180^o = 243.26^o$$



For check

$$\bar{\sigma}_x = \frac{3+1}{2} + \frac{3-1}{2}\cos 63.26^{o-} + 2\sin 63.26^o = 4.24 \text{ MPa} \quad \text{(ok)}$$

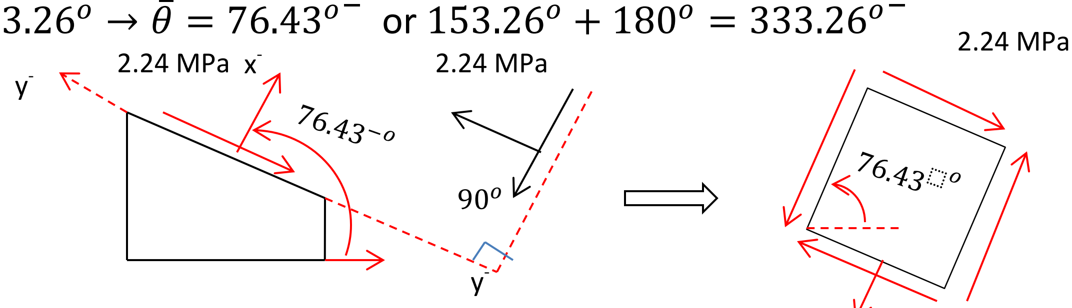
(c) for max shearing stress :

$$\tau_{max} = \sqrt{\left\{(\sigma_x - \sigma_y)/2\right\}^2 + \tau_{xy}^2} = \sqrt{\left((3-1)/2\right)^2 + (2)^2} = \sqrt{5}$$

$$\therefore \tau_{max} = 2.24 \text{ MPa}$$

$$\tan 2\bar{\theta} = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} \rightarrow \tan 2\bar{\theta} = \frac{-(3-1)/2}{2} = -0.5$$

$$\therefore 2\bar{\theta} = 153.26^o \rightarrow \bar{\theta} = 76.43^{o-} \text{ or } 153.26^o + 180^o = 333.26^{o-}$$



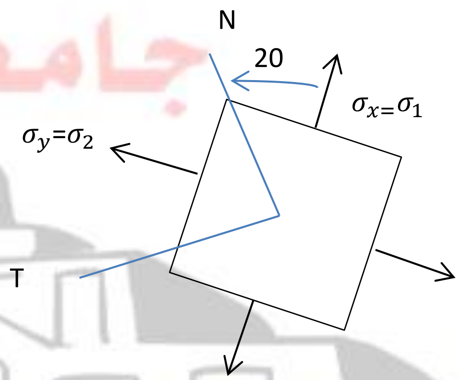
For check :

$$\therefore\ \tau_{\overline{xy}} = \frac{\sigma_x - \sigma_y}{2}\sin 2\bar{\theta} + \tau_{xy}\cos 2\bar{\theta}$$

$$\therefore\ \tau_{\overline{xy}} = \frac{3-1}{2}\sin 253.26^o + 2\cos 253.26^o = +2.24 \text{ MPa (ok)}$$

Ex2: for $\sigma_1 = 50$ MPa and $\sigma_2 = 40$ MPa as shown , Find σ_n and τ_{nt}

Solution :



$\sigma_x = \sigma_1 = 50$ MPa

$\sigma_y = \sigma_2 = 40$ MPa

$\tau_{xy} = 0$

$$\therefore\ \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{50+40}{2} + \frac{50-40}{2}\cos(2*20) - 0$$

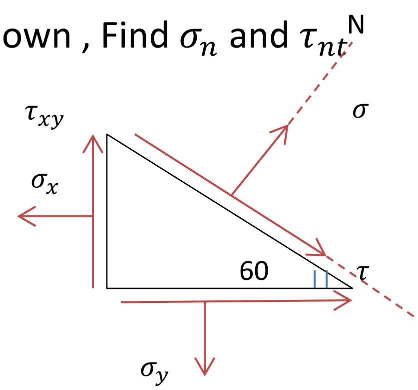
=48.83 MPa

$$\tau_{nt} = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \left(\frac{50-40}{2}\right)\sin(2*20) + 0 = 3.21 \text{ MPa}$$

Ex3: for $\sigma_x = 60$ MPa , $\sigma_y = 60$ MPa , $\tau_{xy} = 40$ MPa as shown , Find σ_n and τ_{nt}

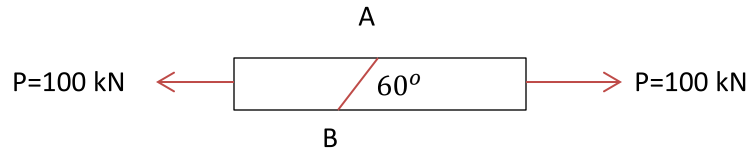
Sol:



$$\sigma = \frac{60+60}{2} + \frac{60-60}{2}\cos 2(30) - 40\sin(2*30) = 25.4 \text{ MPa}$$

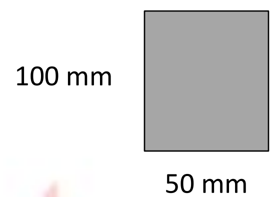
$$\tau = \frac{60-60}{2}\sin 2*30 + 40\cos(2*30) = 20 \text{ MPa}$$

Example4 :For fig. show P=100 kN find σ and τ on the inclined plane as shown (AB)



Sol :

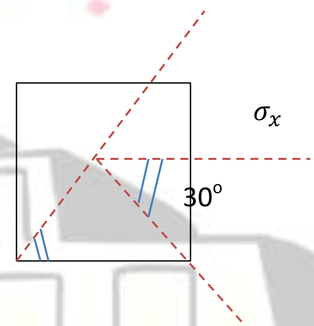
$$\sigma_x = \frac{P}{A} = \frac{100*10^3}{0.1*0.05} = 20\ MN/m^2$$



$\sigma_y = 0$;

$\theta = -30^o$ (into clockwise)

$\tau_{xy} = 0$



$$\therefore\ \sigma_n = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

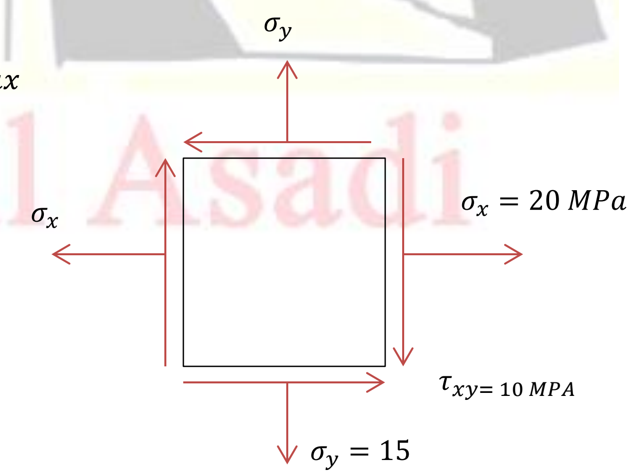
$$= \frac{20+0}{2} + \frac{20-0}{2}\cos\left(2*(-30)\right) - 0$$

$\sigma_n = 15$ MPa

$$\tau_{nt} = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \left(\frac{20-0}{2}\right)\sin\left(2*(-30)\right) + 0 = -8.66 \text{ MPa}$$

Ex5 For the state of stress shown find $\sigma_1, \sigma_2, \theta, \tau_{max}$



Sol:

$$\sigma_1, \sigma_2 = \frac{\sigma_x-\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{20+15}{2} \pm \sqrt{\left(\frac{20-15}{2}\right)^2 + (10)^2}$$

$$\sigma_1, \sigma_2 = 17.5 \pm 10.3 \rightarrow \sigma_1 = 27.8\ MPa$$

$$\sigma_2 = 7.2\ MPa$$

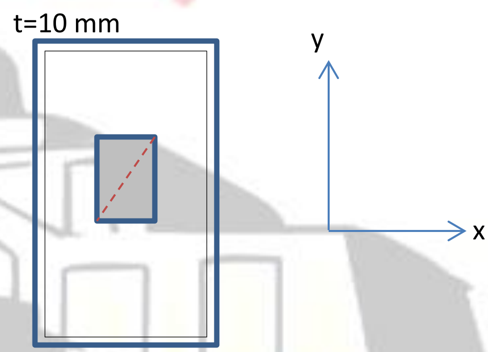
$$\tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{-2 * 10}{20 - 15} = -4$$

$$\therefore\ \theta = -37.98^o + n90 \quad \{ n=0,1,2,\ldots..\}$$

$$\therefore\ \tau_{max} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm\sqrt{\left(\frac{20-15}{2}\right)^2 + 10^2} = +10.3 \text{ MPa}$$

Ex6: for thin walled cylinder as shown below , if P = 1.4 MPa , τ_{max}=-30 MPa , Find (d_{max})

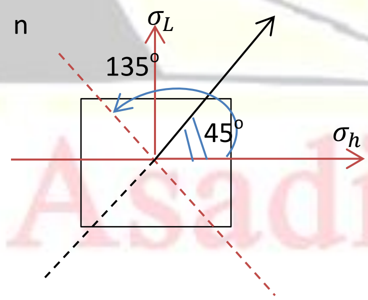


Sol:

$$\sigma_x = \sigma_n = \frac{PD}{2t} = \frac{1.4*d}{2*0.01} = 70\ d$$

$$\sigma_y = \frac{PD}{4t} = \frac{1.4 * d}{4t} = 35\ d$$

$$\therefore\ \tau = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



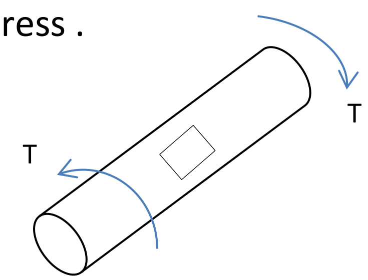
$$-30 = \frac{70d-35d}{2}\sin(2 * 135) + 0$$

$$\therefore\ d = 1.71m$$

EX7: For fig shown , a torsional loading (T) applied to the bar , it produce a state pure shear Determine ; the max. shear stress and principle stress .

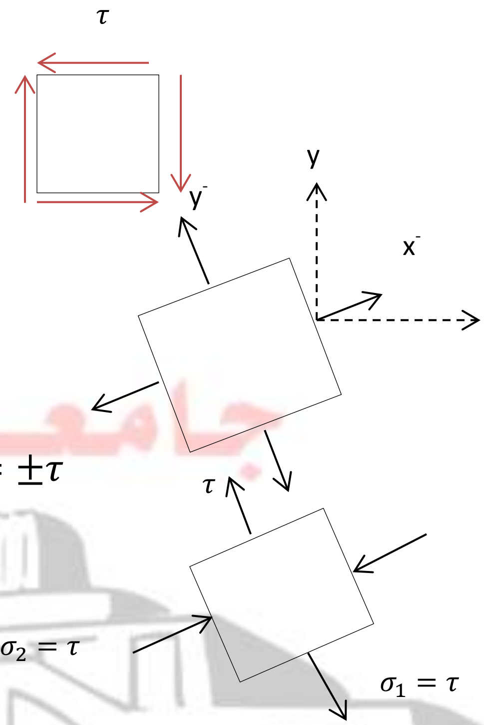
Sol :

$$\sigma_x = 0\ , \sigma_y = 0\ , \tau_{xy} = \tau$$



$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{0^2 + \tau^2}$$

$\therefore\ \tau_{max} = \pm\tau$

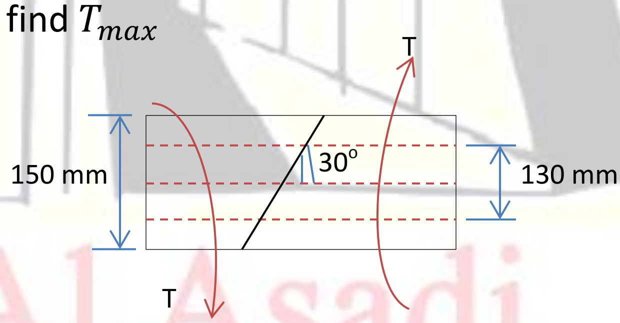


For principle stresses :

$$\tan 2\theta = \frac{-\tau_{xy}}{(\sigma_x - \sigma_y)/2} \rightarrow \tan 2\theta = \infty$$

$$\sigma_{1,2} = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0 \pm \sqrt{0^2 + \tau^2} = \pm\tau$$

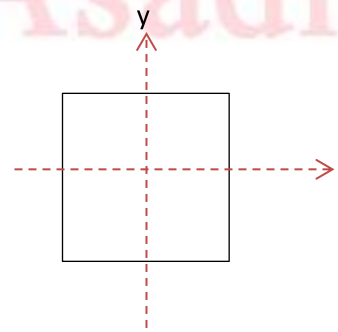
Ex8 for fig. shown , if τ_{all} in the weld = 30 MPa find T_{max}



Sol :

$$J = \frac{\pi}{2}\left[\left(\frac{D}{2}\right)^4 - \left(\frac{d}{2}\right)^2\right]$$

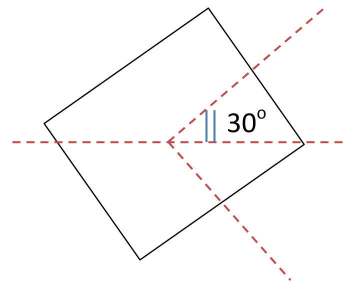
$$J = \frac{\pi}{2}\left[\left(\frac{0.15}{2}\right)^4 - \left(\frac{0.13}{2}\right)^4\right] = 21.66 * 10^6 m^4$$



$$\tau_{xy} = \frac{T.r}{J} = \frac{0.15T}{2 * 21.66 * 10^6} = 3.462 * 10^3 T\ \ N/m^2$$

$$\therefore\ \tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Can be use $\tau = \tau_{weld}$ in section



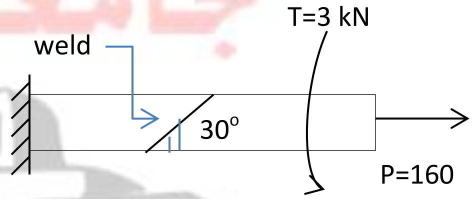
$\therefore\ \tau_{weld} = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$

$= 0 - 3.46 * 10^3 * T\cos\big(2*(-60)\big)$

$30 = -3.46 * 10^3 \cos(-120)T$

$\therefore T = 17.33 * 10^3\ N.m \quad \rightarrow T = 17.33\ N.mm$

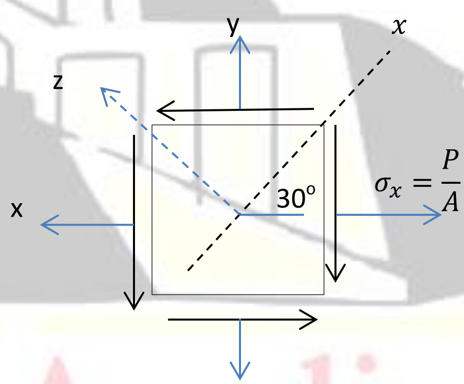
Ex9: For fig . shown and load applied find σ_n and τ_{tn} on weld .D=100 mm



Sol :

$\sigma_x = \frac{P}{A} = \frac{160*10^3}{\frac{\pi}{4}(100)^2} = 20.37 * 10^6 N/m^2$

$\sigma_x = 20.37$ MPa



From torque can estimate τ_{xy}

$\therefore\ \tau = \frac{T.r}{J} \quad , J = \frac{\pi}{32}(0.1)^4 = 9.817 * 10^{-6} m^4$

$\therefore\ \tau = \frac{3*10^3*0.05}{9.817*10^{-6}} = 15.28 * 10^6 N/m^2$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$= \frac{20.37+0}{2} + \frac{20.37-0}{2}\cos(2*120) - 15.28\sin(2*120)$

$\sigma_n = 18.33$ MPa

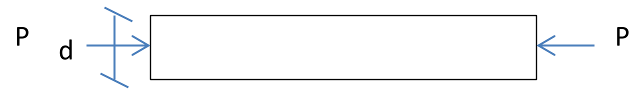
$$\tau_{nt} = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= \left(\frac{20.37}{2}\right)\sin 240 + 15.28\cos 240 = -16.46\ MN/m$

Ex10: For fig . shown if d=50 mm $\sigma_{all} = 80\ MPa$; $\tau_{all} = 30\ MPa$

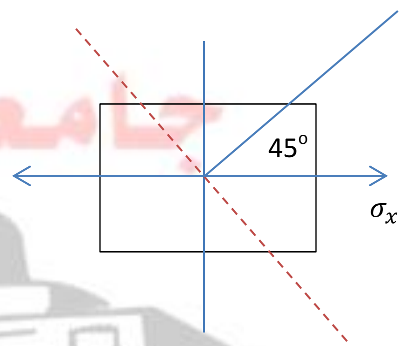
Find P_{max} . (on plane the normal make 45^o with x-axis)

Sol :



$\therefore\ \theta = 45$

$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$



$30 = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$

$\therefore 30 = \frac{\sigma_x}{2} \sin(90) \rightarrow \sigma_x = 60\ MPa$

$\therefore\ \sigma_x < \sigma_{all}$ (i.e. $60\ MPa\ < 80\ MPa$)

$\therefore\ P_{max} = \sigma_x . A = 60 * \frac{\pi(0.05)^2}{4}$=117.8 kN

Graphical method in plane stress (mohr's circle)

- From analytical method we obtain eq. (1) and (2) that used to estime direct stress and shear stress by first rewriting them as :

$\sigma - \frac{\sigma_x+\sigma_y}{2} = \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$ --------(1`)

And

$\tau = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$ ------------- (2`)

Then by squaring both these equatiions , adding simplifying :

$\left(\sigma - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$----------- (3`)

In every given problem σ_x, σ_y *and* τ_{xy} are the three known constants , and σ ; τ are the variable

The eq. above may be written as

$(\sigma - c)^2 + \tau^2 = r^2$ ---------- (4`)

Where :

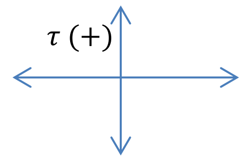
$c = \left(\frac{\sigma_x+\sigma_y}{2}\right)$ and $r^2 = \left\{\left(\sigma_x - \sigma_y\right)/2\right\}^2 + \tau_{xy}^2$ { are const.}

The eq (4`) is the familir expression of analytical geometry $(x-c)^2+y^2=r^2$ for a circle of radius r with this center at (c,0) { i.e. (x_1 ,) for particular orientation of ainclined plan the ordinate of point on the circle a (τ , σ) .

The circle so constructed is called a circle of stress or Mohr's circle of stress .

- Construction of Mohr's circle of stress .
  1- sketch an element for which normal and the shearing stresses are known .

2- set up a rectangular coordinate system as shown .



3- locate the center of the circle with $(\sigma_x + \sigma_y)/2$ a distance from the origin (tensile stress is the ; como stress - ve)

4- from the right face of the element , plot the σ_x, σ_y as the controlling point A on the circle and connect this point with center of circle by dis such as (r) , then draw the circle using the radius fou . If only magnitudes and sign of stresses are of interest the coordinates of points on the circle provide the required in formation .

5- To find direction of stresses acting on any inclined plane draw through point A a line parallel to the inclined plane and locate point B on the circle from B draw vertical line to gate a now point such as d $(\bar{\sigma}, \bar{\tau})$

And

- Important conclusion :

1- σ_1, σ_2 { with no shearing stresses }

2- τ_{max} numerically equal to the radius of the circle also $((\sigma_1 - \sigma_2)/2)$ A normal stress equal $(\sigma_1 + \sigma_2)/2$ act on each of the planes of (τ_{max})

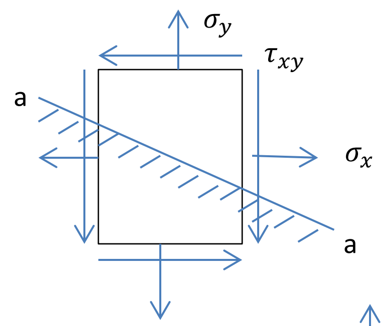
3- If $\sigma_1 = \sigma_2$, Mohr's circle degenerates in to a and no shearing stresses at all develop in the x y plane .

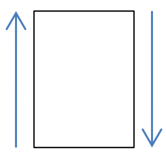
4- If $\sigma_x + \sigma_y = 0$, the center of mohr's circle conici with the origin of the $\sigma - \tau$ coordinate , and the state of pure shear exists .

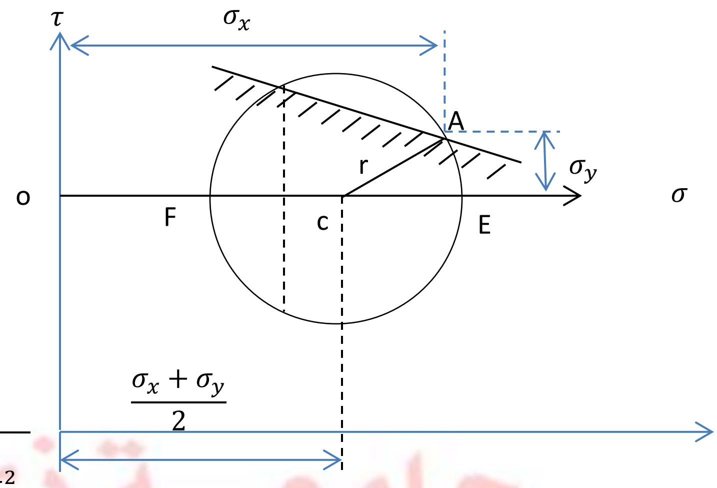
5- The sum of normal stresses on any two perpendicular planes is in variant .

$\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_{x^-} + \sigma_y$

- Can be explain by the fig shown below :

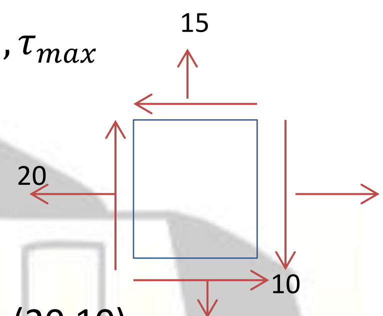






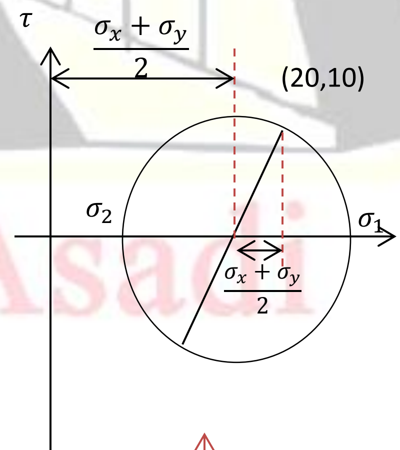
$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Example11 : for given the state of stress in fig. below find $\sigma_1, \sigma_2, \theta$, τ_{max}



Sol :

- Center of circle on the σ axis :
  $(15 + 20)/2 = 17.5\ MPa$
- Point A on a circle from desten on the right face of element is (20,10)
- Radius of circle = $\sqrt{\left(\frac{20-15}{2}\right)^2 + 10^2} = 10.3$



Then drawing the circle

$\therefore\ \sigma_1 = R + 17.5$

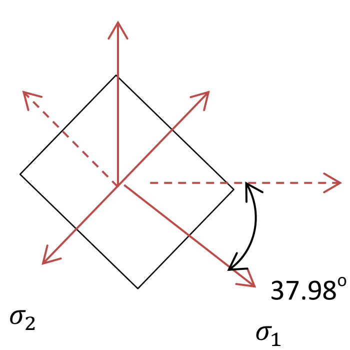
$= 10.3 + 17.5 = 27.8$

$\sigma_2 = 17.5 - R = 17.5 - 10.3 = 7.2\ MPa$

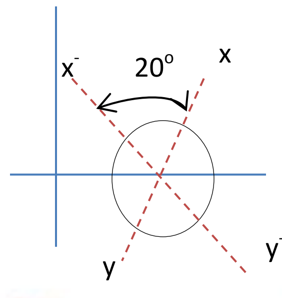
$\tan 2\theta = \frac{10}{2.5}$

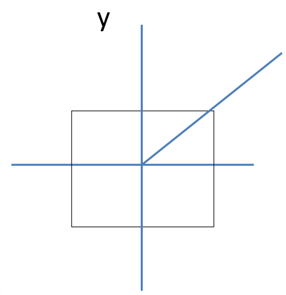
$\therefore\ \theta = -37.98^o$ { clockwise –ve }

$\tau_{max} = R = \pm 10\ MPa \rightarrow\ \tau_{max} = 10.3\ MPa$



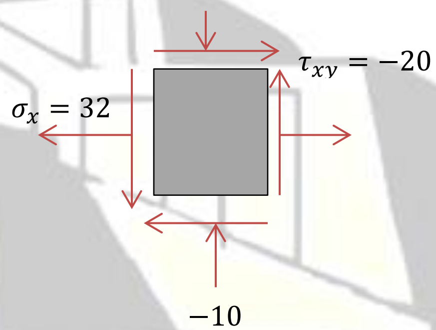
- Notes : the angle between the radii to selected points on mohr's circle is twice the angle between the normal to the actual planes represented by these points .





Therefore can by estimate any stresses on any inclined plane by draw line with angle (2θ) to obtain a point with (σ , τ) { σ , τ is a normal stress and shear stress }

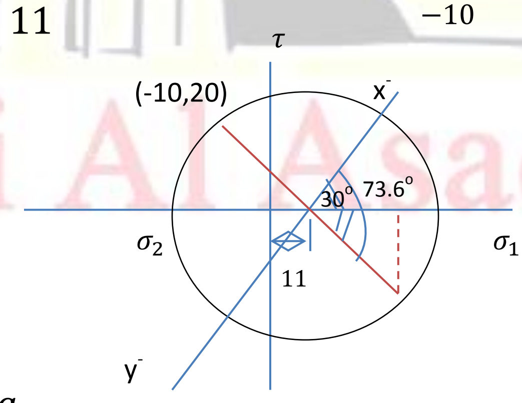
Ex12: for the data given , $\sigma_x = 32\ MPa$, $\sigma_y = -10\ MPa$, $\tau_{xy} = -20\ MPa$ find $\sigma_1, \sigma_2, \theta$, $\tau_{max}, \sigma_{\bar{x}}, \sigma_{\bar{y}}, \tau_{\overline{xy}}$ with $\theta = 36.8$



Sol :

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{32+10}{2}\right)^2 + (-20)^2}$$

$\therefore R = 29$; center c$=\frac{\sigma_x + \sigma_y}{2} = \frac{32-10}{2} = 11$



Can draw a circle

$\therefore\ \sigma_1 = R + \frac{\sigma_x + \sigma_y}{2}$

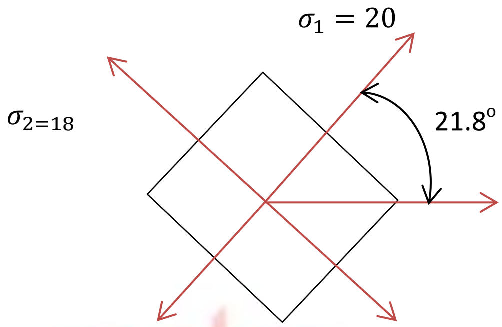
=29+11 = 40 MPa

$$\sigma_2 = R - \frac{\sigma_x + \sigma_y}{2} = 29 - 11 = 18\ MPa$$

$$\tan 2\theta = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{20}{21}$$

θ =21.8 (+ve) anti clockwise

$\therefore\ \tau_{xy} = R = 29\ MPa$



And

$\sigma_{\bar{x}} = R\cos 30 + 11 = 36.11\ MPa$ (+ tens)

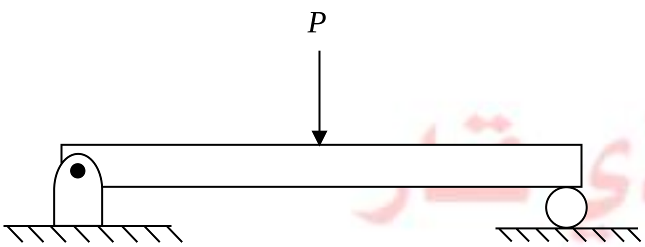
$\sigma_{\bar{y}} = R\cos 30 - 11 = 14.11\ MPa$ (comp)

$\tau_{\overline{xy}} = R\sin 30 = 29\sin 30 = 14.5\ MPa$

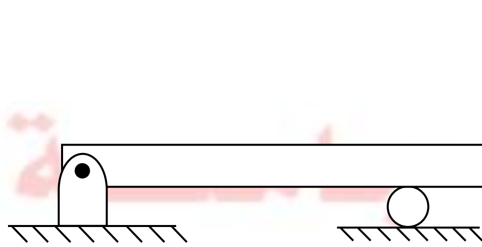
$\therefore\ \tau_{\overline{xy}} = -14.5\ MPa$

Shear and Moment Diagram:

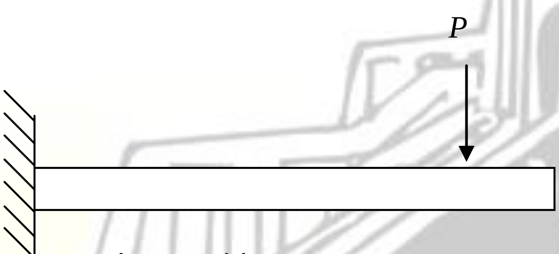
Beams are long straight members that carry loads perpendicular to their longitudinal axis. They are classified according to the way they are supported, e.g. simply supported, cantilevered, or overhanging.



Simply supported beam



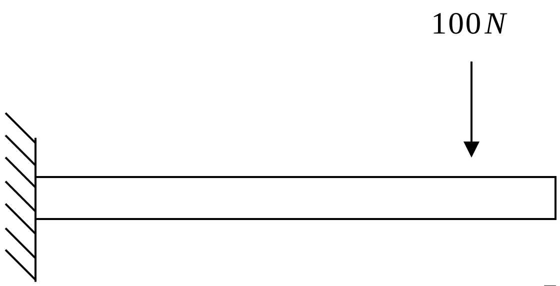
overhanging beam



Cantilevered beam

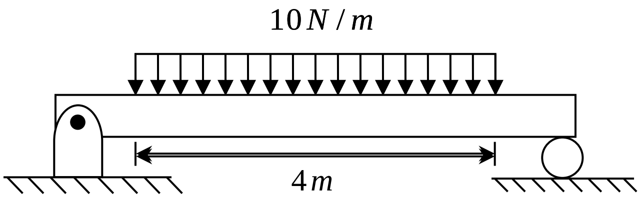
Types of Loading:

Loads commonly applied to a beam may consist of concentrated forces(applied at a point), uniformly distributed loads, in which case the magnitude is expressed as a certain number of newtons per meter of length of the beam, or uniformly varying loads. A beam may also be loaded by an applied couple.

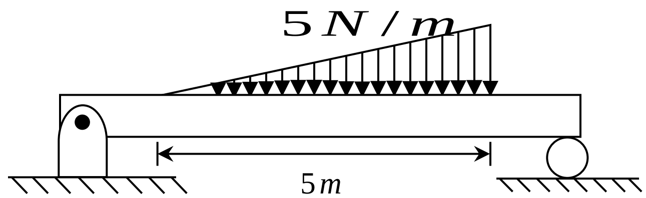


(point load)

(concentrated force)



Uniformly distributed load



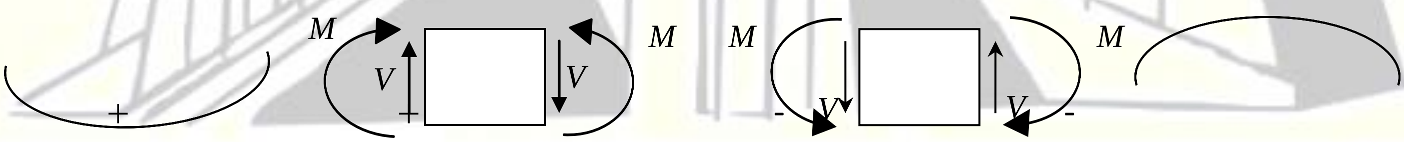
Uniformly varying load

Shearing force and bending moment diagrams show the variation of these quantities along the length of a beam for any fixed loading condition. At every section in a beam carrying transverse loads there will be resultant forces on either side of the section which, for equilibrium, must be equal and opposite.

Shearing force at the section is defined as the algebraic sum of the forces taken on one side of the section. The bending moment is defined as the algebraic sum of the moments of the forces about the section, taken on either side of the section.

Sign Convention:

Forces upwards to the left of a section or downwards to the right of a section are positive. Clockwise moments to the left and counter clockwise to the right are positive.



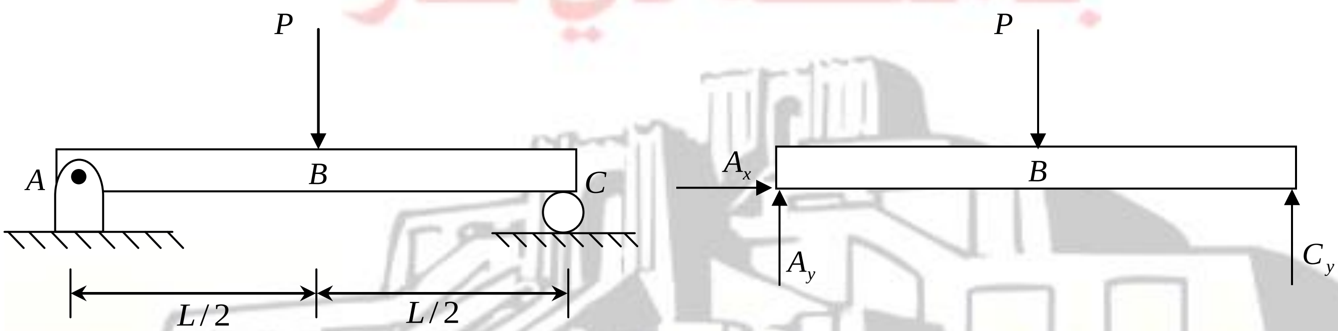
Procedure of Analysis:

The shear and moment diagrams for a beam can be constructed using the following procedure:-

1. Determine all the reactive forces and couple moments acting on the beam, and resolve all the forces into components acting perpendicular and parallel to the beam's axis.
2. Specify separate coordinates x having an origin at the beam's left end extending to regions of the beam between concentrated forces and/or couple moments, or where there is no discontinuity of distributed loading.

3. Section the beam perpendicular to its axis at each distance x, and draw the free body diagram of one of the segments. Be sure V and M are shown acting in their positive sense, in accordance with the sign convention given as above.
4. The shear is obtained by summing forces perpendicular to the beam's axis.
5. The moment is obtained by summing moment about the sectioned end of the segment.
6. Plot the shear diagram(V versus x) and the moment diagram(M versus x). If numerical values of the functions describing V and M are positive, the values are plotted above the x-axis, whereas negative values are plotted below the axis.

Example 1: Draw the shear and moment diagrams for the beam shown below.



$\sum F_x = 0$

$A_x=0$

$\sum M_C = 0$

$P \times L/2 - A_y \times L = 0$

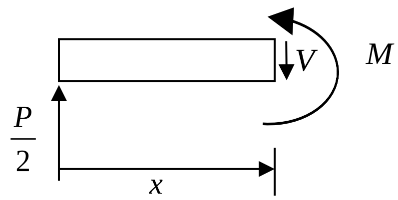
$A_y = P/2$

$\sum F_y = 0$

$C_y + A_y - P = 0$

$C_y = P/2$

- Segment AB



$\sum F_y = 0$

$\frac{P}{2}$-V=0

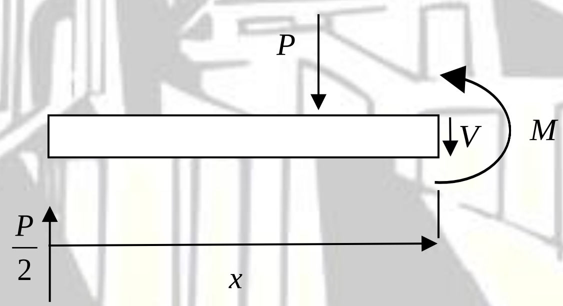
V=$\frac{P}{2}$

$\sum M = 0$

M-$\frac{P}{2}$×x=0

M=$\frac{P}{2}$x

- Segment BC



$\sum F_y = 0$

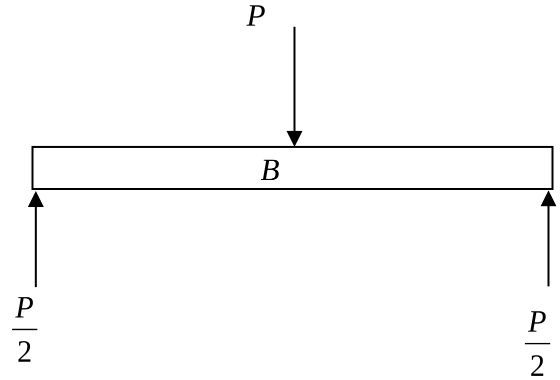
$\frac{P}{2}$-P-V=0

V=-$\frac{P}{2}$

$\sum M = 0$

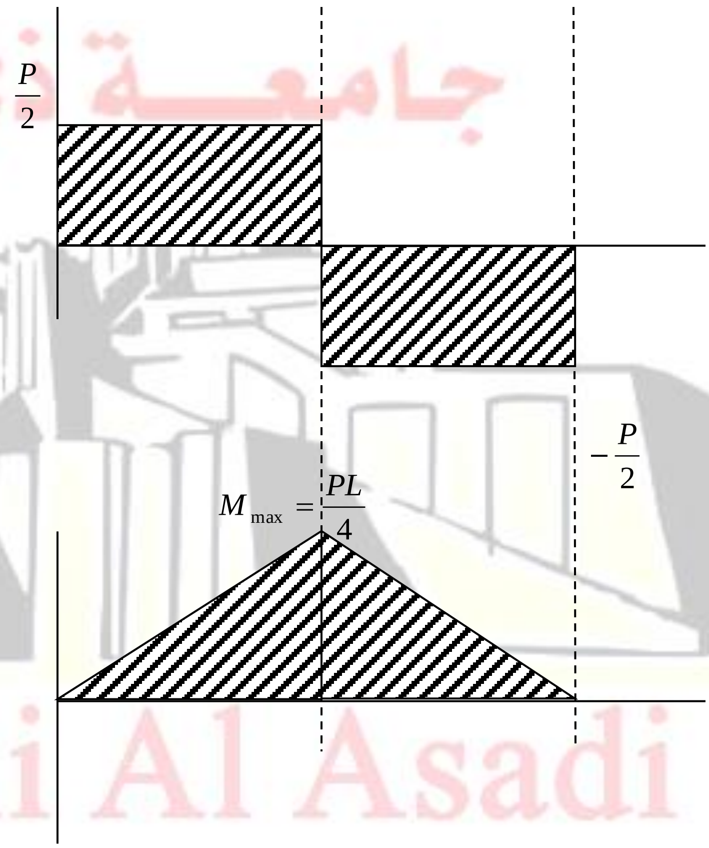
M-$\frac{P}{2}$×x+P(x-$\frac{L}{2}$)=0

M=$\frac{P}{2}$(L-x)

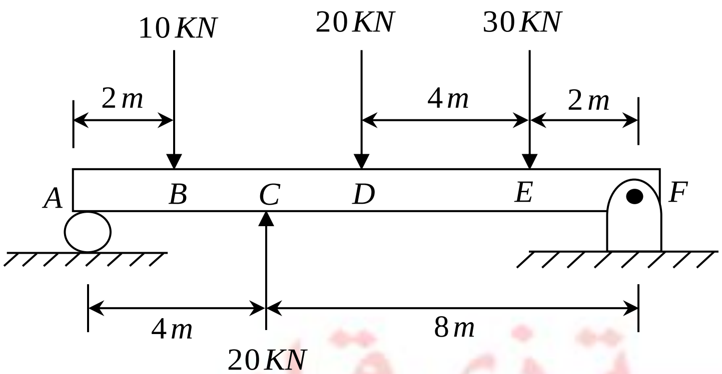


S.F. diagram

B.M. diagram



Example 2: Draw the shear and moment diagrams for the beam shown below.



$\sum F_x = 0$

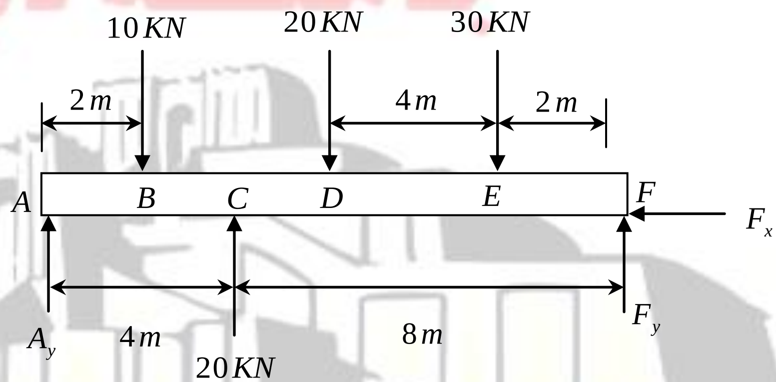
$F_x=0$

$\sum M_F = 0$

$-A_y \times 12 + 10 \times 10 - 20 \times 8 + 20 \times 6$

$+30 \times 2 = 0$

$A_y = 10$ KN



$\sum F_y = 0$

$10-10+20-20-30+F_y=0$

$F_y = 30$ KN

- Segment AB $\quad 0 \le x \le 2$

$\sum F_y = 0$

$10-V=0$

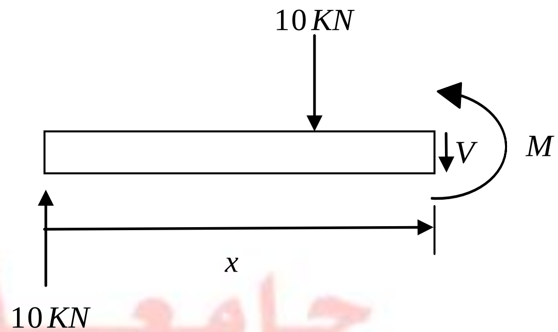
$V=10$ KN

$\sum M = 0$

M-10×x=0

M=10x

- Segment BC $2 \le x \le 4$



$\sum F_y = 0$

10-10-V=0

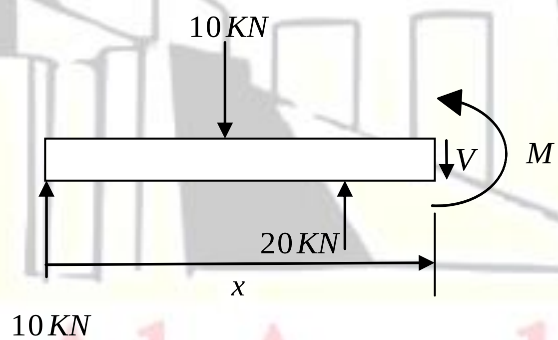
V=0

$\sum M = 0$

M-10x+10(x-2)=0

M=20 KN.m

- Segment CD $4 \le x \le 6$



$\sum F_y = 0$

10-10+20-V=0

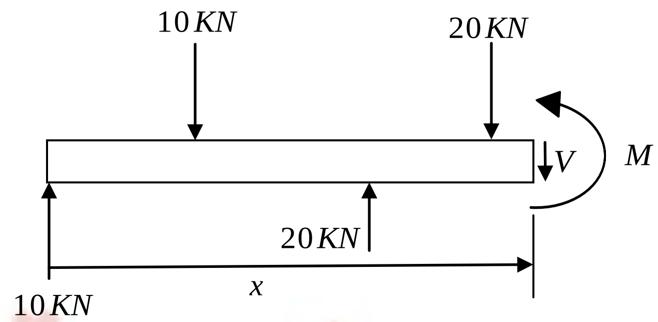
V=20 KN

$\sum M = 0$

M-10x+10(x-2)-20(x-4)=0

M=20(x-3)

- Segment DE $6 \le x \le 10$



$\sum F_y = 0$

10-10+20-20-V=0

V=0

$\sum M = 0$

M-10x+10(x-2)-20(x-4)+20(x-6)=0

M=60 KN.m

- Segment EF $10 \le x \le 12$

$\sum F_y = 0$

10-10+20-20-30-V=0

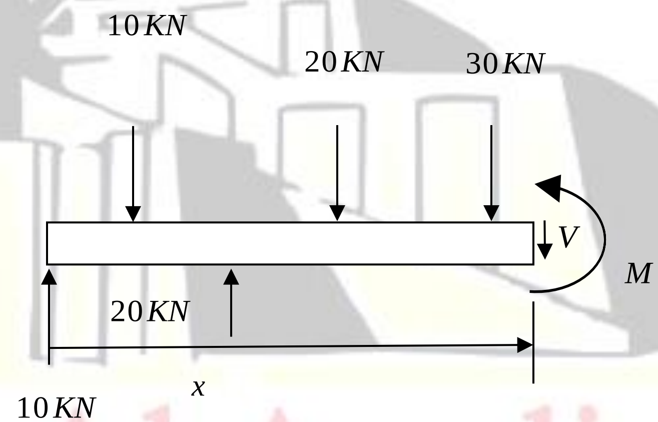
V=-30 KN

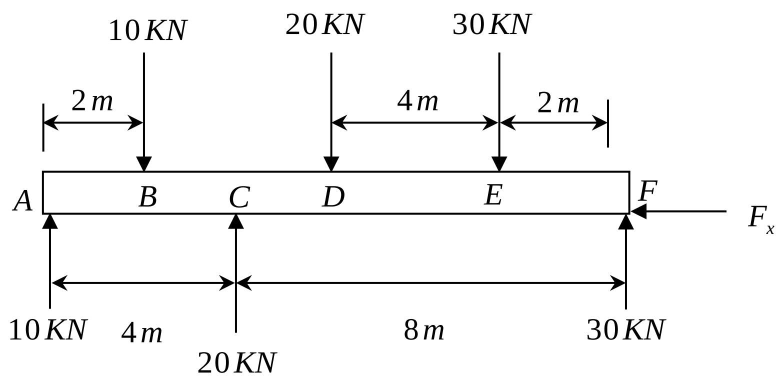
$\sum M = 0$

M-10x+10(x-2)-20(x-4)+20(x-6)

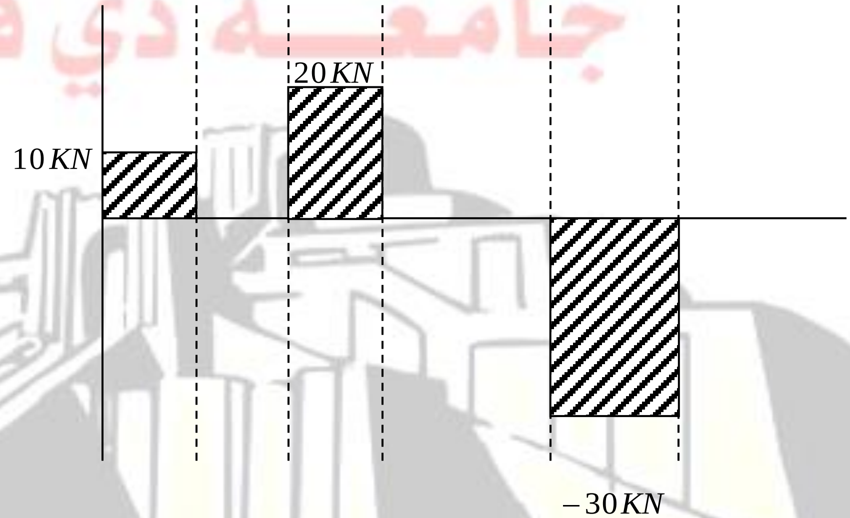
+30(x-10)=0

M=30(12-x)

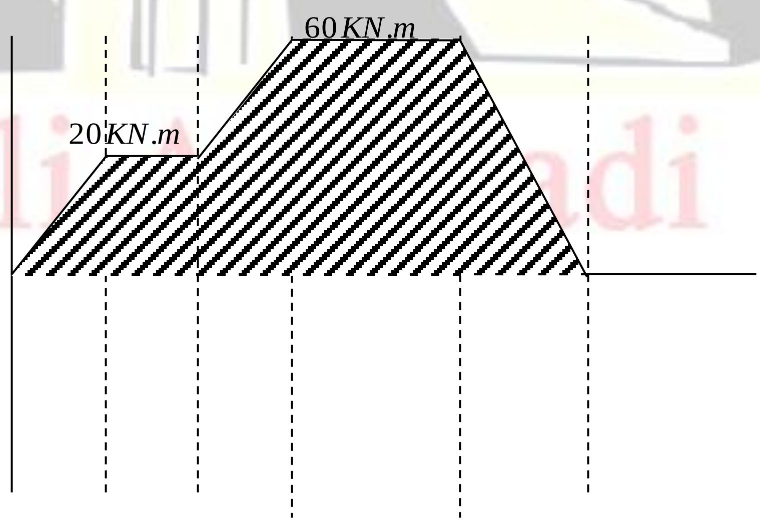




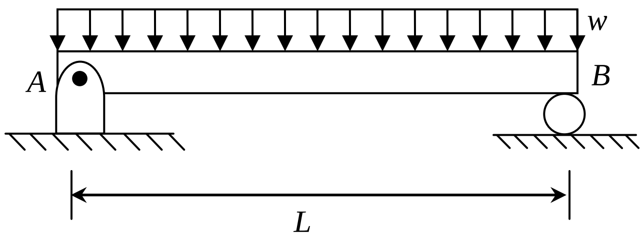
S.F Diagram

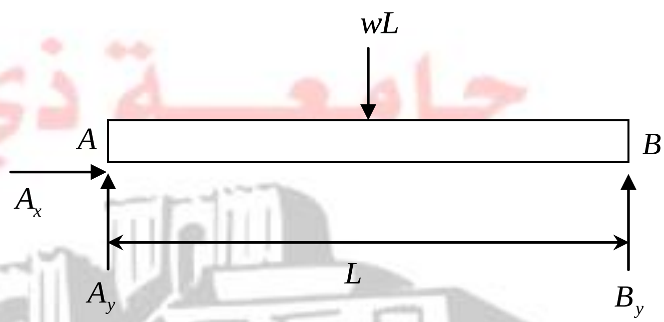


B.M. Diagram



Example 3: Draw the shear and moment diagrams for the beam shown below.





$\sum F_x = 0$

$A_x=0$

$\sum M_B = 0$

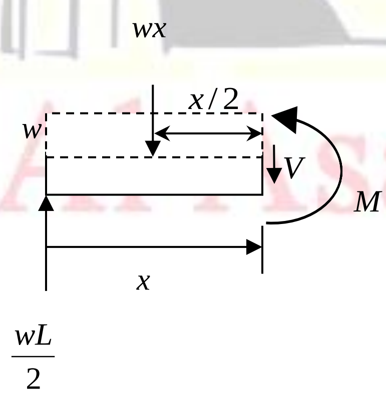
$wL\frac{L}{2}-A_yL=0$

$A_y=\frac{wL}{2}$

$\sum F_y = 0$

$\frac{wL}{2}+B_y-wL=0$

$B_y=\frac{wL}{2}$



$\sum F_y = 0$

$\frac{wL}{2}-wx-V=0$

$V=-w(x-\frac{L}{2})$

$\sum M = 0$

$$M-\frac{wL}{2}x+wx\left(\frac{x}{2}\right)=0$$

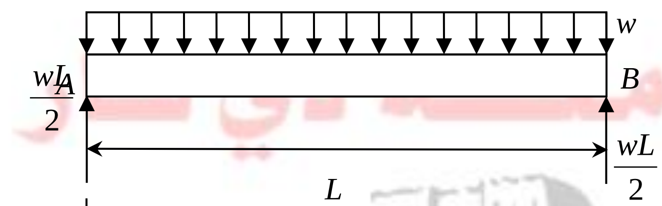
$$M=\frac{w}{2}(xL-x^2)$$

Maximum moment occur when $\frac{dM}{dx}=0$

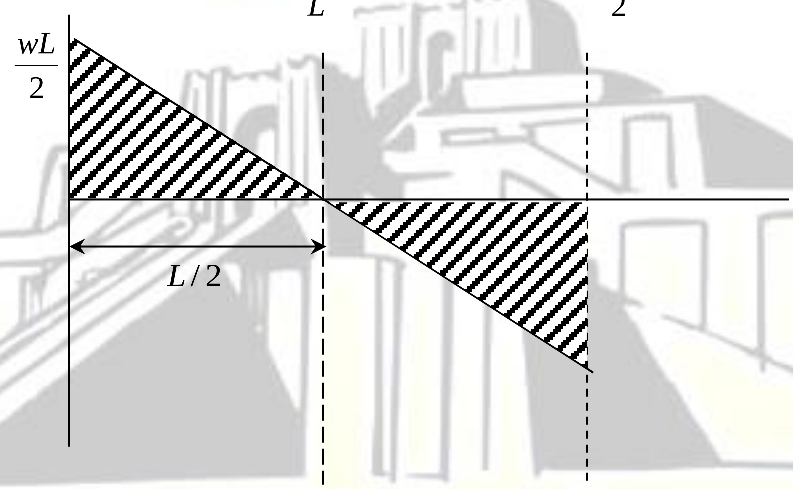
$$\frac{dM}{dx}=\frac{w}{2}(L-2x)=0$$

$$L-2x=0$$

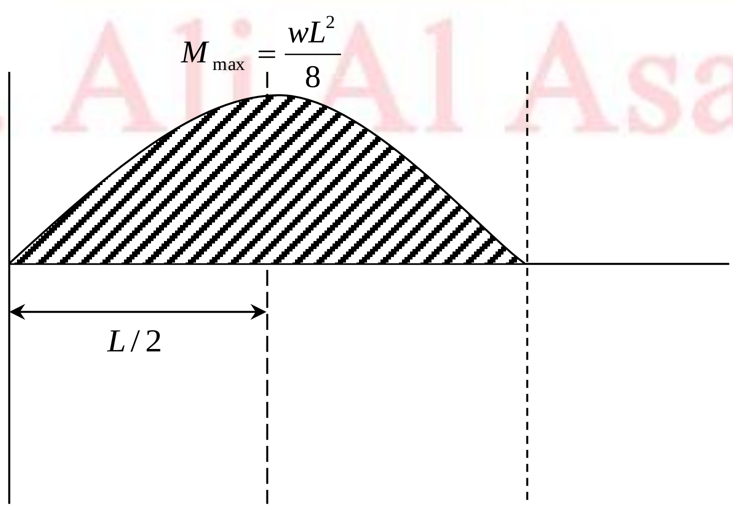
$$x=\frac{L}{2}$$



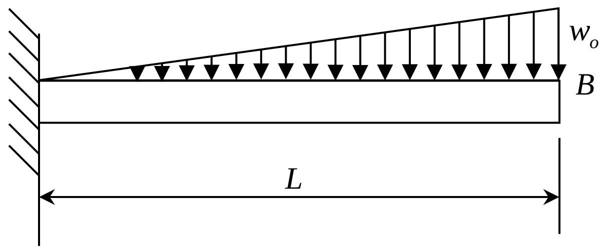
S.F Diagram

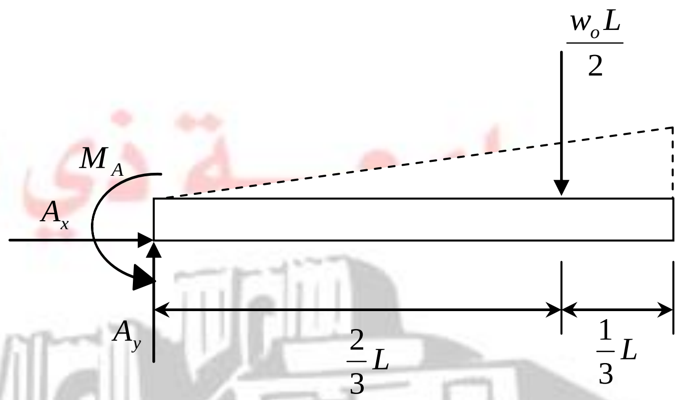


B .M Diagram



example 4: Draw the shear and moment diagrams for the beam shown below.





$\sum F_x = 0$

A_x=0

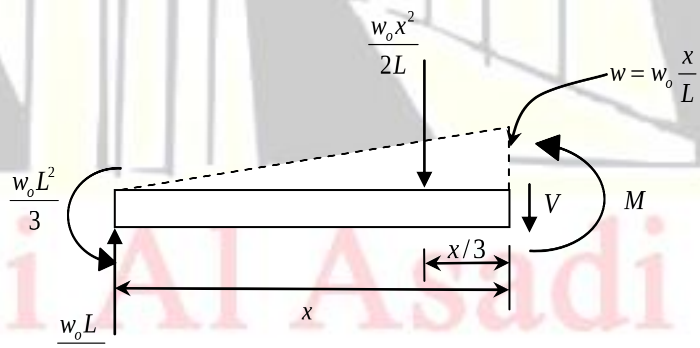
$\sum F_y = 0$

$A_y = \frac{w_o L}{2}$

$\sum M_A = 0$

$M_A - \frac{w_o L}{2} \frac{2}{3} L = 0$

$M_A = \frac{w_o L^2}{3}$



$\sum F_y = 0$

$\frac{w_o L}{2} - \frac{w_o x^2}{2L} - V = 0$

$V = \frac{w_o}{2}(L - \frac{x^2}{L})$

$V_{max} = \frac{w_o L}{2}$

Maximum shear force occur at $\frac{dV}{dx} = 0$

$\frac{dV}{dx} = -\frac{w_o x}{L} = 0$

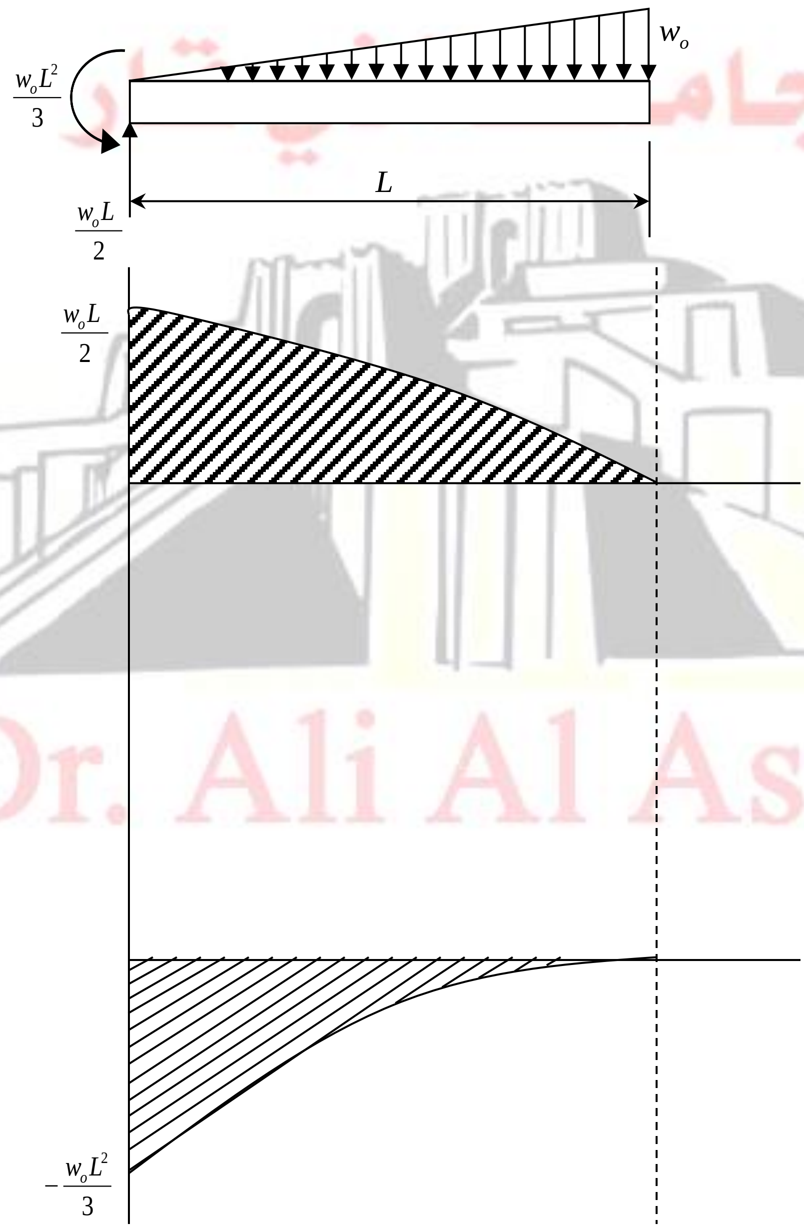
x=0

$$\sum M = 0$$

$$M + \frac{w_o L^2}{3} - \frac{w_o L}{2} x + \frac{w_o x^2}{2L} \; \frac{1}{3} x = 0$$

$$M = \frac{w_o}{6L} (3L^2 x - x^3 - 2L^3)$$

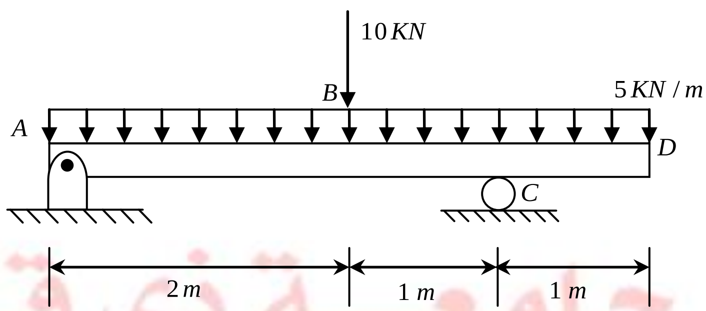
$$M_{max} = -\frac{w_o L^2}{3}$$



S.F Diagram

B .M Diagram

Example 5: The horizontal beam ***AD*** is loaded by a uniform distributed load of ***5 KN per meter*** of length and is also subjected to the concentrated force of ***10 KN*** applied as shown below. Determine the shearing force and bending moment diagrams.



$\sum F_x = 0$

$A_x=0$

$\sum M_A = 0$

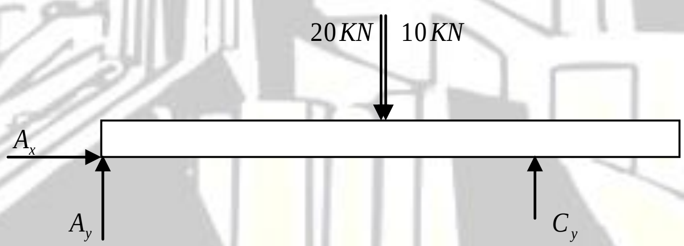
$C_y \times 3 - 30 \times 2 = 0$

$C_y = 20$ KN

$\sum F_y = 0$

$A_y + 20 - 30 = 0$

$A_y = 10$ KN



- Segment AB $0 \le x \le 2$

$\sum F_y = 0$

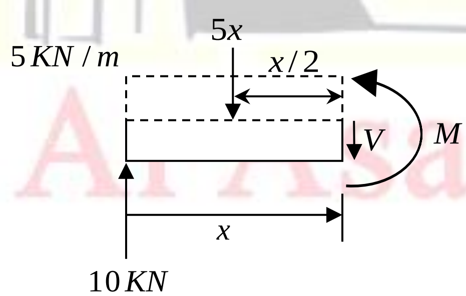
$10 - 5x - V = 0$

$V = 10 - 5x$

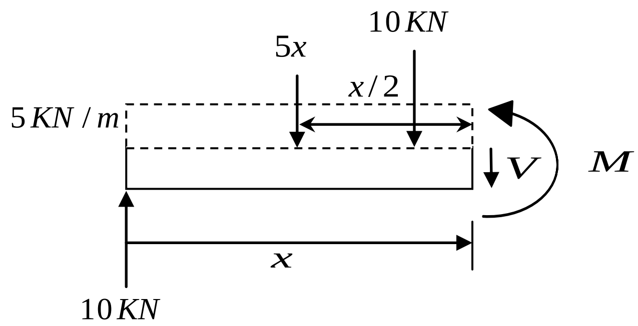
$\sum M = 0$

$M - 10x + 5x\frac{x}{2} = 0$

$M = 5x(2 - \frac{x}{2})$



- Segment BC $2 \le x \le 3$



$\sum F_y = 0$

10-5x-10-V=0

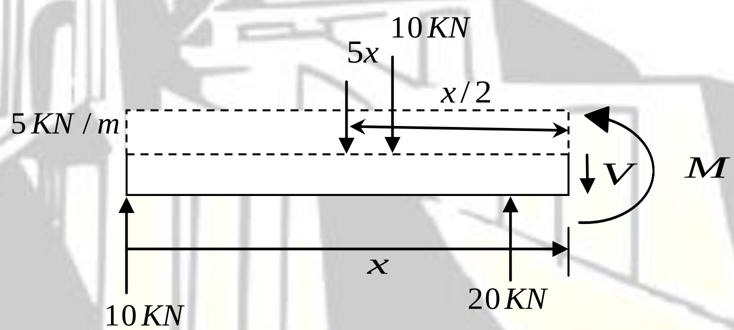
V=-5x

$\sum M = 0$

M-10x+5x $\frac{x}{2}$ +10(x-2)=0

M=20- $\frac{5}{2}$ x^2

- Segment CD $3 \le x \le 4$



$\sum F_y = 0$

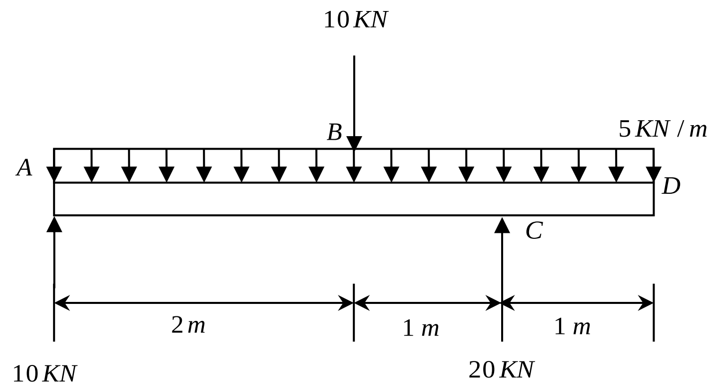
10-5x-10+20-V=0

V=20-5x

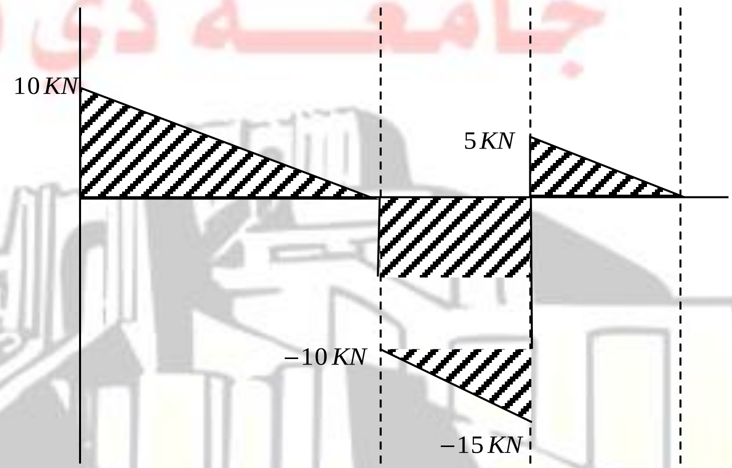
$\sum M = 0$

M-10x+5x $\frac{x}{2}$ +10(x-2)-20(x-3)=0

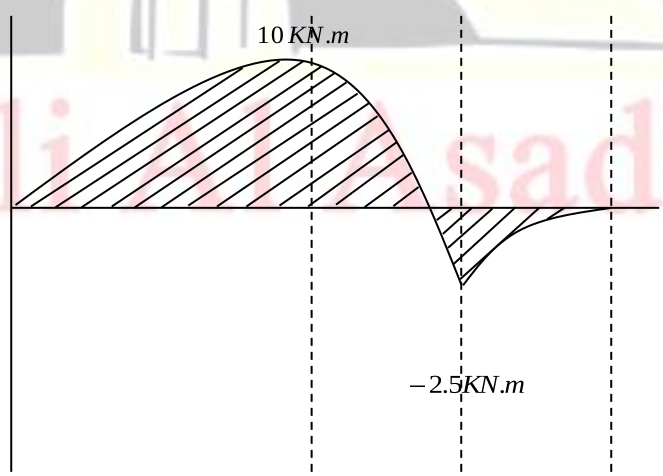
M=-40+20x- $\frac{5}{2}$ x^2



S.F Diagram



B.M Diagram



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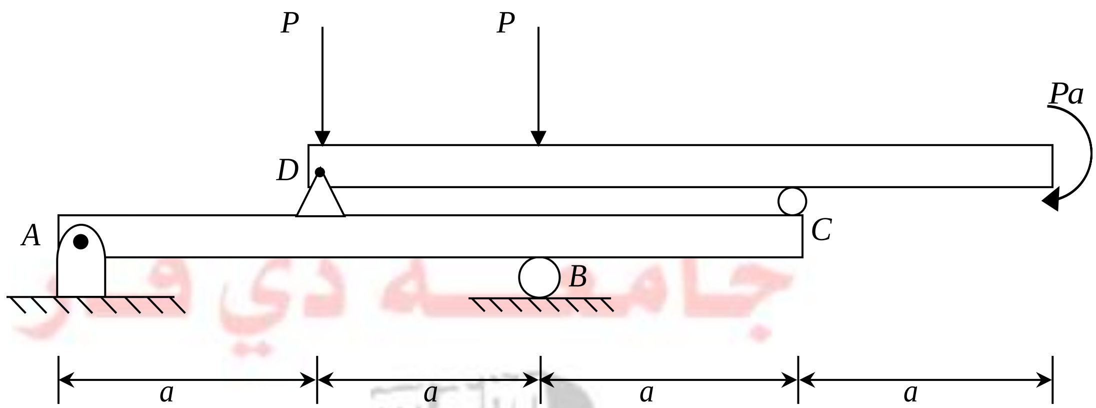
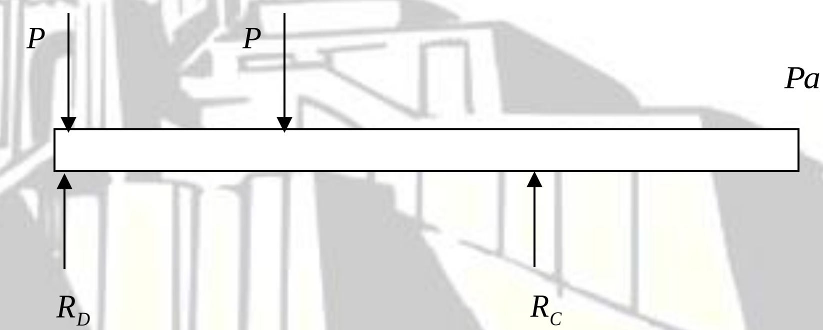
Example 6: A beam ***ABC*** is simply supported at ***A*** and ***B*** and has an overhang ***BC***. The beam is loaded by two forces ***P*** and a clockwise couple of moment ***Pa*** that act through the arrangement shown. Draw the shear force and bending moment diagrams for beam ***ABC***.





$\sum M_D = 0$

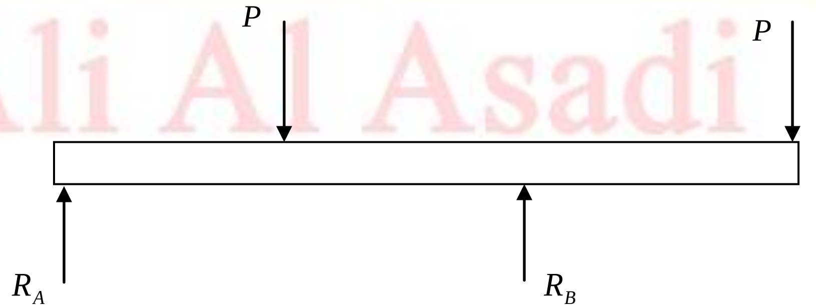
$-Pa+R_C(2a)-Pa=0$

$R_C=P$

$\sum F_y = 0$

$R_D+P-P-P=0$

$R_D=P$



$\sum M_A = 0$

$R_B(2a)-Pa-P(3a)=0$

$R_B=2P$

$\sum F_y = 0$

$R_A+2P-P-P=0$

$R_A=0$

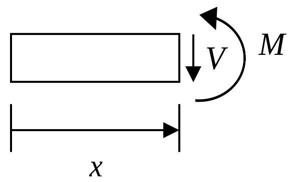
- Segment AD $0 \le x \le a$

  $\sum F_y = 0$

  V=0

  $\sum M = 0$

  M=0



- Segment DB $a \le x \le 2a$

  $\sum F_y = 0$

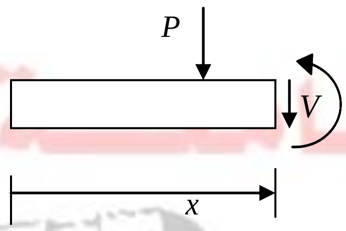
  -V-P=0

  V=-P

  $\sum M = 0$

  M+P(x-a)=0

  M=P(a-x)



- Segment DB $2a \le x \le 3a$

  $\sum F_y = 0$

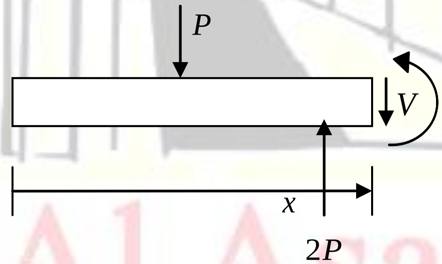
  2P-P-V=0

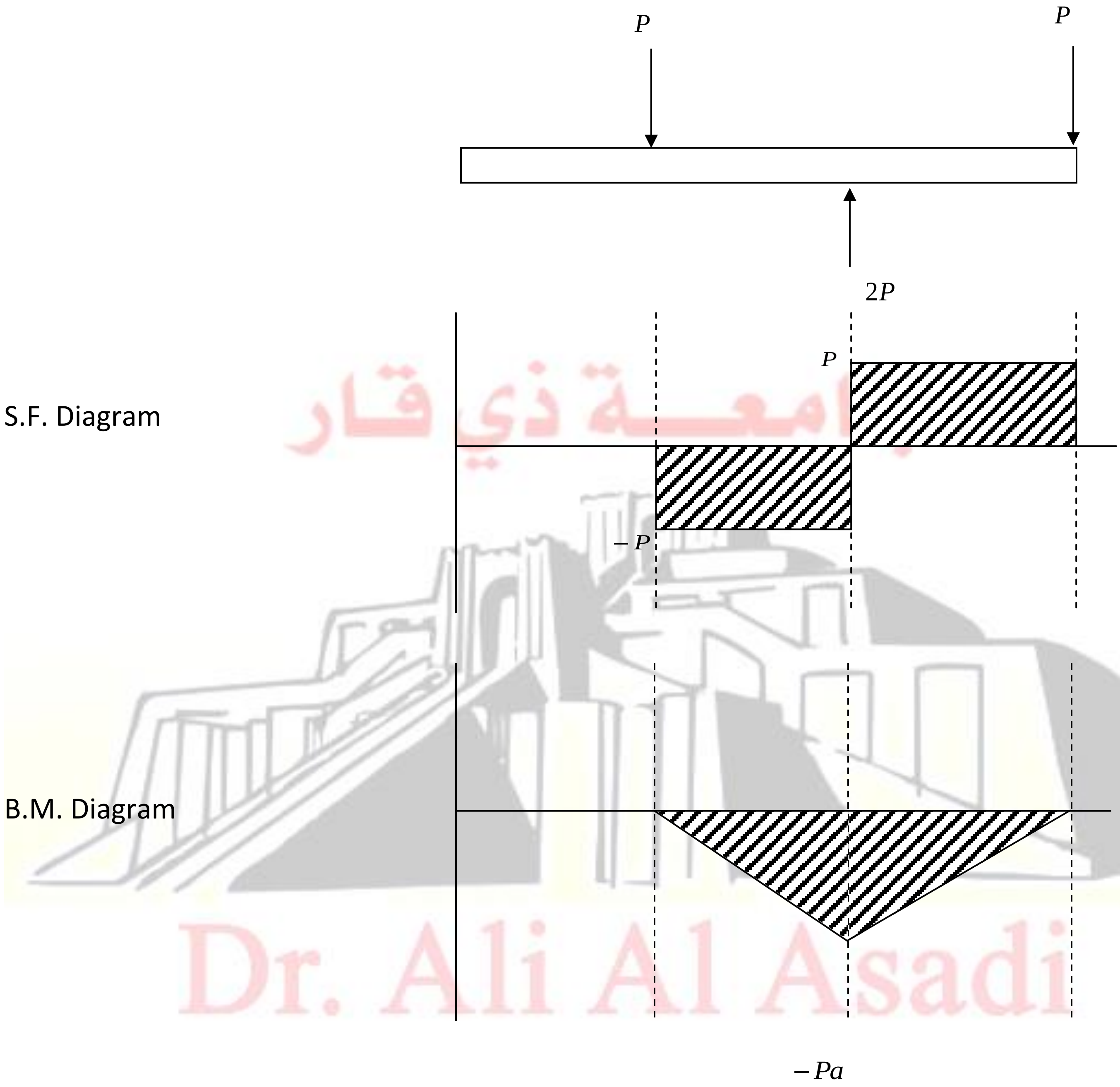
  V=P

  $\sum M = 0$

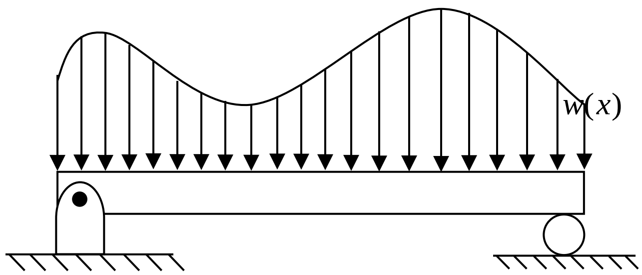
  M+P(x-a)-2P(x-2a)=0

  M=P(x-3a)




P
P
2P
S.F. Diagram
P
− P
B.M. Diagram
− Pa

Graphical Method for Constructing Shear and Moment Diagram:



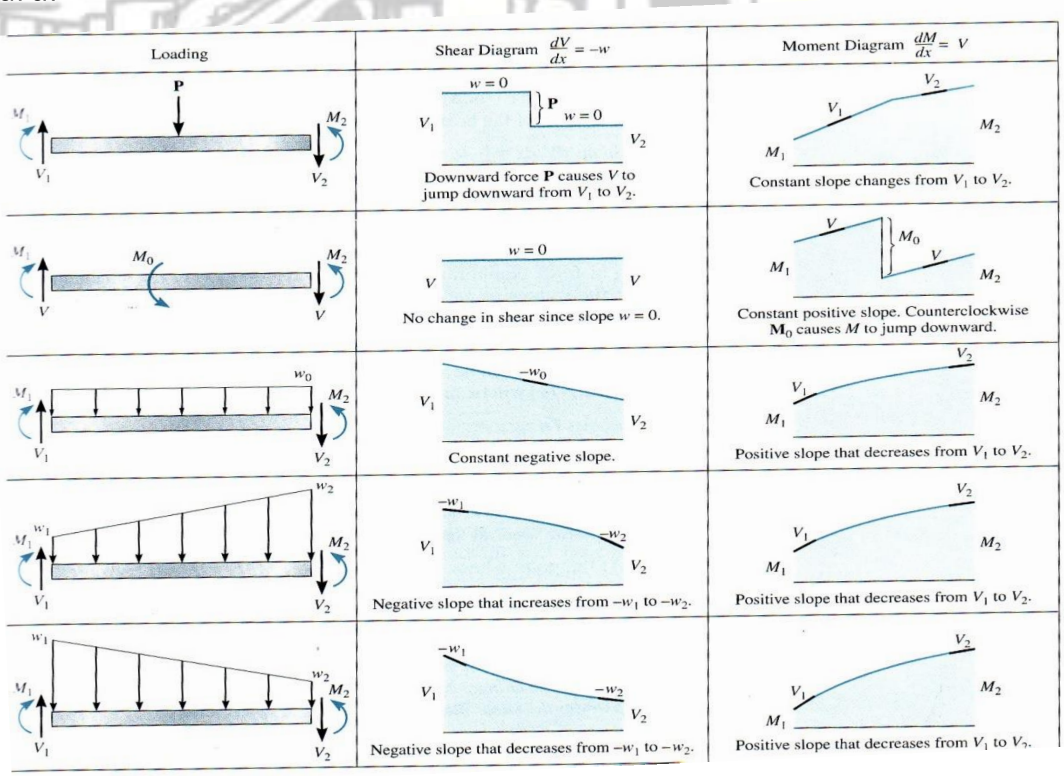
$$\frac{dV}{dx} = -w(x)$$

Slope of shear diagram at each point=-distributed load intensity at each point.

$$\frac{dM}{dx} = V$$

Slope of moment diagram at each point=shear at each point.

- When the force acts downward on the beam, ΔV is negative so the shear will jump downward. Likewise, if the force acts upward, the jump will be upward.
- If moment M_o is applied clockwise on the beam, ΔM is positive so the moment diagram will jump upward. Likewise, when M_o acts counterclockwise, the jump will be downward.



Example 1: Draw the shear and moment diagrams for the beam shown below.

$\sum F_x = 0$

$A_x=0$

$\sum F_y = 0$

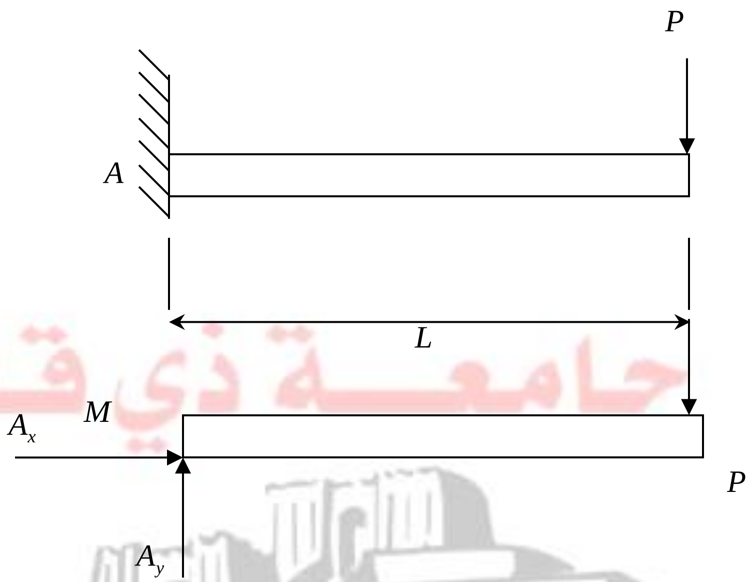
$A_y-P=0$

$A_y=P$

$\sum M_A = 0$

$M-PL=0$

$M=PL$

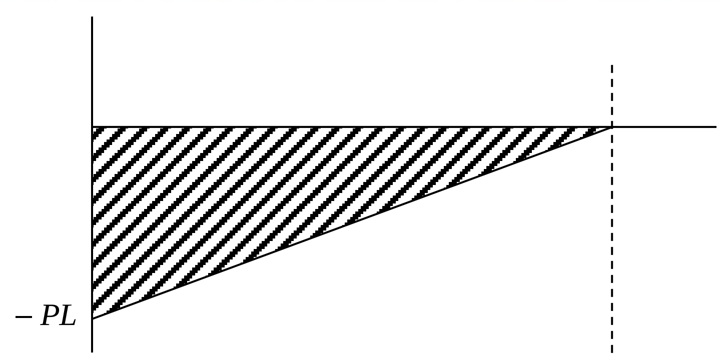


At x=0 V=P

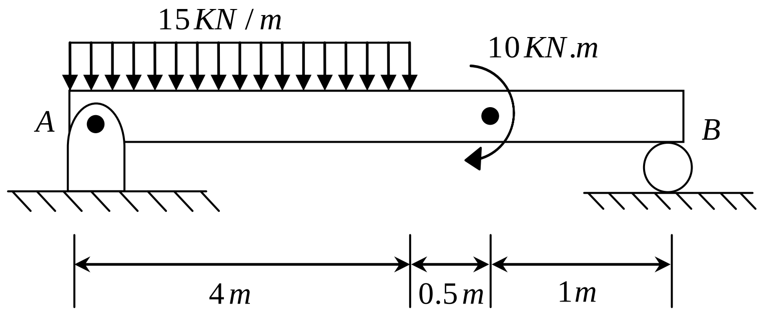
At x=L V=P

At x=0 M=-PL

At x=L M=0



Example 2: Draw the shear and moment diagrams for the beam shown below.



$\sum M_A = 0$

$B_y \times 5.5 - 10 - 60 \times 2 = 0$

$B_y = 23.63$ KN

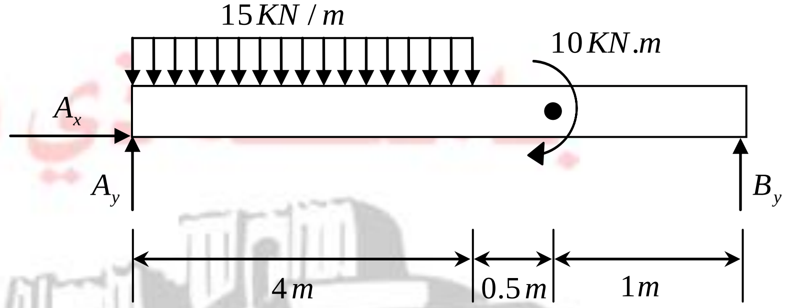
$\sum F_y = 0$

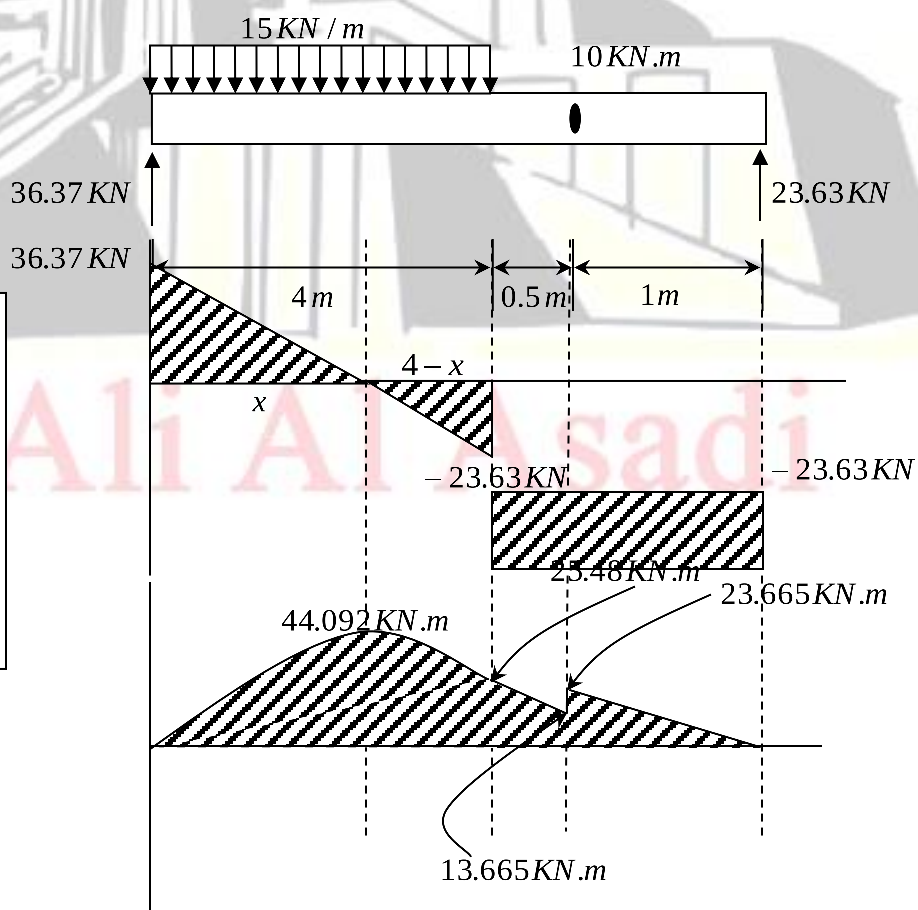
$23.63 + A_y - 60 = 0$

$A_y = 36.37$ Kn

$\sum F_x = 0$

$A_x = 0$





$$\frac{x}{36.37} = \frac{4-x}{23.63}$$

$23.63x = 36.37 \times 4 - 36.37x$

$x = 2.4246$ m

Maximum bending moment occur when V=0, at x=2.4246 m

Example 3: Draw the shear and moment diagrams for the beam shown below.

$\sum M_C = 0$

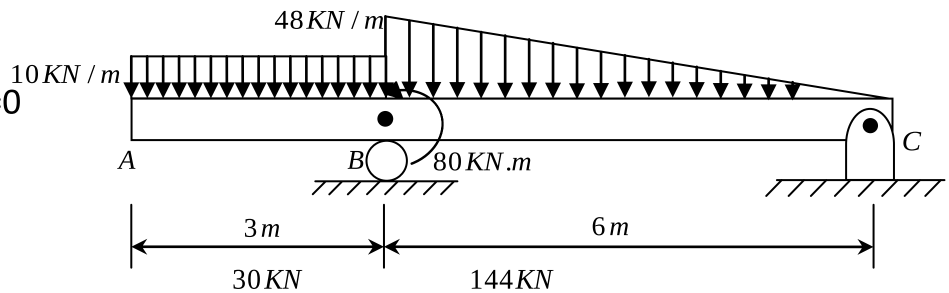
$80 - R_B \times 6 + 30 \times 7.5 + 144 \times 4 = 0$

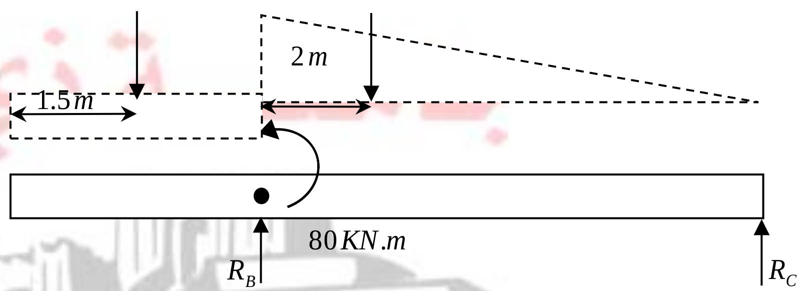
$R_B = 146.83$ KN

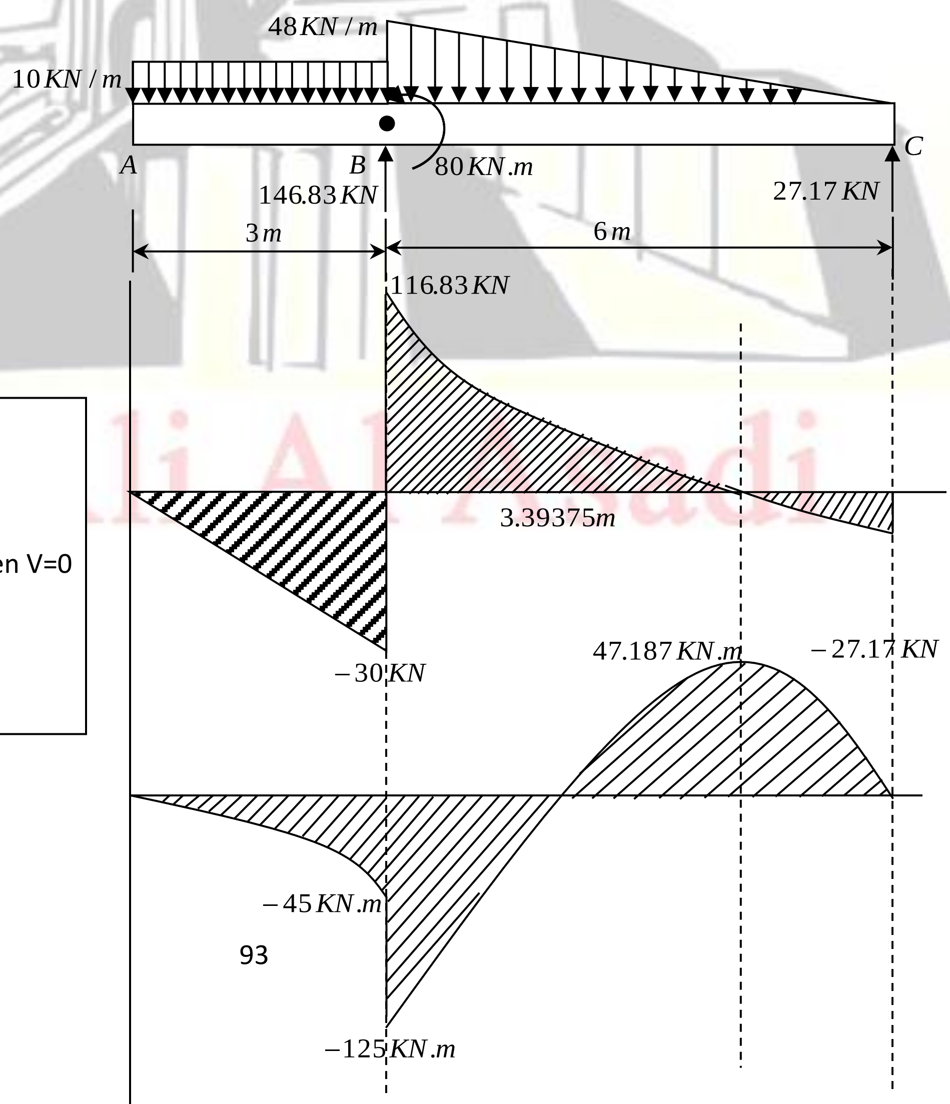
$\sum F_y = 0$

$R_C + 146.83 - 30 - 144 = 0$

$R_C = 27.17$ KN







For segment $3 \le x \le 6$

$V = 116.83 + 4(x-3)^2 - 48(x-3)$

Maximum bending moment occur when V=0

$0 = 116.83 + 4(x-3)^2 - 48(x-3)$

$x^2 - 18x + 74.2075 = 0$

Example 4: Draw the shear and moment diagrams for the beam shown below.

$\sum F_x = 0$

$A_x=0$

$\sum M_A = 0$

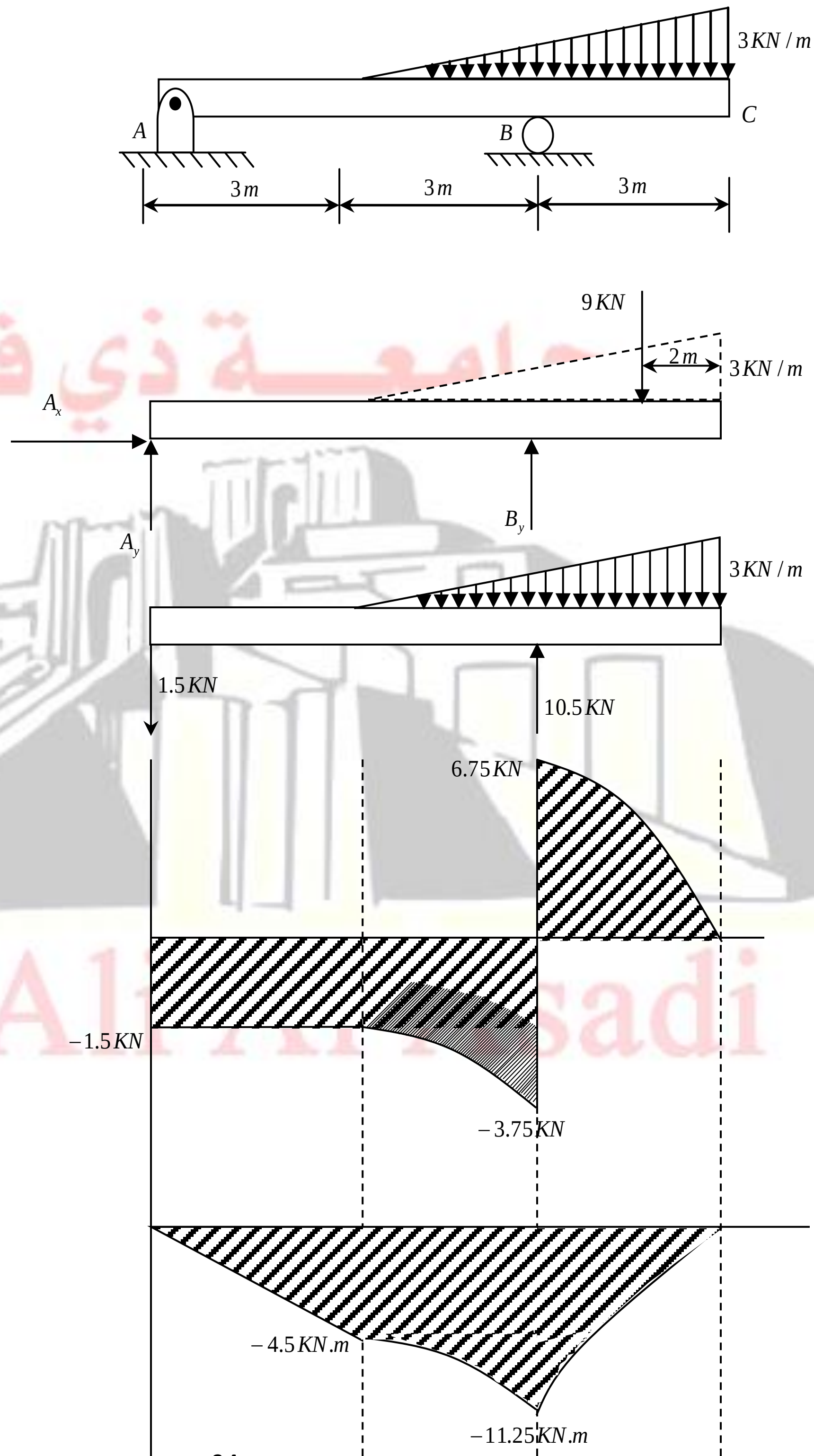
$B_y \times 6 - 9 \times 7 = 0$

$B_y = 10.5$ KN

$\sum F_y = 0$

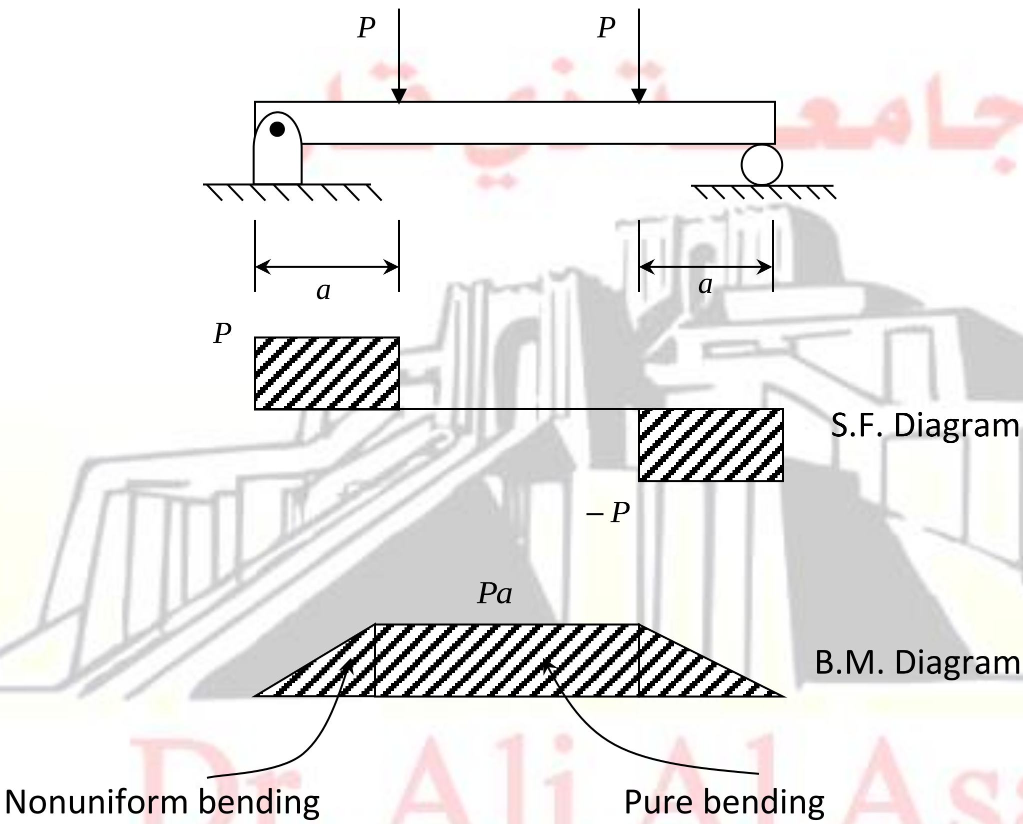
$A_y + 10.5 - 9 = 0$

$A_y = -1.5$ KN



Stresses in Beams:

Pure bending refers to flexure of a beam under a constant bending moment. Therefore, pure bending occurs only in regions of a beam where the shear force is zero. Nonuniform bending refers to flexure in the presence of shear forces, which means that the bending moment changes as we move along the axis of the beam.

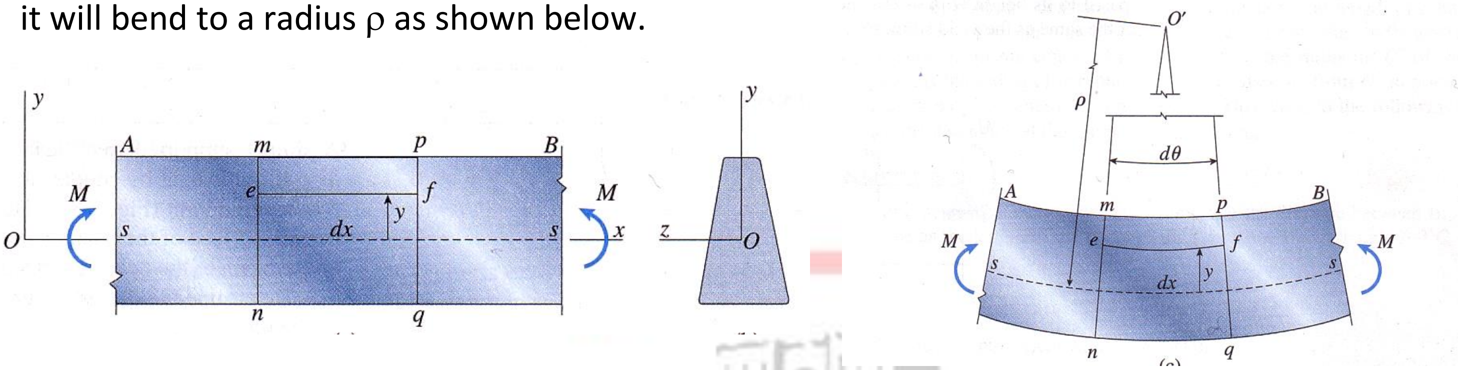


Assumptions:

1. The beam is initially straight and unstressed.
2. The material of the beam is perfectly homogeneous.
3. The elastic limit is nowhere exceeded.
4. Young's modulus for the material is the same in tension and compression.
5. Plane cross-sections remain plane before and after bending.

6. Every cross-section of the beam is symmetrical about the plane of bending i.e. about an axis perpendicular to the N.A.
7. There is no resultant force perpendicular to any cross-section.

If we now considered a beam initially unstressed and subjected to a constant bending moment along its length, i.e. pure bending as would be obtained by applying equal couples at each end, it will bend to a radius ρ as shown below.



As a result of this bending the top fibers of the beam will be subjected to compression and the bottom to tension. Its reasonable to suppose, that somewhere between the two there are points at which the stress is zero, these points is termed the neutral axis. The neutral axis will always pass through the centre of area or centroid.

The length L_1 of the line ***ef*** after bending takes place is:

$L_1=(\rho-y)d\theta$

$d\theta=\frac{dx}{\rho}$

$L_1=(1-\frac{y}{\rho})dx$

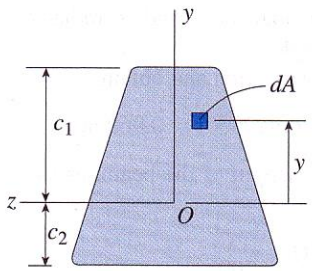
The original length of line ***ef*** is dx

$$\text{Strain}(\varepsilon_x)=\frac{L_1-originallength}{originallength}=\frac{(1-\frac{y}{\rho})dx-dx}{dx}=-\frac{y}{\rho}$$

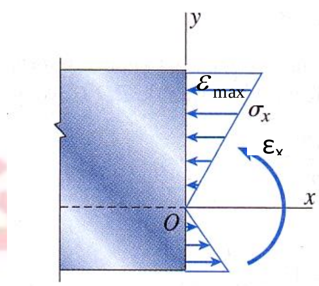
$\varepsilon_x=-ky$

where k is the curvature.

The longitudinal normal strain will vary linearly with y from the neutral axis. A contraction ($-\varepsilon_x$) will occur in fibers located above the neutral axis (+y), whereas elongation ($+\varepsilon_x$) will occur in fibers located below the neutral axis (-y).



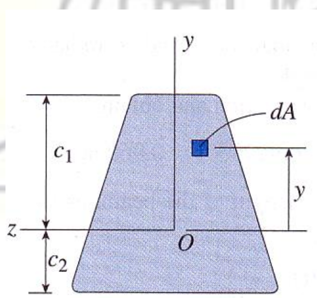
N.A



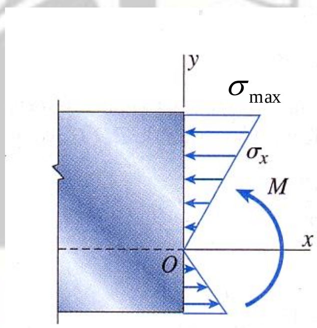
$\varepsilon_x = -\left(\frac{y}{c_1}\right)\varepsilon_{max}$

By using Hook's law $\sigma_x = E\varepsilon_x$

$\sigma_x = -Eky = -\frac{E}{\rho}y$

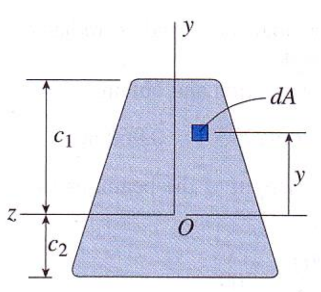


N.A



$\sigma_x = -\left(\frac{y}{c_1}\right)\sigma_{max}$

Normal stress will vary linearly with y from the neutral axis. Stress will vary from zero at the neutral axis to a maximum value σ_{max} a distance c_1 farthest from neutral axis.



N.A.

$dF = \sigma_x dA$

$$M = \int_A y\,dF = \int_A (\sigma_x dA) y$$

$$= \int_A \left(-\frac{y}{c_1}\sigma_{\max}\right) y\,dA$$

$$M = \frac{\sigma_{\max}}{c_1}\int_A y^2 dA$$

$\int_A y^2 dA = I$ moment of inertia

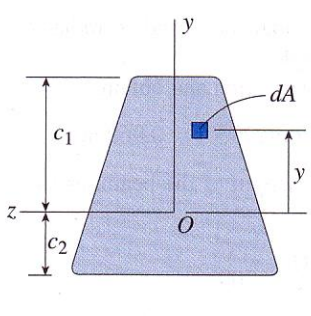
$$\sigma_{\max} = \frac{Mc_1}{I}$$

σ_{max}: The maximum normal stress in the member, which occurs at a point on the cross sectional area farthest away from the neutral axis.

M: The resultant internal moment.

I: The moment of inertia of the cross sectional area computed about the neutral axis.

c_1: The perpendicular distance from the neutral axis to a point farthest away from the neutral axis, where σ_{max} acts.



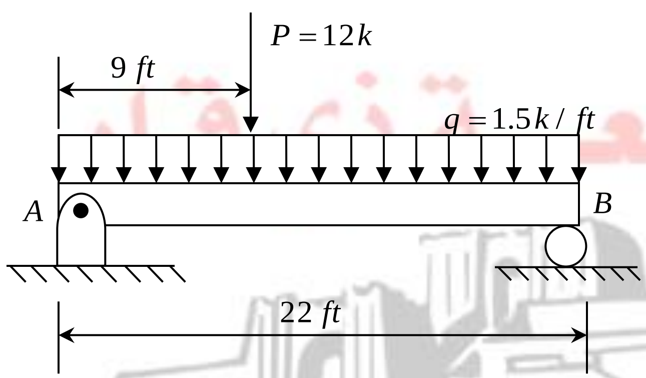
$$\sigma_1 = -\frac{Mc_1}{I} \qquad , \sigma_2 = \frac{Mc_2}{I}$$

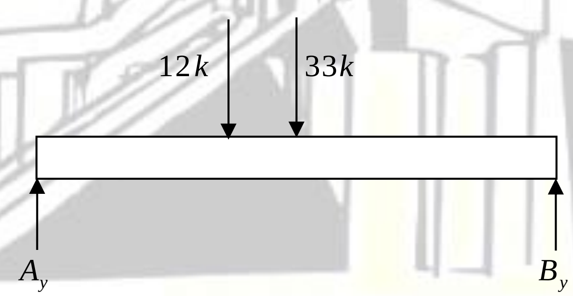
$\sigma_1 = -\frac{M}{S_1}$, $\sigma_2 = \frac{M}{S_2}$

$S_1 = \frac{I}{c_1}$, $S_2 = \frac{I}{c_2}$

The quantities S_1 and S_2 are known as the section moduli of the cross sectional area.

Example 1: A simple beam ***AB*** of span length ***L=22 ft*** supports a uniform load of intensity ***q=1.5 k/ft*** and a concentrated load ***P=12 k***. The uniform load includes an allowance for the weight of the beam. The concentrated load acts at a point ***9 ft*** from the left hand end of the beam. The beam is constructed of glued laminated wood and has a cross section of width ***b=8.75 in*** and height ***h=27 in***. Determine the maximum tensile and compressive stresses in the beam due to bending.





$\sum M_A = 0$

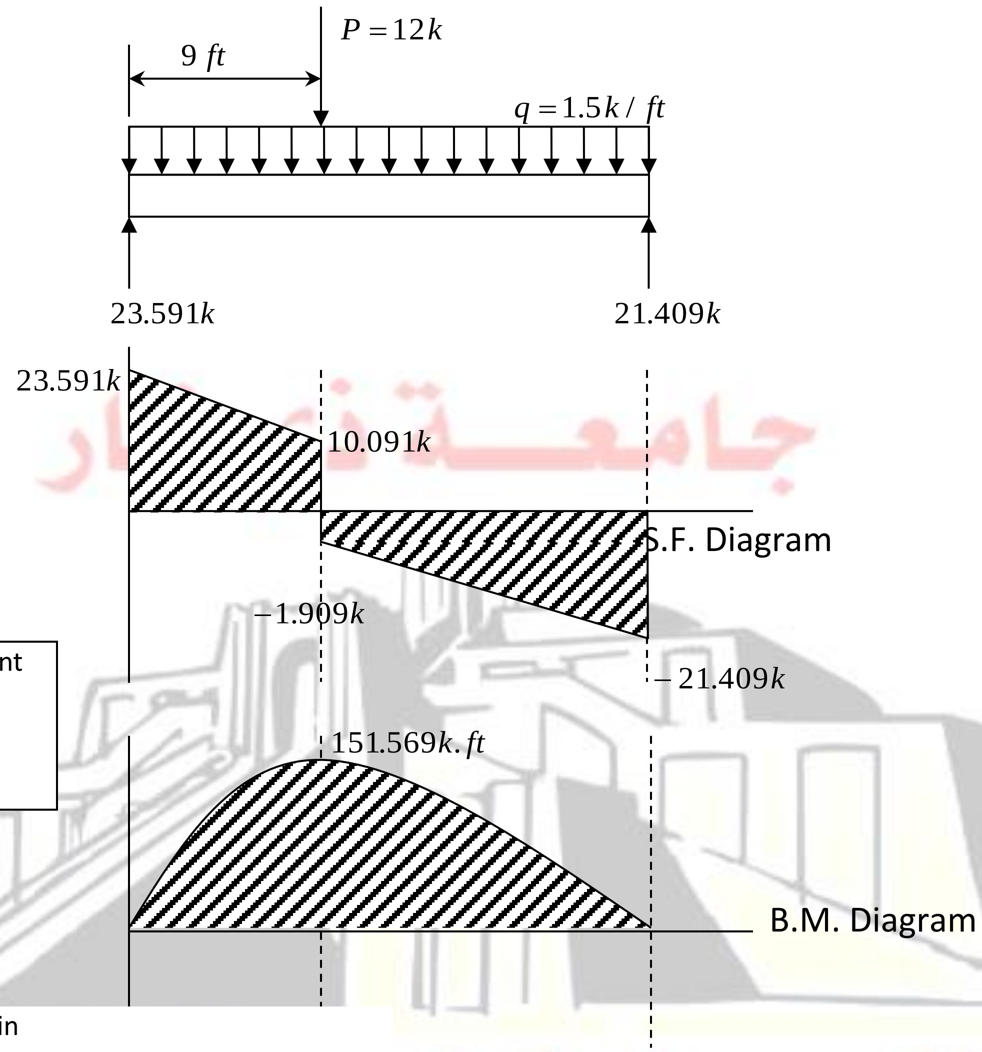
$B_y \times 22 - 12 \times 9 - 33 \times 11 = 0$

$B_y = 21.409$ k

$\sum F_y = 0$

$A_y + 21.409 - 12 - 33 = 0$

$A_y = 23.591$ k



Maximum bending moment

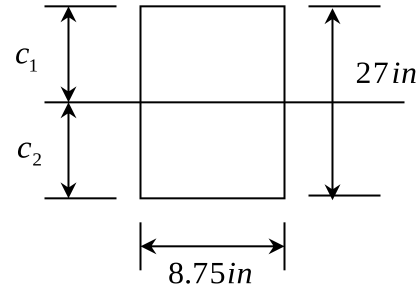
M_{max}=151.569 k.ft

=151.569×12

c_1=c_2=13.5 in

$\sigma_1 = -\frac{Mc_1}{I}$

$I = \frac{bh^3}{12} = \frac{8.75\times(27)^3}{12} = 143521875 in^4$



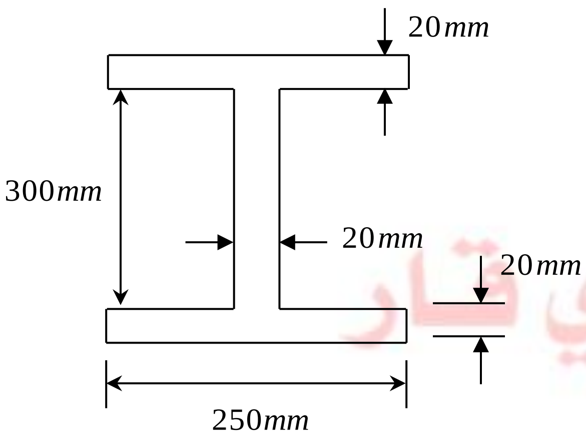
$\sigma_1 = -\frac{1818.828\times10^3\times13.5}{143521875}$

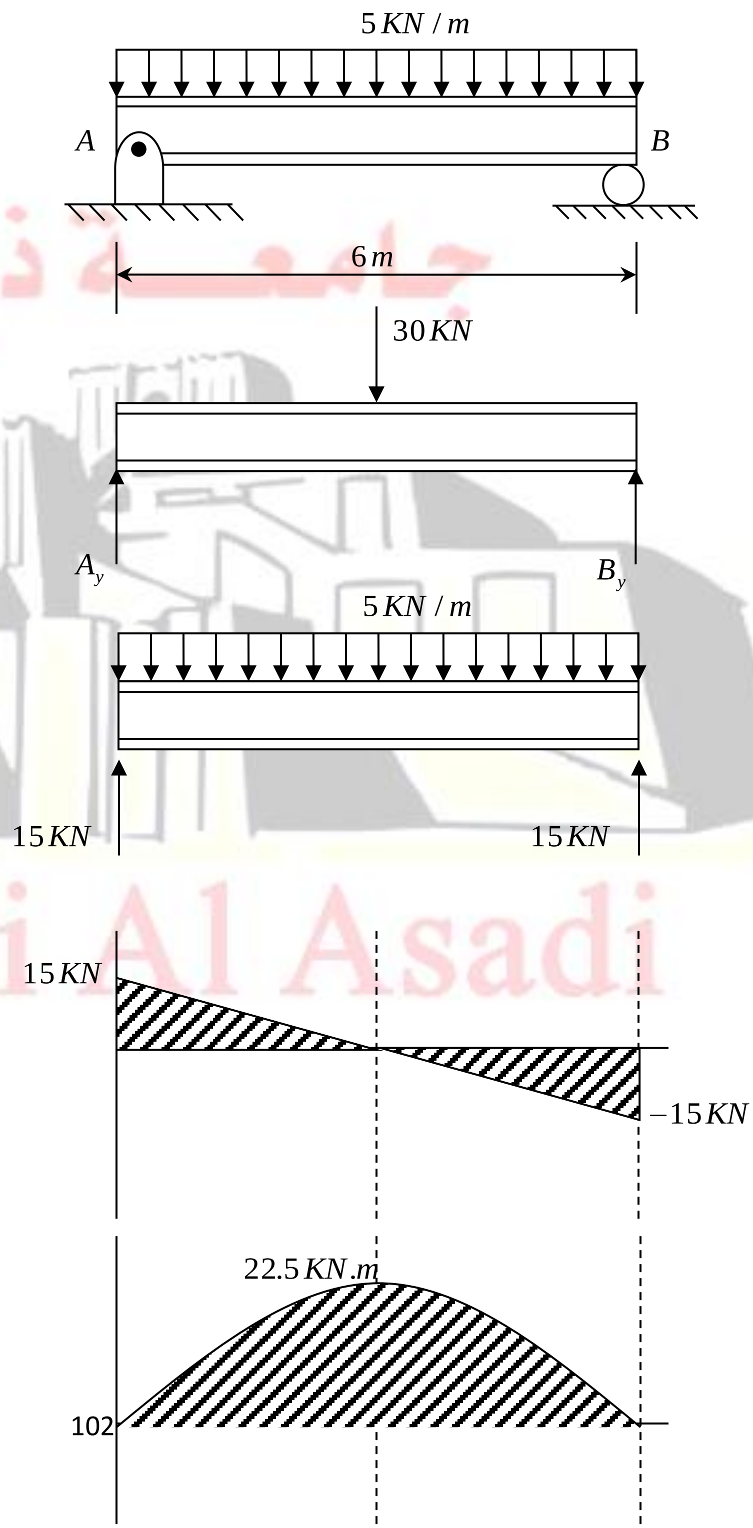
=-1710.8317 psi

$\sigma_1 = \frac{1818.828\times10^3\times13.5}{143521875}$

=1710.8317 psi

Example 2: The simply supported beam has the cross sectional area shown below. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.





$\sum M_A = 0$

$B_y \times 6 - 30 \times 3 = 0$

$B_y = 15$ KN

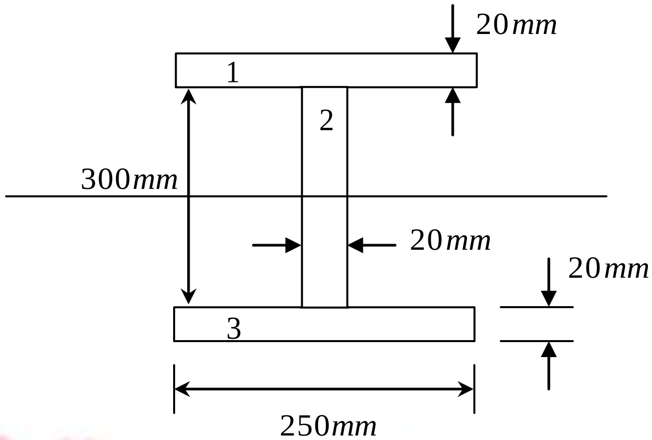
$\sum F_y = 0$

$A_y + 15 - 30 = 0$

$A_y = 15$ KN

Maximum bending moment

$M_{max} = 22.5$ KN.m



$c_1=c_2=170$ mm

$I_1=\frac{bh^3}{12}+Ad^2$

$I_1=\frac{250\times10^{-3}\times(20\times10^{-3})^3}{12}+(250\times10^{-3}\times20\times10^{-3})\times(160\times10^{-3})^2$

$I_1=128.16667\times10^{-6}$ m^4

$I_3=I_1=128.16667\times10^{-6}$ m^4

$I_2=\frac{bh^3}{12}=\frac{(20\times10^{-3})\times(300\times10^{-3})^3}{12}=45\times10^{-6}$ m^4

$I= I_1+ I_2+ I_3=128.16667\times10^{-6}+128.16667\times10^{-6}+45\times10^{-6}$

$I=301.333\times10^{-6}$ m^4

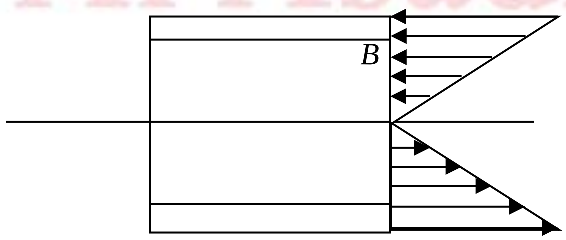
$\sigma_{max}=\frac{Mc_1}{I}=\frac{22.5\times10^3\times170\times10^{-3}}{301.333\times10^{-6}}$

=12.693598 MPa.

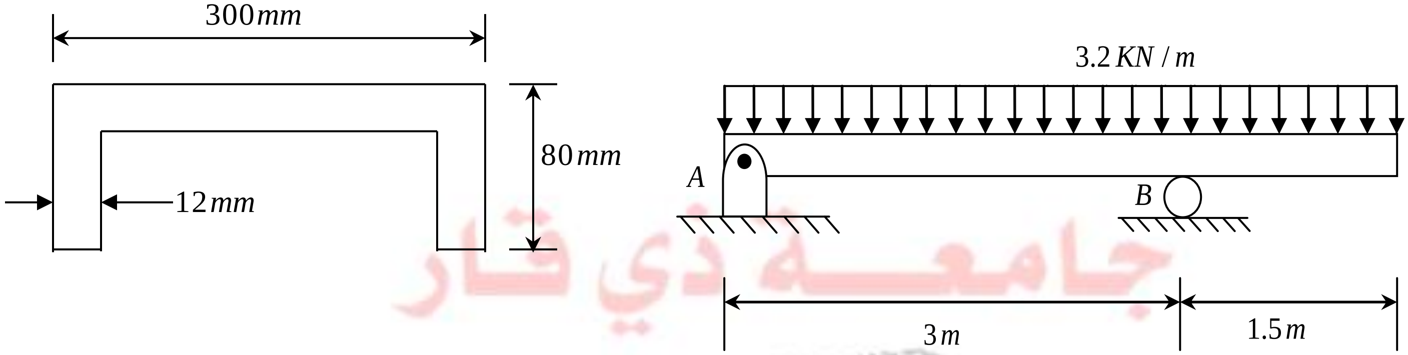
$\sigma_B=\frac{My_B}{I}$

$=\frac{22.5\times10^3\times150\times10^{-3}}{301.333\times10^{-6}}$

=11.200233 MPa.



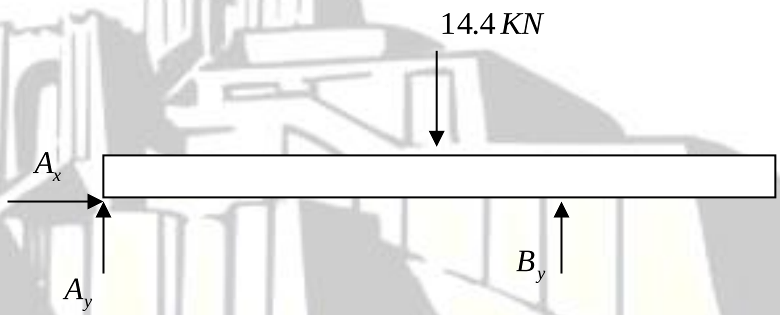
Example 3: The beam shown below has a cross section of channel shape with width ***b=300 mm*** and height ***h=80 mm***, the web thickness is ***t=12 mm***. Determine the maximum tensile and compressive stresses in the beam due to uniform load.



$\sum M_A = 0$

$B_y \times 3 - 14.4 \times 2.25 = 0$

$B_y = 10.8$ KN



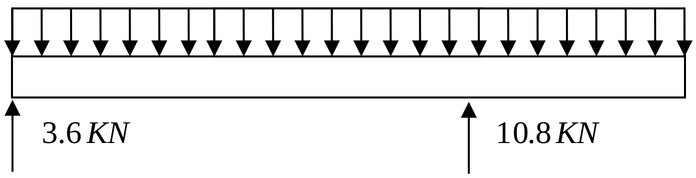
$\sum F_y = 0$

$A_y + 10.8 - 14.4 = 0$

$A_y = 3.6$ KN

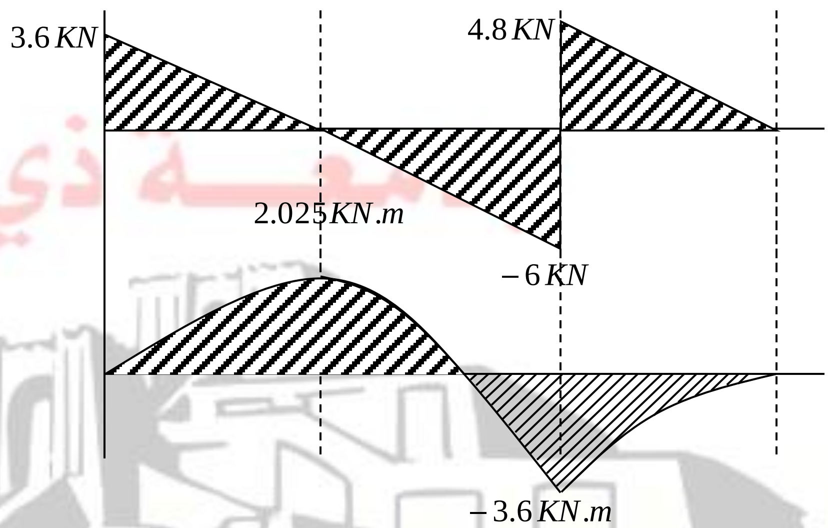
$\sum F_x = 0$

$A_x = 0$



M_1=2.025 KN.m

M_2=3.6 KN.m

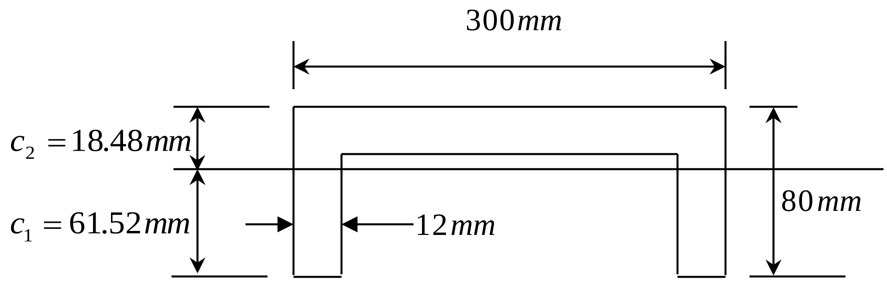


$$y_c = \frac{\sum \bar{y}A}{\sum A}$$

No. of Area	A(m^2)	$\bar{y}$ (m)	$\bar{y}A$ (m^3)
1	960×10^{-6}	40×10^{-3}	38400×10^{-9}
2	3312×10^{-6}	74×10^{-3}	245088×10^{-9}
3	960×10^{-6}	40×10^{-3}	38400×10^{-9}
	$\sum A=5232\times10^{-6}$		$\sum \bar{y}A=321888\times10^{-9}$

$y_c = \dfrac{321888 \times 10^{-9}}{5232 \times 10^{-6}}$ $= 61.52 \times 10^{-3}$ m

$y_c = 61.52$ mm



$I_1 = \dfrac{bh^3}{12} + Ad^2$

$I_1 = \dfrac{12 \times 10^{-3}(80 \times 10^{-3})^3}{12} + 960 \times 10^{-6} \times (21.52 \times 10^{-3})^2 = 0.95658 \times 10^{-6}\ m^4$

$I_3 = I_1 = 0.95658 \times 10^{-6}\ m^4$

$I_2 = \dfrac{bh^3}{12} + Ad^2$

$I_2 = \dfrac{276 \times 10^{-3}(12 \times 10^{-3})^3}{12} + 3312 \times 10^{-6} \times (12.48 \times 10^{-3})^2 = 0.55558 \times 10^{-6}\ m^4$

$I = I_1 + I_2 + I_3 = 2.46874 \times 10^{-6}\ m^4$

$(\sigma_t)_1 = \dfrac{M_1 c_2}{I} = \dfrac{2.025 \times 10^3 \times 61.52 \times 10^{-3}}{2.46874 \times 10^{-6}} = 50.462179$ MPa

$(\sigma_t)_2 = \dfrac{M_2 c_1}{I} = \dfrac{3.6 \times 10^3 \times 18.48 \times 10^{-3}}{2.46874 \times 10^{-6}} = 26.94815$ MPa

$(\sigma_t)_{max} = 50.462179$ MPa

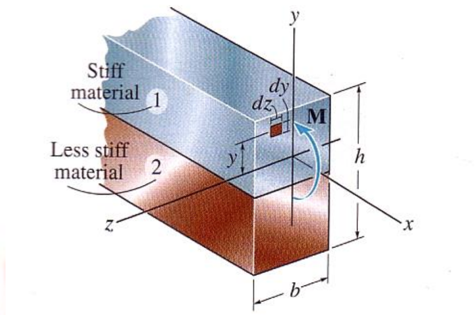
$(\sigma_c)_1 = -\dfrac{M_1 c_1}{I} = -\dfrac{2.025 \times 10^3 \times 18.48 \times 10^{-3}}{2.46874 \times 10^{-6}} = -15.158339$ MPa

$(\sigma_c)_2 = -\dfrac{M_2 c_2}{I} = -\dfrac{3.6 \times 10^3 \times 61.52 \times 10^{-3}}{2.46874 \times 10^{-6}} = -89.71054$ MPa

$(\sigma_c)_{max} = -89.71054$ MPa

Composite Beams:

Composite beams are made from different materials in order to efficiently carry a load.



Normal stress in material 1 is determined from $\sigma=E_1\varepsilon$

Normal stress in material 2 is determined from $\sigma=E_2\varepsilon$

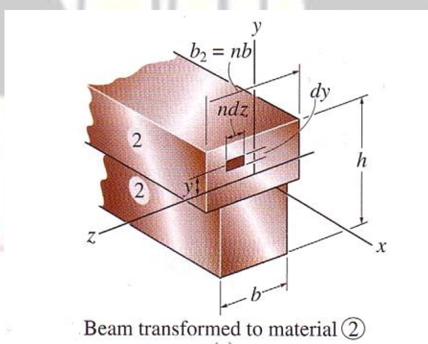
$dA=dydz$

The force dF acting on the area dA of the beam is

$dF=\sigma dA=(E_1\varepsilon)dydz$

If the material 1 is being transformed into material 2

$b_2=nb$



Beam transformed to material ②

$d\overline{F}=\overline{\sigma}d\overline{A}=(E_2\varepsilon)ndydz$

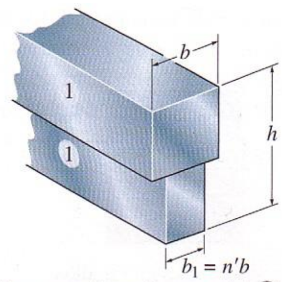
$dF=d\overline{F}$

$(E_1\varepsilon)dydz=(E_2\varepsilon)ndydz$

$n=\frac{E_1}{E_2}$

n: transformation factor (modular ratio).

If the material 2 is being transformed into material 1



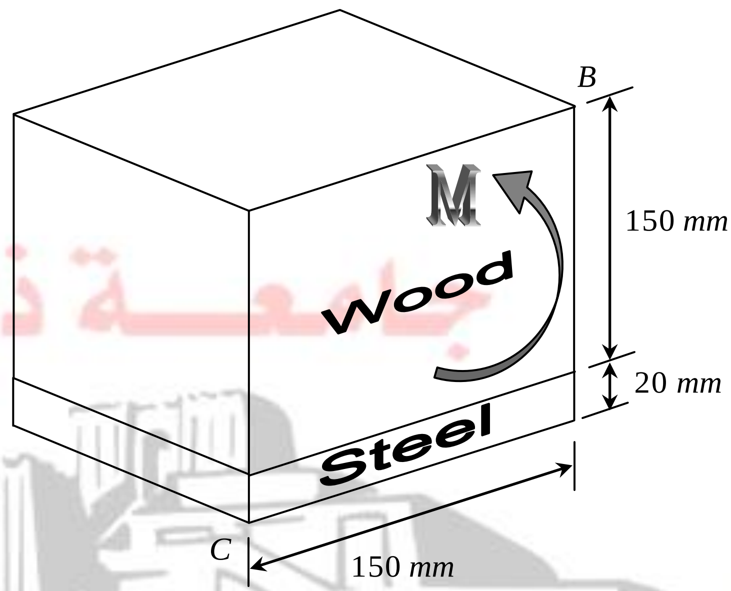
Beam transformed to material ①

$b_1 = \bar{n}$ b where $\bar{n} = \frac{E_2}{E_1}$

For the transformed material

$\sigma = n\bar{\sigma}$

Example 1: A composite beam is made of wood and reinforced with a steel strap located on its bottom side. It has the cross sectional area shown below. If the beam is subjected to a bending moment of ***M=2 KN.m*** determine the normal stress at point ***B*** and ***C***. Take E_w***=12 GPa*** and E_{st}***=200 GPa***.

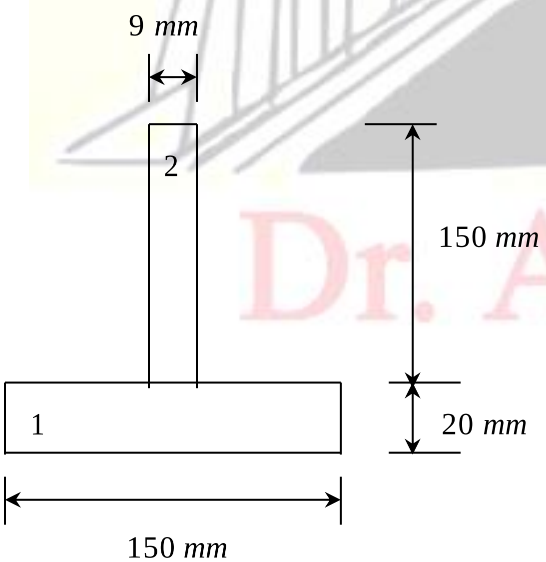


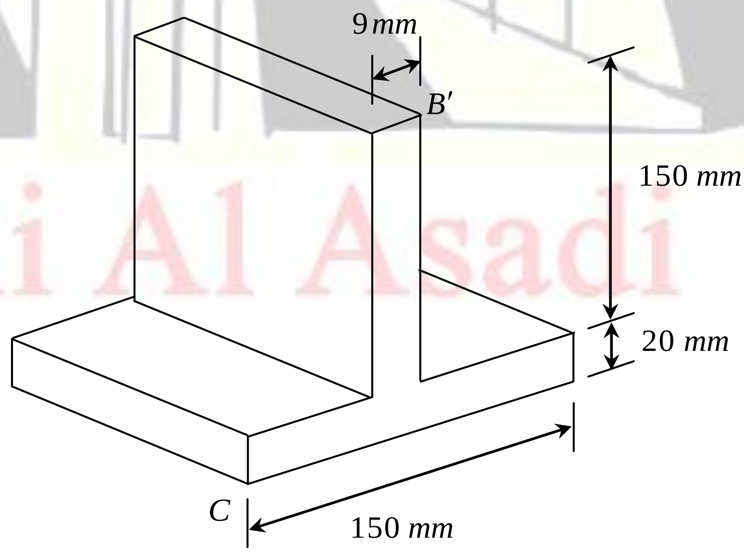
$$n = \frac{E_w}{E_{st}}$$

$$n = \frac{12}{200} = 0.06$$

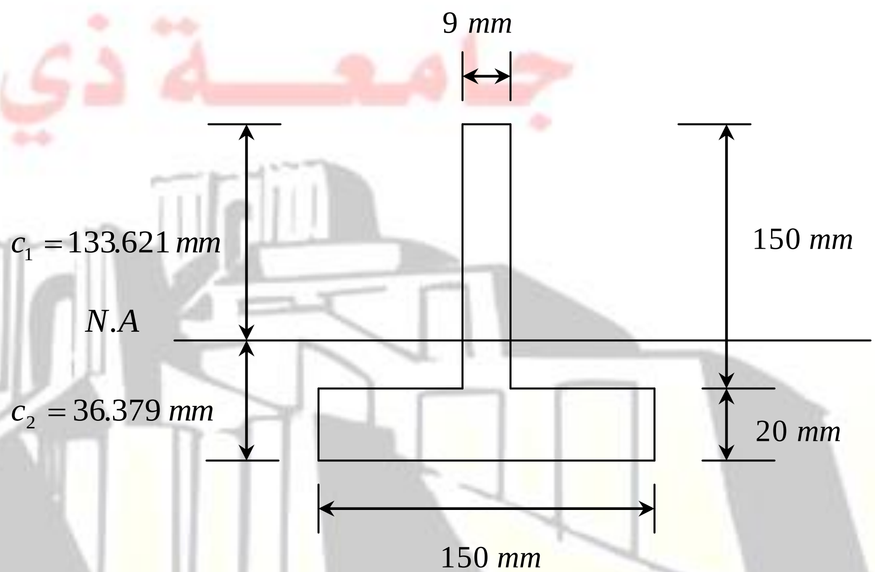
$$b_{st} = n \times b_w$$

$$b_{st} = 0.06 \times 150 = 9\ mm$$





No. of Area	$A(m^2)$	$\bar{y}$ (m)	$\bar{y}A$ (m^3)
1	3000×10^{-6}	10×10^{-3}	30000×10^{-9}
2	1350×10^{-6}	95×10^{-3}	128250×10^{-9}
	$\sum A=4350\times10^{-6}$		$\sum \bar{y}A=158250\times10^{-9}$



$$y_c = \frac{\sum y'A}{\sum A}$$

$$y_c = \frac{158250\times10^{-9}}{4350\times10^{-3}} = 36.379\times10^{-3}\ m$$
$$= 36.379\ mm$$

$I=I_1+I_2$

$$I_1 = \frac{bh^3}{12} + Ad^2$$

$$I_1 = \frac{150\times10^{-3}\times(20\times10^{-3})^3}{12} + 3000\times10^{-6}\times(26.379\times10^{-3})^2$$

$I_1=2.187554\times10^{-6}\ m^4$

$$I_2 = \frac{bh^3}{12} + Ad^2$$

$$I_2 = \frac{9\times10^{-3}\times(150\times10^{-3})^3}{12} + 1350\times10^{-6}\times(58.621\times10^{-3})^2$$

$I_2=7.170419\times10^{-6}\ m^4$

$I=9.35797\times10^{-6}\ m^4$

$$\sigma_{B'} = -\frac{My}{I}$$

$$\sigma_{B'} = -\frac{2\times10^{3}\times133.621\times10^{-3}}{9.35797\times10^{-6}} = -28.557689\,MPa$$

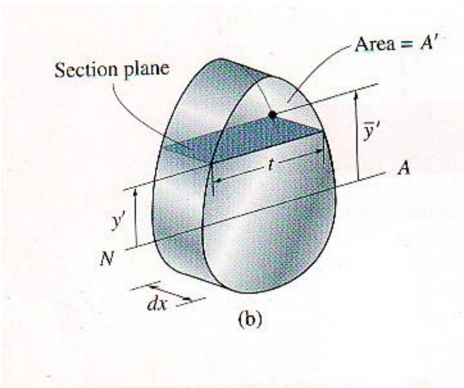
$$\sigma_{B} = n\times\sigma_{B'}$$

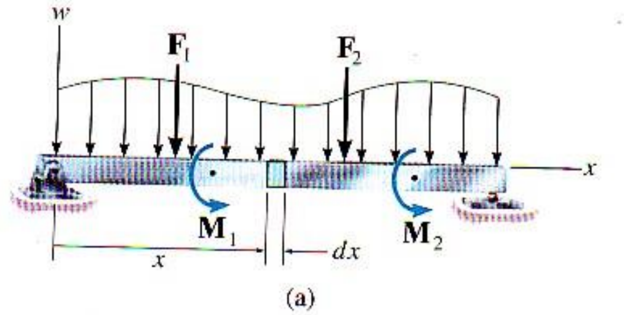
$$\sigma_{B} = 0.06\times(-28.557689) = -1.71346134\,MPa$$

$$\sigma_{B} = \frac{My}{I}$$

$$\sigma_{B} = \frac{2\times10^{3}\times36.379\times10^{-3}}{9.35797\times10^{-6}} = 7.774976\,MPa$$

Shear Stresses in Beams





$$\tau = \frac{V}{It}\int_{A'} y\,dA$$

$$\int_{A'} y\,dA = \bar{y}'A' = Q$$

$$\tau = \frac{VQ}{It}$$

τ :- the shear stress in the member at the point located a distance y' from the neutral axis.

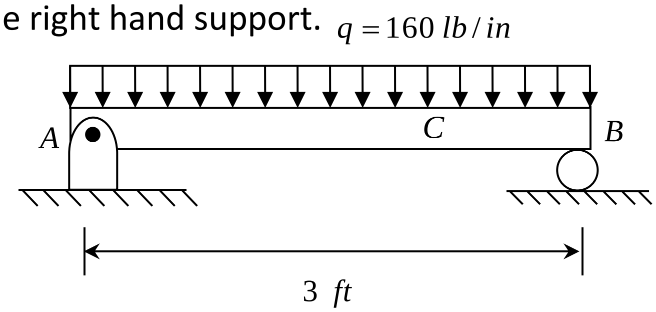
V:-the internal resultant shear force.

I:-the moment of inertia of the entire cross sectional area computed about the neutral axis.

t:-the width of the members cross sectional area, measured at the point where τ is to be determined.

$Q = \bar{y}'A'$, where A' is the top (or bottom) portion of the members cross sectional area, defined from the section where t is measured, and $\bar{y}'$ is the distance to the centroid of A', measured from the neutral axis.

Example 1: A metal beam with span ***L=3 ft*** is simply supported at points ***A*** and ***B***. The uniform load on the beam is ***q=160 lb/***in. The cross section of the beam is rectangular with width ***b=1 in*** and height ***h=4 in***. Determine the normal stress and shear stress at point ***C***, which is located ***1 in*** below the top of the beam and ***8 in*** from the right hand support.



$\sum M_A = 0$

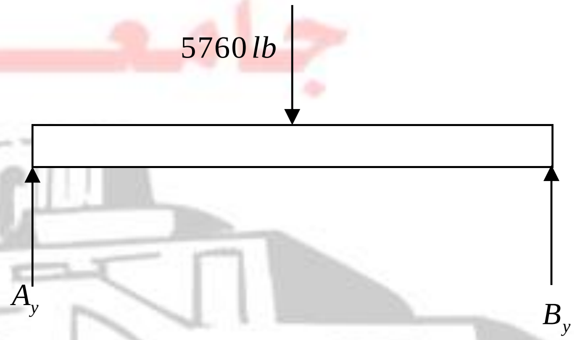
$B_y \times 3 \times 12 - 5760 \times 1.5 \times 12 = 0$

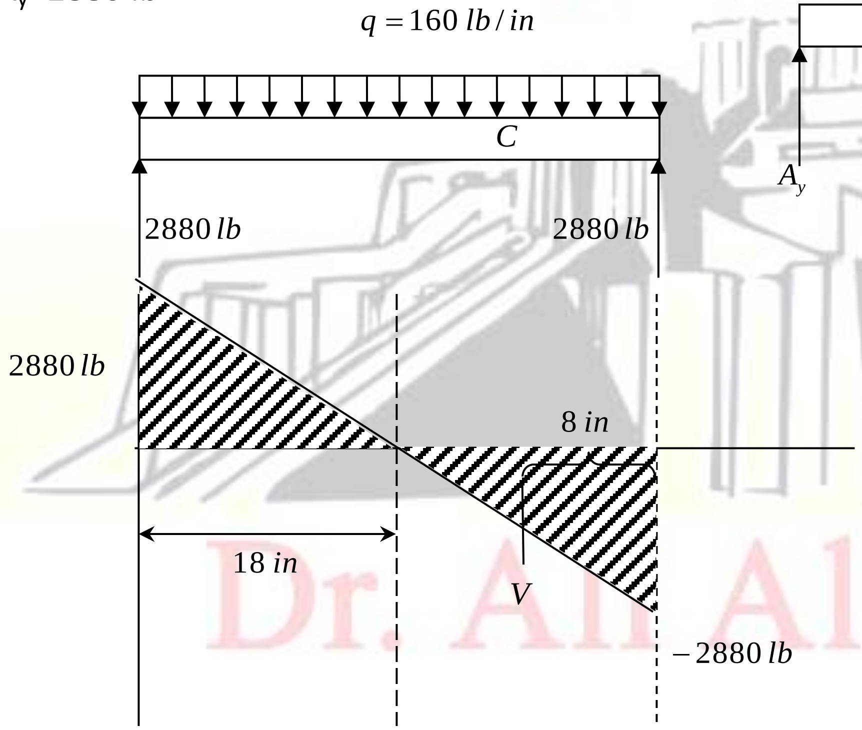
$B_y = 2880$ lb

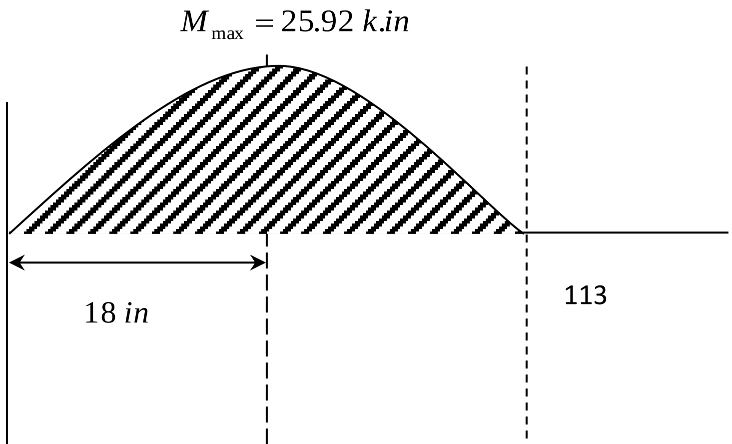
$\sum F_y = 0$

$A_y + 2880 - 5760 = 0$

$A_y = 2880$ lb





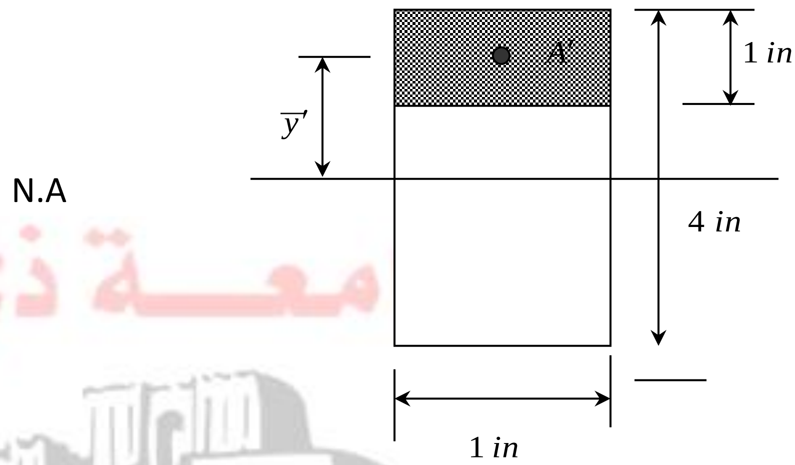


At point C x=28 in from left end

from shear force diagram

$$\frac{V}{10}=\frac{2880}{18}$$

V=1600 lb



$$M=\frac{1}{2}\times 2880\times 18-\frac{1}{2}\times 1600\times 10$$

M=17.92 k.in

$$I=\frac{bh^3}{12}=\frac{1\times(4)^3}{12}=5.3333\,in^4$$

A' =1×1=1 in^2

$\overline{y}'$ =1.5 in

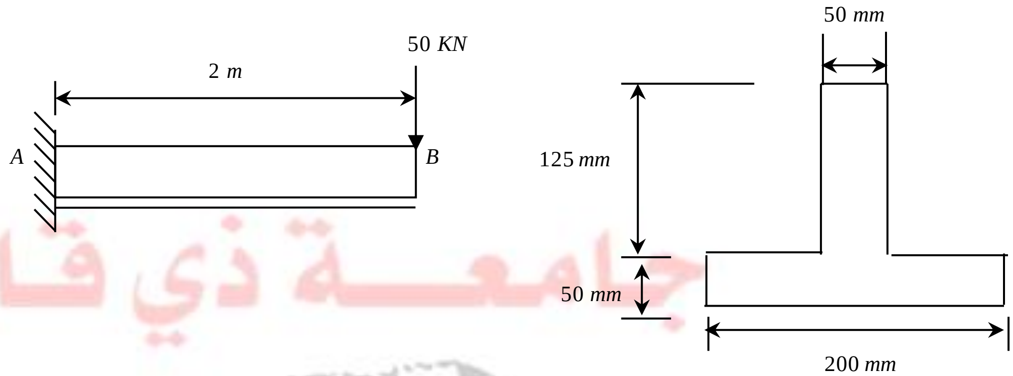
Q$=\overline{y}'A'$ =1.5×1=1.5 in^3

$$\sigma_C=-\frac{My}{I}=-\frac{17920\times 1}{5.3333}=-3.36\ ksi$$

$$\tau=\frac{VQ}{It}$$

$$\tau=\frac{1600\times 1.5}{5.3333\times 1}=450\ psi$$

Example 2: Consider the cantilever beam subjected to the concentrated load shown below. The cross section of the beam is of T-shape. Determine the maximum shearing stress in the beam and also determine the shearing stress ***25 mm*** from the top surface of the beam of a section adjacent to the supporting wall.



$\sum M_A = 0$

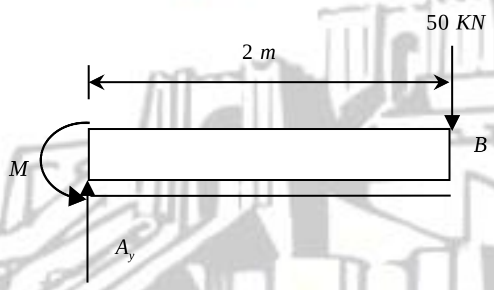
M-50×2=0

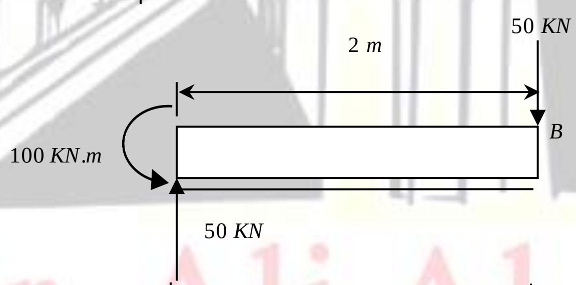
M=100 KN.m

$\sum F_y = 0$

A_y-50=0

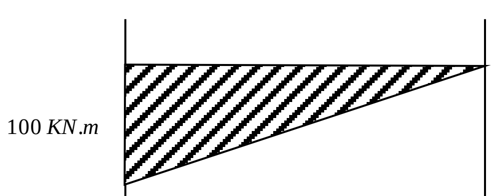
A_y=50 KN





S.F. Diagram

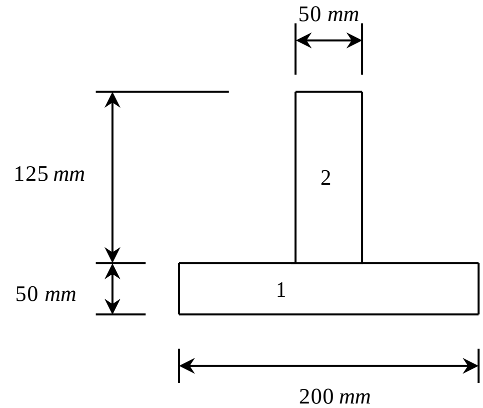
B.M. Diagram



From shear and bending moment diagrams

V=50 KN

M=100 KN.m

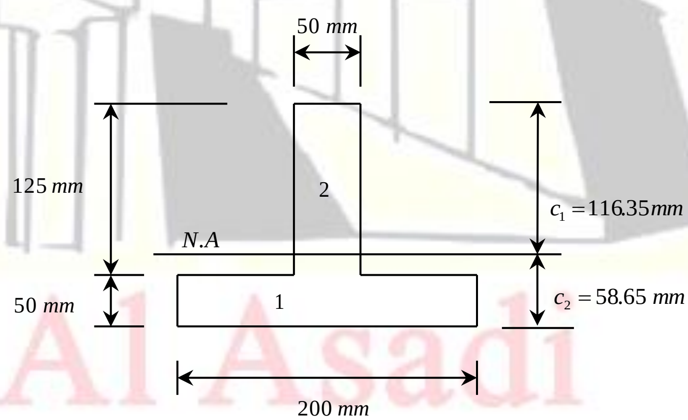


No. of Area	A(m^2)	$\bar{y}$ (m)	$\bar{y}A$ (m^3)
1	10000×10^{-6}	25×10^{-3}	250000×10^{-9}
2	6250×10^{-6}	112.5×10^{-3}	703125×10^{-9}
	$\sum A=16250\times10^{-6}$		$\sum \bar{y}A=953125\times10^{-9}$

$$y_c = \frac{\sum y'A}{\sum A}$$

$$y_c = \frac{953125\times10^{-9}}{16250\times10^{-3}} = 58.65\times10^{-3} \ m$$

$$= 58.65 \ mm$$



$I = I_1 + I_2$

$$I_1 = \frac{bh^3}{12} + Ad^2$$

$$I_1 = \frac{200\times10^{-3}\times(50\times10^{-3})^3}{12} + 10000\times10^{-6}\times(33.65\times10^{-3})^2$$

$I_1 = 13.40655833\times10^{-6} \ m^4$

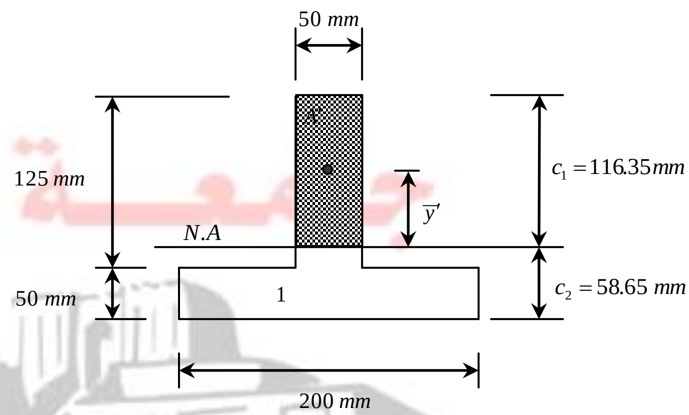
$$I_2 = \frac{bh^3}{12} + Ad^2$$

$$I_2 = \frac{50\times10^{-3}\times(125\times10^{-3})^3}{12} + 6250\times10^{-6}\times(53.85\times10^{-3})^2$$

I_2=26.26191146×10^{-6} m^4

I=39.6684×10^{-6} m^4

Q= $\overline{y}'A'$



A'=50×10^{-3}×116.35×10^{-3}

=0.0058175 m^2

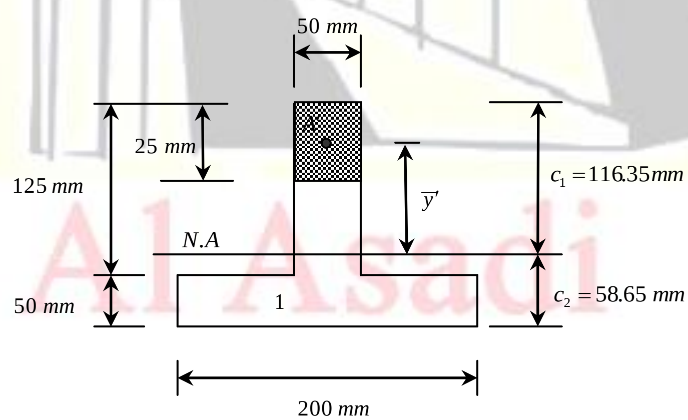
$\overline{y}'$=58.175×10^{-3} m

Q=0.000338433 m^3

$$\tau_{max} = \frac{VQ}{It}$$

$$\tau_{max} = \frac{50\times10^3\times0.000338433}{39.6684\times10^{-6}\times50\times10^{-3}} = 8.5315553MPa$$

Q= $\overline{y}'A'$



A'=50×10^{-3}×25×10^{-3}

=0.00125 m^2

$\overline{y}'$=103.85×10^{-3} m

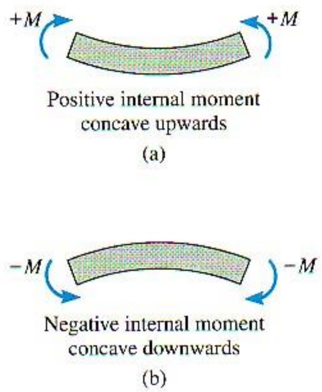
Q=0.000129812 m^3

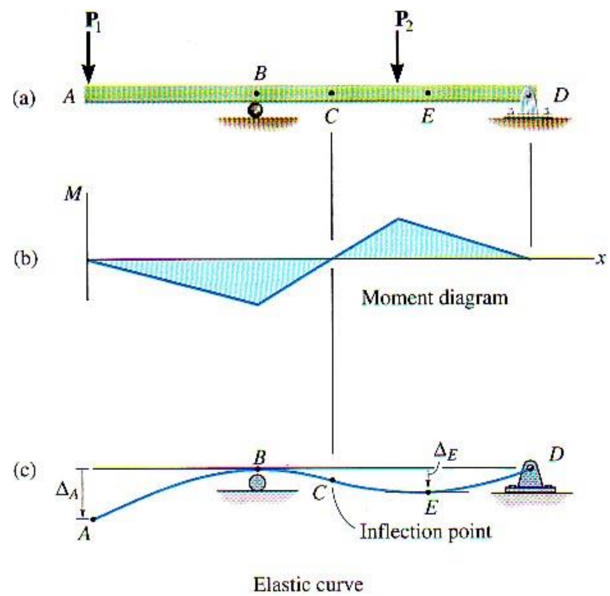
$$\tau = \frac{VQ}{It}$$

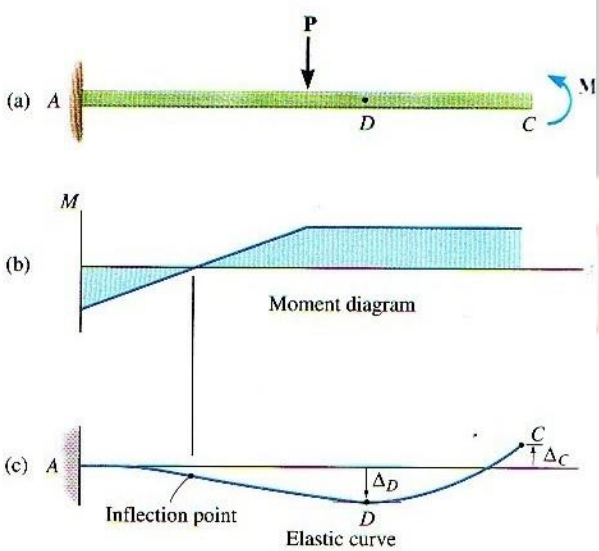
$$\tau = \frac{50\times10^3\times0.000129812}{39.6684\times10^{-6}\times50\times10^{-3}} = 3.272441MPa$$

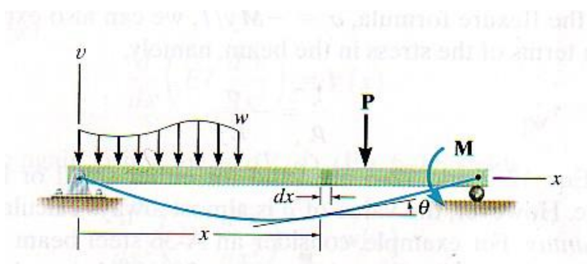
Slop and Deflection in Beams

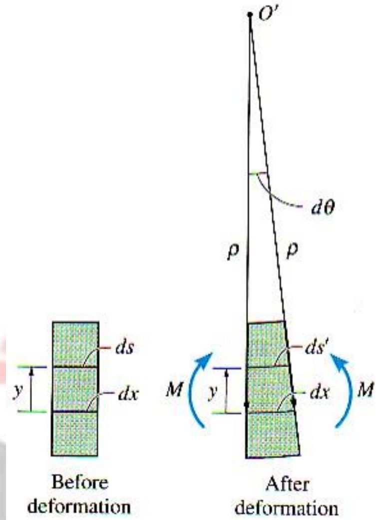
The elastic curve :-the deflection diagram of the longitudinal axis that passes through the centroid of each cross sectional area of the beam.











x-axis extends positive to the right.

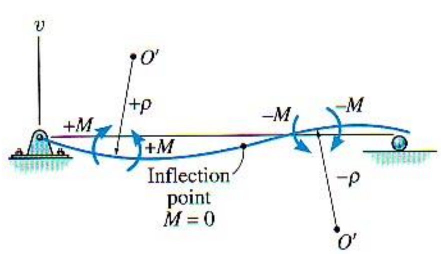
v-axis extends positive upward from the x-axis.

$$\frac{1}{\rho} = -\frac{\varepsilon}{y}$$

$$\varepsilon = \frac{\sigma}{E}$$

$$\sigma = -\frac{My}{I}$$

$$\frac{1}{\rho} = \frac{M}{EI}$$



When M is positive, ρ extends above the beam, i.e. ρ in the positive v direction. When M is negative, ρ extends below the beam, or in the negative v direction.

Integration Method

The elastic curve for a beam can be expressed mathematically as v=f(x)

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{[1+(dv/dx)^2]^{3/2}}$$

$$\frac{M}{EI} = \frac{d^2v/dx^2}{[1+(dv/dx)^2]^{3/2}}$$

The slop of the elastic curve which is determined from dv/dx will be very small, and its square will be negligible compared with unity.