Multistage and Compound Amplifiers

Basic Definitions:

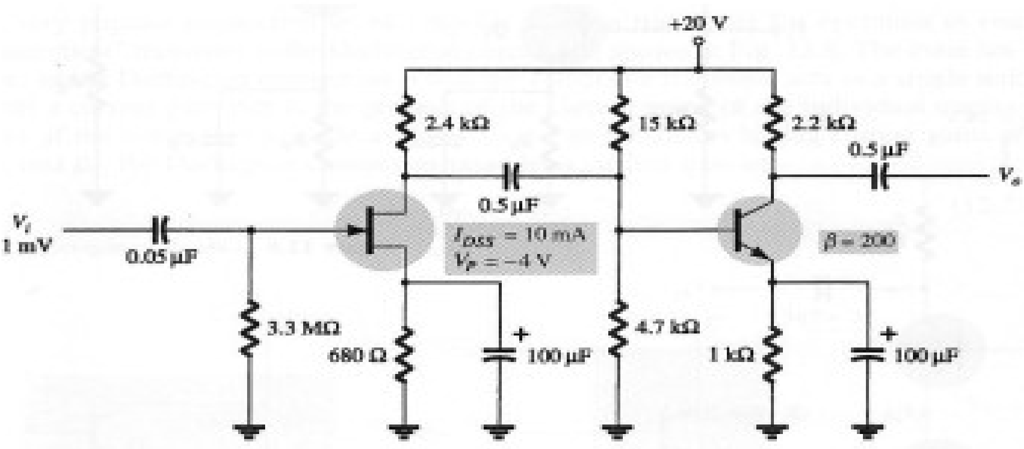
Amplifiers that create voltage, current, and/or power gain through the use of two or more stages (**devices**) are called **multistage** (**compound**) amplifiers The circuitry used to connect the output of one stage of a multistage amplifier to the input of the next stage is called the coupling method. In general, there are three coupling methods: **RC** coupling, **direct** coupling, and **transformer** coupling.

Cascade Amplifiers:

A popular connection of amplifier stages is the cascade connection. Basically, a cascade connection is a series connection with the output of one stage then applied as input to the second stage. A combination of FET and/or BJT stages can be used to provide high gain and high input impedance, as demonstrated by the following example.

Example 19-1:

Calculate the voltage gain, output voltage, input impedance, and output impedance for the cascade amplifier of Fig. 19-1. Calculate the output voltage resulting if a 10-kΩ load is connected to the output.



Solution:

For the FET amplifier (*stage*1),

$$V_{GS} = -1.9\,V\,,\; I_D = 2.8\,mA\,,$$

$$g_m = \frac{2I_{DSS}}{|V_P|}\left(1-\frac{V_{GS}}{V_P}\right) = \frac{2(10m)}{|-4|}\left(1-\frac{-1.9}{-4}\right) = 2.6\,mS.$$

For the BJT amplifier (*stage*2),

$V_B = 4.7\,V,\ V_E = 4.0\,V,\ V_C = 11.2\,V,\ I_E = 4.0\,mA,$

$$r_e = \frac{0.026}{I_E} = \frac{26m}{4m} = 6.5\,\Omega$$

Since $r_i(Stage\,2) = 15k \parallel 4.7k \parallel 200(6.5) = 953.6\,\Omega,$ the gain of *stage*1 (when loaded by *stage*2) is:

$$A_{v1} = -g_m[R_D \parallel r_i(Stage\,2)] = -2.6m(2.4k \parallel 953.6) = -1.77$$

The voltage gain of *stage*2 is:

$$A_{v2} = \frac{-R_C}{r_e} = \frac{-2.2k}{6.5} = -338.46$$

The overall voltage gain is then:

$$A_v = A_{v1}A_{v2} = (-1.77)(-338.46) = 599.1$$

The output voltage is then:

$$V_o = A_v V_i = (599.1)(1\,mV) \approx 0.6\,V.$$

The input impedance of the amplifier is that of *stage*1,

$$Z_i = R_G = 3.3\,M\Omega.$$

While the output impedance of the amplifier is that of *stage*2,

$$Z_o = R_C = 2.2\,k\Omega.$$

If a 10-kΩ load is connected to the output, the resulting voltage across the load is:

$$V_L = \frac{R_L * V_o}{R_L + Z_o} = \frac{(10k)(0.6)}{10k + 2.2k} = 0.49\,V.$$

Frequency Response of Cascade Amplifiers:

When amplifier stages are cascaded to form a multistage amplifier, the dominant frequency response is determined by the response of the individual stages. In general, there are two cases to consider:

Different Cutoff Frequencies:

- When the lower-cutoff frequency, f_L, of each amplifier stage is different, the dominant lower-cutoff frequency, f_L' , equals the cutoff frequency of the stage with highest f_L.
- When the higher-cutoff frequency, f_H, of each amplifier stage is different, the dominant higher-cutoff frequency, f_H' , equals the cutoff frequency of the stage with lowest f_H.
- The overall bandwidth of a multistage amplifier is the difference between the dominant lower-cutoff frequency and the dominant higher-cutoff frequency.

$$BW = f_H' - f_L'$$

Equal Cutoff Frequencies:

- When the lower-cutoff frequencies of each stage in a multistage amplifier are all the same, the dominant lower-cutoff frequency is increased by a factor of $1/\sqrt{2^{1/n}-1}$ as shown by the following formula:

$$f_L' = \frac{f_L}{\sqrt{2^{1/n}-1}}$$

n: the number of stages in the multistage amplifier.

- When the higher-cutoff frequencies of each stage are the same, the dominant higher cutoff frequency is reduced by a factor of $\sqrt{2^{1/n}-1}$ as shown by the following formula:

$$f_H' = f_H\sqrt{2^{1/n}-1}$$

Example 19-2:

Fig. 19-2 shows an amplifier consisting of a common-emitter stage driving an emitter follower (a common-collector) stage. The transistors have the following parameter values: Q_1: $r_{e1} = 15\ \Omega$, $\beta_1 = 180$, $r_{o1} \approx \infty$, and Q_2: $r_{e2} = 25\ \Omega$, $\beta_2 = 100$, $r_{o2} \approx \infty$.
Find $A_{vs} = V_o/V_s$, and f_L.

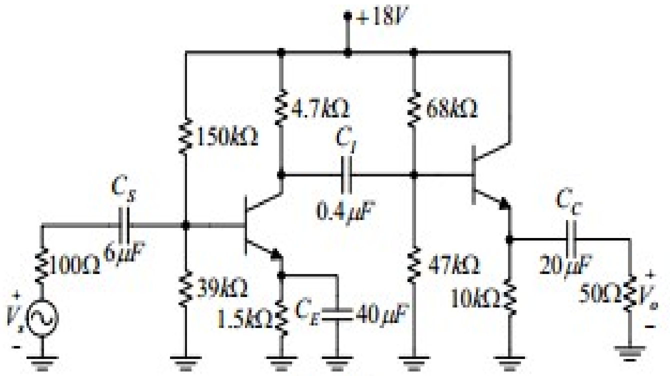


Fig. 19-2

Solution:

$$r' = R_E \parallel R_L = 10k \parallel 50 \approx 50\,\Omega,$$

$$r_i(Stage\,2) = R_1 \parallel R_2 \parallel \beta_2(r_{e2} + r') = 68 \parallel 47k \parallel (100)(25+50) = 5.9\,k\Omega,$$

$$A_{v1} = \frac{-R_C \parallel r_i(Stage\,2)}{r_{e1}} = \frac{-4.7k \parallel 5.9k}{15} = -174.4$$

$$r'_s = \frac{R_1 \parallel R_2 \parallel R_C}{\beta_2} = \frac{68k \parallel 47k \parallel 4.7k}{100} \approx 40\,\Omega,$$

$$A_{v1} = \frac{r'}{r'_s + r_{e2} + r'} = \frac{50}{40+25+50} = 0.435$$

$$r_i(Stage\,1) = R_1 \parallel R_2 \parallel \beta_1 r_{e1} = 150k \parallel 39k \parallel (180)(15) = 2.48\,k\Omega,$$

$$A_{vs} = \frac{r_i(Stage\,1)}{r_i(Stage\,1) + R_s} A_{v1}.\,A_{v2} = \frac{2.48k}{2.48k+100}(-174.4)(0.435) = -72.9$$

$$f_{L_s} = \frac{1}{2\pi(R_s + r_i(Stage\,1))C_S} = \frac{1}{2\pi(100+2.48k)6\mu} = 10.3\,HZ$$

$$R_{e1} = R_E \parallel \left(r_{e1} + \frac{R_1 \parallel R_2 \parallel R_s}{\beta_1}\right) = 1.5k \parallel \left(15 + \frac{150k \parallel 39k \parallel 100}{180}\right) \approx 15\,\Omega,$$

$$f_{L_E} = \frac{1}{2\pi R_{e1} C_1} = \frac{1}{2\pi (15) 40\,\mu} = 265.3\ Hz.$$

$$f_{L_1} = \frac{1}{2\pi [R_C + r_i(Stage\,2)] C_1} = \frac{1}{2\pi (4.7 + 5.9\,kk) 0.4\,\mu} = 37.5\ HZ$$

$$R_{e2} = R_E \parallel (r_{e2} + r'_s) = 10k \parallel (25 + 40) \approx 65\,\Omega,$$

$R_L + R_{e2}$

C

C

2π

$f_{L_C} = \frac{1}{\ }$

$$f_{L_C} = Max.(f_{L_s}, f_{L_E}, f_{L_1}, f_{L_C}) = 265.3\ Hz.$$

Cascode Amplifiers:

A cascode connection has one transistor one top of (in series with) another. Fig. 19-3a shows a cascode configuration with common-emitter (CE) stage feeding a commonbase (CB) stage. This arrangement is designed to provide a high input impedance with low voltage gain to insure that the input miller capacitance is at a minimum with the CB stage providing good high-frequency operation. A practical BJT version of a cascode amplifier is provided in Fig. 19-3b.

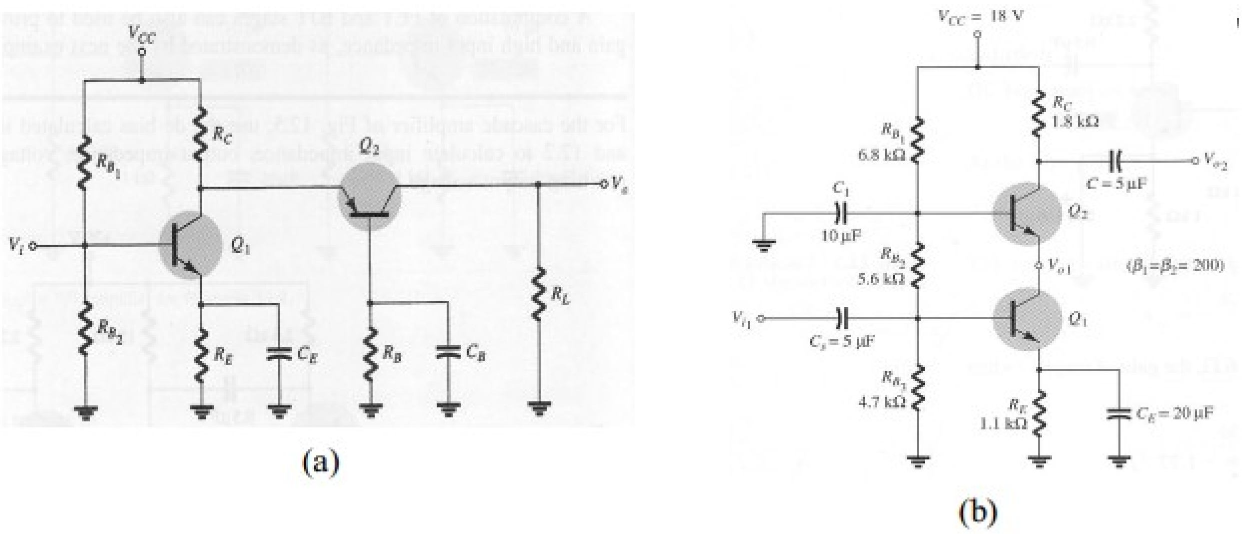


Fig. 19-3

Example 19-3:

Calculate the voltage gain for the cascode amplifier of Fig. 19-3b.

Solution:

From dc analysis,

$$V_{B1} = 4.9\,V\,,V_{B1} = 10.8\,V\,,\circ\,I_{E1} \approx I_{C1} = I_{E2} \approx I_{C2} = 3.8\,mA\,.$$

The dynamic resistance of each transistor is then:

$$r_{e1} \approx r_{e2} = \frac{0.026}{I_E} = \frac{26\,m}{3.8\,m} = 6.8\,\Omega\,.$$

The voltage gain of *stage*1 (common-emitter) is approximately:.

$$A_{v1} = \frac{-r_{e2}}{r_{e1}} = -1$$

The voltage gain of *stage*2 (common-base) is:.

$$A_{v2} = \frac{R_C}{r_{e1}} = \frac{1.8\,k}{6.8} = 265\,.$$

Resulting in an overall cascode amplifier gain of.

$$A_v = A_{v1} A_{v2} = (-1)(265) = -265.$$

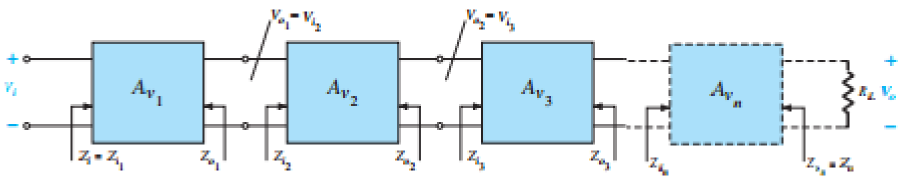
Cascaded Systems

The two-port systems approach is particularly useful for cascaded systems where:

$$A_{VT} = A_{V1} \cdot A_{V2} \cdot A_{V3} \cdot \ldots$$

And the total current gain is give by:

$$A_{iT} = -A_{vT} \frac{Z_{i1}}{R_L}$$



$$A_{iT} = A_{i1} * A_{i2} * A_{i3} * \ldots . A_{in}$$

$$(A_{vT}) = -(A_{iT}) \cdot \left(\frac{Z_L}{Z_{i1}}\right)$$

$$(A_{pT}) = (A_{vT}) \cdot (A_{iT})$$ overall power gain of the system.

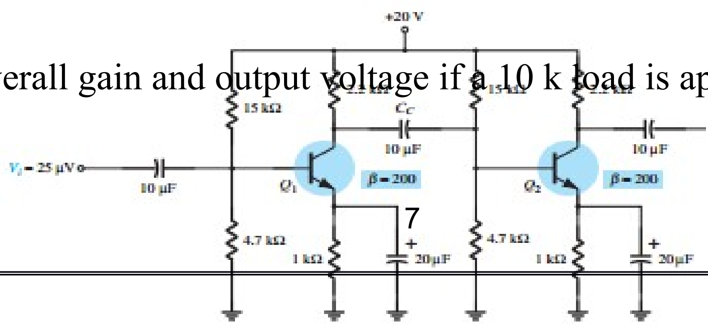
RC-coupled BJT amplifiers:

The name is derived from the capacitive coupling capacitor *Cc* and the fact that the load on the first stage is an *RC* combination. The input impedance of the second stage acts as a load on the first stage.

Example:

a. Calculate the no-load voltage gain and output voltage of the *RC*-coupled transistor amplifiers.

b. Calculate the overall gain and output voltage if a 10 k load is applied to the second stage.



c. Calculate the input impedance of the first stage and the output impedance of the second stage.

Solution:

$$V_B = 4.7\,V\,,\ V_E = 4\,V\,,\ V_C = 11\,V\,,\ I_E = 4\,mA$$

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{4\,mA} = 6.5\,\Omega$$

$$Z_{i2} = R_1 \parallel R_1 \parallel \beta r_e$$

$$A_{v1} = -\frac{R_c \parallel (R_1 \parallel R_1 \parallel \beta r_e)}{r_e} = -\frac{(2.2\,k) \parallel (15\,k \parallel 4.7\,k \parallel (200)(6.5))}{6.5\,\Omega} = -102.3$$

For the unloaded second stage

$$A_{v2(NL)} = \frac{-R_c}{r_e} = \frac{-2.2\,k}{6.5} = -338.46$$

$$A_{vT(NL)} = A_{v1}A_{v2(NL)} = (-102.3)(-338.46) = 34.6 \times 10^3$$

The output voltage :

$$V_o = A_{vT(NL)} V_i = (34.6 * 10^3) * (25\,\mu V) \cong 865\,mV$$

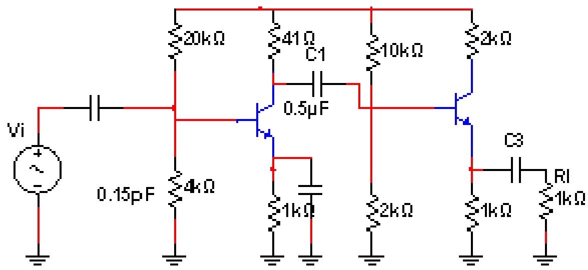
b)

$$Z_{i1} = R_1 \parallel R_2 \parallel \beta r_e = 4.4\,k \parallel 15\,k \parallel (200)(6.5) = 953.6\,\Omega$$

$$Z_{o2} = R_c = 2.2\,k\,\Omega$$

Example:

Q1=Q2: $h_{fe}=\beta=50\,,\,h_{ie}=\beta r_e=0.5\,k\,\Omega$



Z_i:

$Z_{i1}=20\parallel 4\,k\parallel 0.5\,k=0.435\,k\,\Omega$

Z_o:

$Z_o(at\,(V_i=0))=R_{C2}=2\,k\,\Omega$

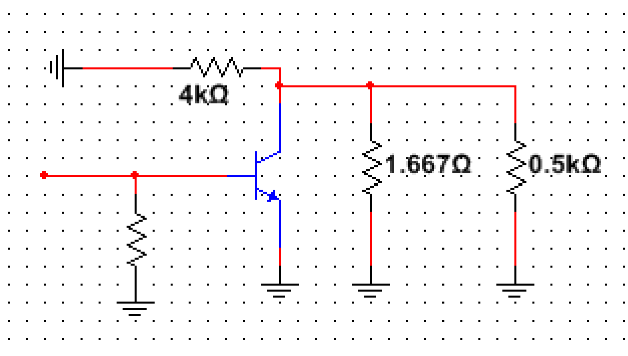
A_i:

$I_{b1}=\dfrac{R' I_{i1}}{R'+h_{ie}}$ $if\ R'=R_1\parallel R_2,\ R=$ {R} rsub {3} || {R} rsub {4}

$I_{b1}=\dfrac{3.333\,k\,I_{i1}}{3.33\,k+0.5\,k}=0.87\,I_{i1}$

$I_{o1}\cong\dfrac{-R_{C1}(I_{c1})}{R_{C1}+0.385\,k}$ current

divider rule



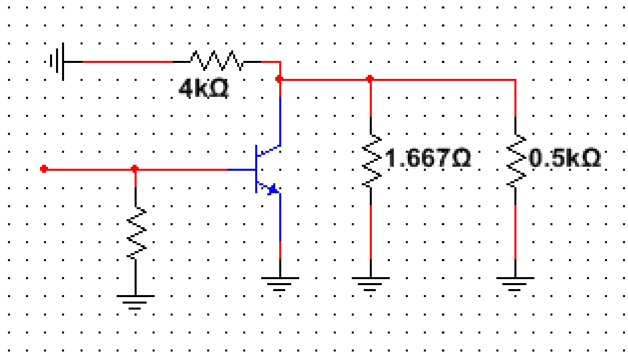
$$= \frac{-4k(I_{c1})}{4k+0.385k} = \frac{-4k\, h_{fe} I_{b1}}{4.385k} = \frac{-4(50)(0.87 I_{i1})}{4.385k}$$

$$A_{i1} = \frac{I_{o1}}{I_{i1}} = -39.68$$

$$I_{b2} = \frac{R\, I_{i2}}{R + h_{ie}} = \frac{1,667k(I_{i2})}{1.667k + 0.5k} = 0.769 I_{i2}$$

$$I_{C2} = h_{fe} I_{b2} = 50\,?(0.769 I_{i2}) = 38.45 I_{i2}$$

I_{o2}



$$2k\frac{(h_{fe} I_{b2})}{2k+1k} = \frac{2k(38.45 I_{i2})}{3k} = 25.63 I_{i2}$$

$$\frac{-R_{C2}(I_{c2})}{R_{C2}+R_L} =$$

$$A_{i2} = \frac{I_{o2}}{I_{i2}} = 25.63$$

$$A_{iT} = A_{i1} \cdot A_{i2} = (-39.68)(25.63) \cong 1017.$$

A_V:

$$A_V = \frac{h_{fe} R_L}{h_{ie}} = \frac{-R_L}{r_e},\ A_{V1} = \frac{-50(0.3509k)}{0.5k} = -35.09$$

$$A_{V2} = \frac{-50(R_{C2}) \parallel R_L}{0.5k} = \frac{-50(0.667k)}{0.5k} = -66.7$$

Note:

$$R_{L1} = R_{C1} \parallel R_3 \parallel R_4 \parallel h_{ie} = 0.3509k$$

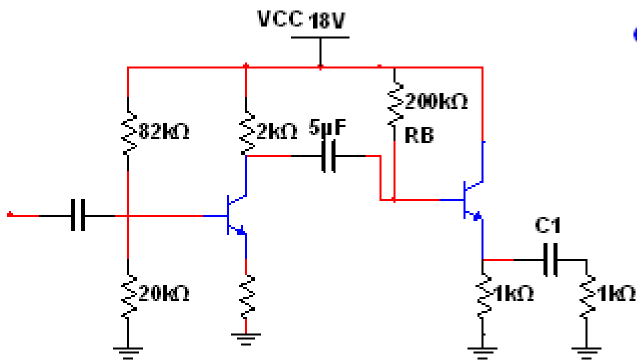
$$A_{VT} = A_{V1} \cdot A_{V2} = (-35.09)(-66.7) \cong 2340.50$$

Or

$$(A_{vT}) = (A_{iT}) \cdot \left(\frac{Z_L}{Z_{i1}}\right) = \frac{(1017)(1k)}{0.435} = 2337.93$$

Example:

Calculate the input and output impedance A_V, A_i of the two stage amplifier.



$Q1: \beta = 100, r_e = 10.74\,\Omega$

$Q2: \beta = 100, r_e = 4.51\,\Omega$

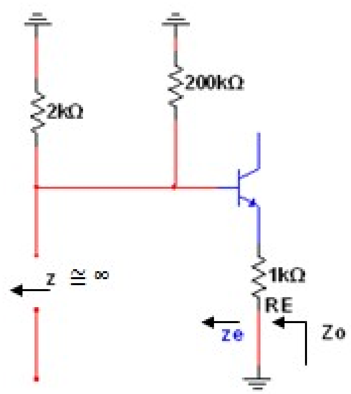
Z_i:

$$Z_i = R_1 // R_2 // \beta r_e = 82k // 20k // 1.074k\,\Omega \cong \beta r_e = 1.074k\,\Omega$$

Z_o:

$$Z_e = \left(\frac{R_s}{\beta}\right) + r_e, \; R_s = 2k \parallel 200k \cong 2k$$

$$Z_e = \left(\frac{2k}{\beta}\right) + r_e = 20 + 4.51 = 24.51\,\Omega$$



A_V:

$$R_C \parallel R_B \parallel \beta(R_{E2} \parallel Z_L)$$

$$A_V = \frac{-R_L}{r_{e1}} =$$

$2k \parallel 200k \parallel 100(1k \parallel 1k)$

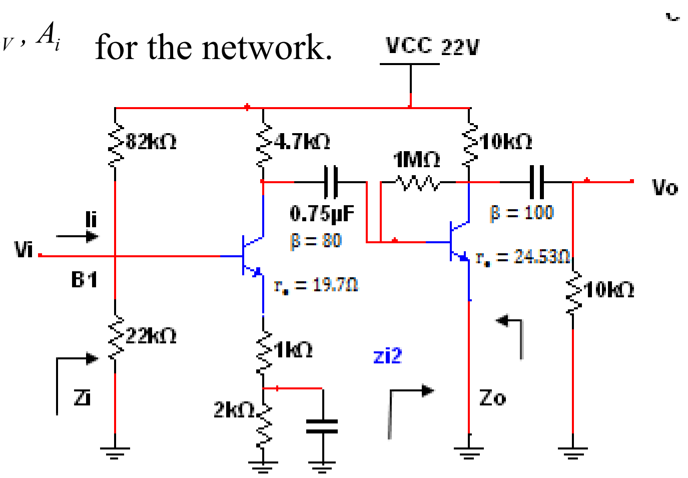
$V_{be2} \cong 0V, V_{be2} \cong V_O$ with $A_V = \frac{V_{O2}}{V_O 1} = 1$

$A_{VT} = A_{V1} \cdot A_{V2} = (-186.22)(1) = -186.22$

$A_{iT} = |A_{vT}| \frac{Z_{i1}}{Z_L} = 186.22 \frac{1.074k}{1k} \cong 200$

Example:

Determine Z_i, Z_o, A_V, A_i for the network.



$R' = R_{B2} \parallel \beta(R_{E1} \parallel R_{E2}) = 22k \parallel 80(3k) = 22 \parallel 240k \cong 22k$

$V_{B1} = \frac{R' * V_{CC}}{R' + R_{B1}} \cong \frac{R_{B2} V_{CC}}{R_{B2} + R_{B1}} = \frac{22k(22)}{22k + 8.2k} = 4.65V$

$V_{E1} = V_{B1} - V_{BE} = 4.65 - 0.7 = 3.95V$

$I_{E1} = \frac{V_{E!}}{R_{E1} + R_{E2}} = \frac{3.95}{3k} = 1.32mA$

$$r_{e1} = \frac{26\,mV}{I_{E1}} = \frac{26\,mV}{1.32\,m} = 19.7\,\Omega$$

$$Z_i = R_{B1} \parallel R_{B2} \parallel \beta R_{E1} = 82k \parallel 22k \parallel 80k = 14.26\,k\Omega$$

Z_o:

$$Z_o \,|\, at\,(V_i = 0) = R_{C2} = 2\,k\Omega$$

A_V:

$$V_{b1} = V_i \qquad A_{V1} = \frac{-R_L}{R_{E1} + r_{e1}} = \frac{-(R_C \parallel Z_{i2})}{R_{E1} + r_{e1}}$$

$$Z_{i2} = \frac{R_F}{|A_{V2}|} \parallel \beta r_e,\; Z_{i2} = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}}$$

$$A_{V2} = \frac{-R_L}{r_{e2}} = \frac{-R_{C2} \parallel Z_L}{r_{e2}} = \frac{-5k}{24.53} = -203.83$$

$$Z_{i2} = \frac{10^6}{203.83} \parallel 100(24.53) = 4.906 \parallel 2.453k = 1.6353\,k\Omega$$

$$A_{V1} = \frac{-(94.7k) \parallel 1.635k}{1k + 0.0197k} = \frac{-1.213}{1.0197} = -1.19$$

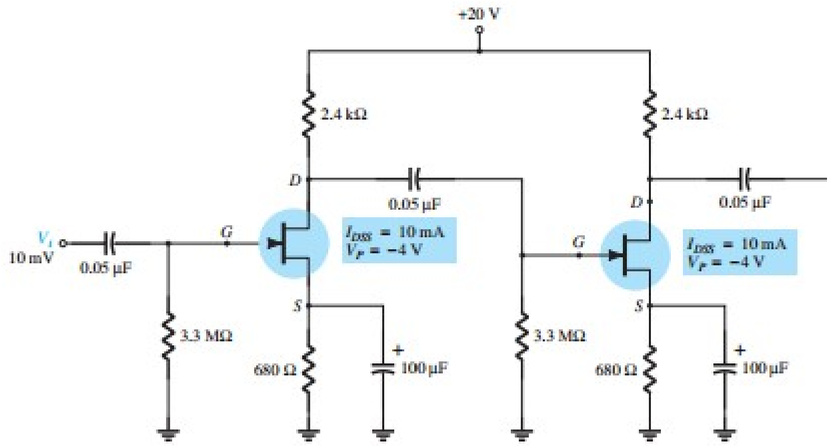
$$A_{VT} = A_{V1} \cdot A_{V2} = (-1.19)(-203.83) \approx 242.56$$

$$A_i : A_{iT} = |A_{VT}| \cdot \frac{|Z_{i1}|}{|Z_L|} = \frac{(242.56)(14.26k)}{10k} \approx 345.81$$

$$A_p = |A_{iT}| \cdot |A_{VT}| = (345.81)\times(242.56) \approx 83.9\times 10^3$$

Example:

Calculate the dc bias, voltage gain, input impedance, output impedance, for the cascade amplifier shown in Fig. and calculate load voltage if a 10kΩ load is connected.



Solution:

$$V_{GSQ} = -1.9V,\ I_{DQ} = 2.8\,mA\ \ g_{mo} = \frac{2I_{DSS}}{|V_P|} = \frac{2(10\,mA)}{|-4V|} = 5\,mS$$

$$g_m = g_{mo}\left(1-\frac{V_{GSQ}}{V_P}\right) = 5\,ms\left(1-\frac{-1.9V}{-4V}\right) = 2.6\,mS$$

$$A_{V2} = -g_m R_D = -(2.6\,mS)(2.4\,k) = -6.24$$

For the first stage:

$$A_{V1} = -g_m R_D \parallel R_{G2} = 2.4\,k \parallel 3.3\,M \cong 2.4\,k$$

$$A_V = A_{V1} A_{V2} = (-6.2)(-6.2) = 38.4$$

$$V_o = A_V V_i = (38.4)(10\,m) = 384\,mV$$

The cascade amplifier input impedance:

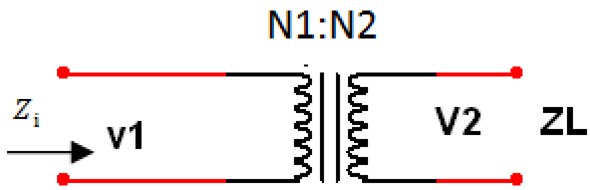
$$Z_i = R_G = 3.3\,M\,\Omega\ (Remake\ Z_i = R_{G1}, Z_o = R_{D2})$$

$Z_o = R_D$ =2.4kΩ (assuming that rd=∞Ω)

The voltage across a 10kΩ load is

$$V_o = \frac{R_L}{Z_o + R_L} V_i = \frac{10\,k}{2.4\,k + 10\,k}(384\,mV) = 310\,mV$$

Transformer-Coupled Transistor Amplifier:



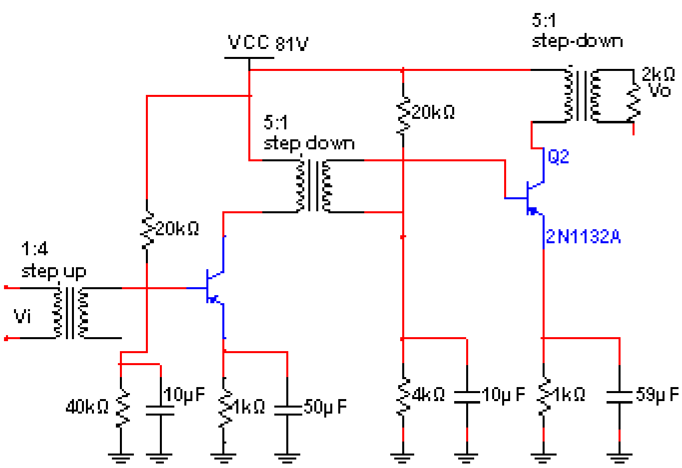
$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a \ (transformation\ ratio)$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

$$Z_i = a^2 Z_L$$

Example:

For the network find A_V

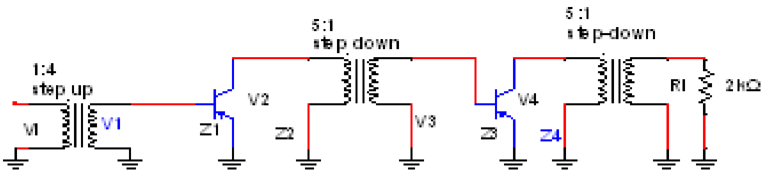


$h_{fe} = \beta = 50$

$h_{ie} = \beta r_e = 2\,k\Omega$

$h_{oe} = 20\,MA/V$

For maximum power transfer the impedance Z_2, Z_4 should be equal to the output impedance :



$Z_o \cong \frac{1}{h_{oe}} = \frac{1}{20\,\mu} = 50\,k\,\Omega \;\; Z_2, Z_4 = Z_0$

$Z_i = a^2 Z_L,\; Z_4 = a^2 R_L = 5^2 * 2\,k = 50 \;\; Z_2 = 50\; where\, are\; Z_1, Z_3 \cong h_{ie} = 2\,k\,\Omega.$

$V_1 = \frac{N_2}{N_1} V_i = 4\,V_i$

$A_{v1} = \frac{V_2}{V_1} = \frac{-h_{fe} Z_L}{h_{ie}} = \frac{-h_{fe}\left(\cong \frac{1}{-h_{oe}} /\!/ Z_2\right)}{h_{ie}} = \frac{-50\,(50\,k /\!/ 50\,k)}{2\,k} = -625\; so\, that$

$V_2 = -625\,V_1 = -625\,(4\,V_i) = -2500\,V_i$

$V_3 = \frac{N_2}{N_1} V_2 = \frac{1}{5} V_2 = \frac{1}{5}(-2500\,V_i) = -500\,V_i$

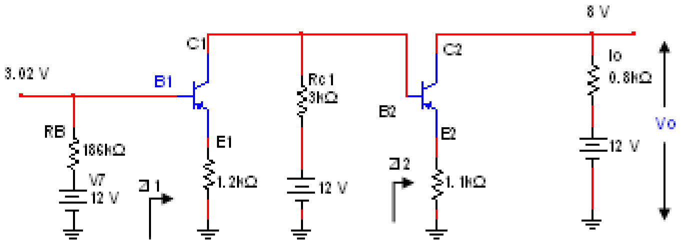
$A_{v2} = \frac{V_4}{V_3} = \frac{-h_{fe} Z_L}{h_{ie}} = \frac{-50\,(50\,k \,\|\, 50\,k)}{2\,k} = -625$

$V_4 = -625\,V_3 = -625\,(-500\,V_i) = 312.50 * 10^3\,V_i$

$V_L = \frac{1}{5} V_4 = \frac{1}{5}(312.50 * 10^3\,V_i)$

$$A_{vT} = \frac{V_L}{V_i} = 62.50 \times 10^3$$

Direct-Coupled Transistor Amplifiers:



$$\beta_1 = 40\,,\ r_{e1} = 13.47\,\Omega\,,\ \beta_2 = 100\,,\ r_{e2} = 5.2\,\Omega$$

Dc-conditions:

$$V_{c2} = 8\,V\,,\ I_o = \frac{12-8}{0.8\,k} = 5\,mA$$

$$I_{c2} \cong I_{E2} = 5\,mA$$

$$V_{E2} = (5\,mA) * (101\,k) = 5.5\,V$$

$$V_{BE} = 0.7\,V$$

$$V_{B2} = V_{c1} = 5.5 + 0.7 = 6.2\,V$$

$$I_{c2} \cong \beta_2 I_{B2}$$

$$I_{B2} = \frac{I_{c2}}{\beta_2} = \frac{5\,mA}{3\,k} = 1.93\,mA$$

$$I_{3k} \gg I_{B2}$$

$$I_{c1} \cong I_{3k} = 1.93\,mA$$

$$V_{E2} = (1.93\,m)(1.2\,k) = 2.32\,V$$

$V_{B1} = V_{E1} + V_{BE1} = 2.32 + 0.7 = 3.02\,V$

Ac-analysis:

$Z_i \cong \beta R_E$ for emitter-follower con.

$Z_{i1} = \beta_1 R_{E1} = 40(1.2\,k) = 48\,k\,\Omega$

$Z_{i2} = \beta_2 R_{E2} = 100(1.1\,k) = 110\,k\,\Omega$

$$A_{v1} = \frac{-R_{L1}}{R_{E1}} = \frac{-R_{c1} /\!/ \beta_2 R_{E2}}{R_{E1}} = \frac{3\,k /\!/ 110\,k}{1.2\,k} \cong \frac{-3\,k}{1.2\,k} = -2.5$$

$$A_{v2} = \frac{-R_{L2}}{R_{E2}} = \frac{-R_{c2}}{R_{E2}} = \frac{-0.8\,k}{1.1\,k} = -0.7273$$

$A_{vT} = A_{v1} A_{v2} = (-2.5)(-0.7273) = 1.818$

$$A_i = (A_v)\left(\frac{Z_{i1}}{Z_L}\right) = \frac{(1.818)(48\,k)}{0.8\,k} = 109.08$$

$A_{PT} = (A_v).(A_i) = (1.819)(109.08) = 198.3$

Example 19-4:

The silicon transistors in Fig. 19-4 have the following parameter values:
Q_1: $\beta_1 = 100$, $r_{o1} \approx \infty$, $C_{bc} = 4$ pF, $C_{be} = 10$ pF and Q_2: $\alpha_2 \approx 1$, $r_{o2} \approx \infty$.
Find I_{C1}, I_{C2}, V_{C1}, V_{C2}, A_{vs}, and f_{Hi1}.

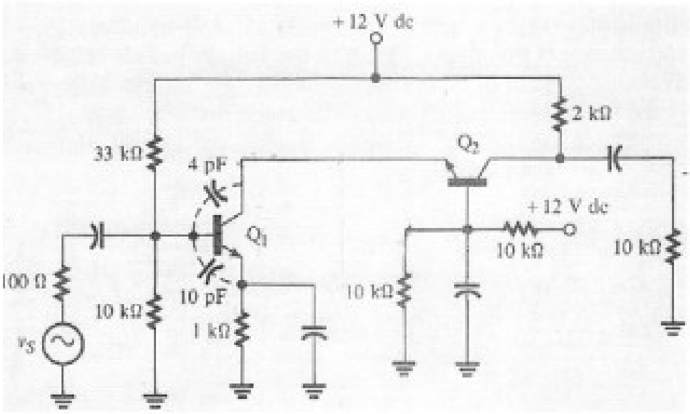


Fig. 19-4

Solution:

$$V_{B1} = \frac{10k(12)}{10k+33k} = 2.8V,\ V_{E1} = V_{B1} - 0.7 = 2.8 - 0.7 = 2.1\,V.$$

$$I_{C1} \approx I_{E1} = \frac{V_{E1}}{R_{E1}} = \frac{2.1}{1k} = 2.1\,mA = I_{E2} \approx I_{C2},\ V_{B2} = \frac{10k(12)}{10k+10k} = 6V,$$

$$V_{C1} = V_{E2} = V_{B2} - 0.7 = 6 - 0.7 = 5.3V,$$

$$V_{C2} = V_{CC} - I_{C2}R_{C2} = 12 - 2.1m(2k) = 7.8V.$$

$$I_{E1} \approx I_{E2} \Rightarrow r_{e1} \approx r_{e2} = \frac{0.026}{I_E} = \frac{26m}{2.1m} = 12.4\,\Omega,$$

$$A_{v1} = \frac{-r_{e2}}{r_{e1}} = -1 \quad , \quad A_{v2} = \frac{R_C \parallel R_L}{r_{e1}} = \frac{2k \parallel 10k}{12.4} = 134.4.$$

$$r_i(Stage\,1) = 33k \parallel 10k \parallel (100)(12.4) = 1.07k\,\Omega,$$

$$A_{vs} = \frac{r_i(Stage\,1)}{r_i(Stage\,1) + R_s} A_{v1} \cdot A_{v2} = \frac{1.07k}{1.07k + 100}(-1)(134.4) = -123.$$

$$R_{TH_{i1}} = r_i(Stage\,1) \parallel R_S = 1.07k \parallel 100 = 91.5\,\Omega .$$

$$C_{i1} = C_{be} + C_{Mi} = C_{be} + C_{be}(1 - A_{v1}) = 10\,p + 4\,p(2) = 18\,pF ,$$

$$f_{Hi1} = \frac{1}{2\pi R_{TH_{i1}} C_{i1}} = \frac{1}{2\pi(91.5)(18\,p)} = 96.7\,MHz .$$

Darlington Amplifiers:

A very popular connection of two BJTs for operation as one "superbeta" transistor is the Darlington connection as shown in Fig. 19-5. The main feature of the Darlington connection is that the composite transistor acts as a single unit with current gain that is the product of the current gains of the individual transistors. That is

$$\beta_D = \beta_1 \beta_2 = \beta^2$$

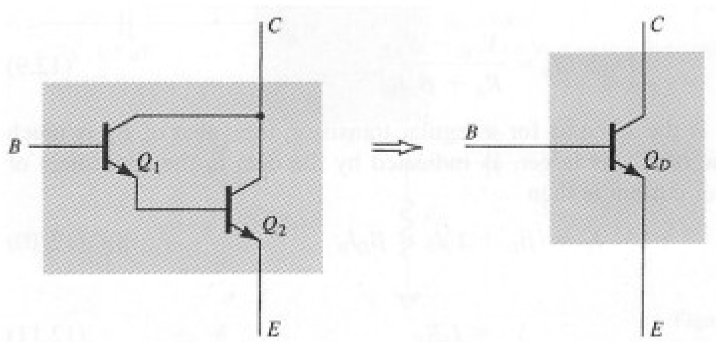


Fig. 19-5

Example 19-5:

For the Darlington emitter-follower circuit shown in Fig. 19-6a, determine I_B, I_C, I_E, V_E, V_B, V_C, Z_b, Z_i, Z_o, A_i, and A_v. Use $\beta_D = 8000$, $V_{BE} = 1.6$ V, and $r_i = 5$ kΩ.

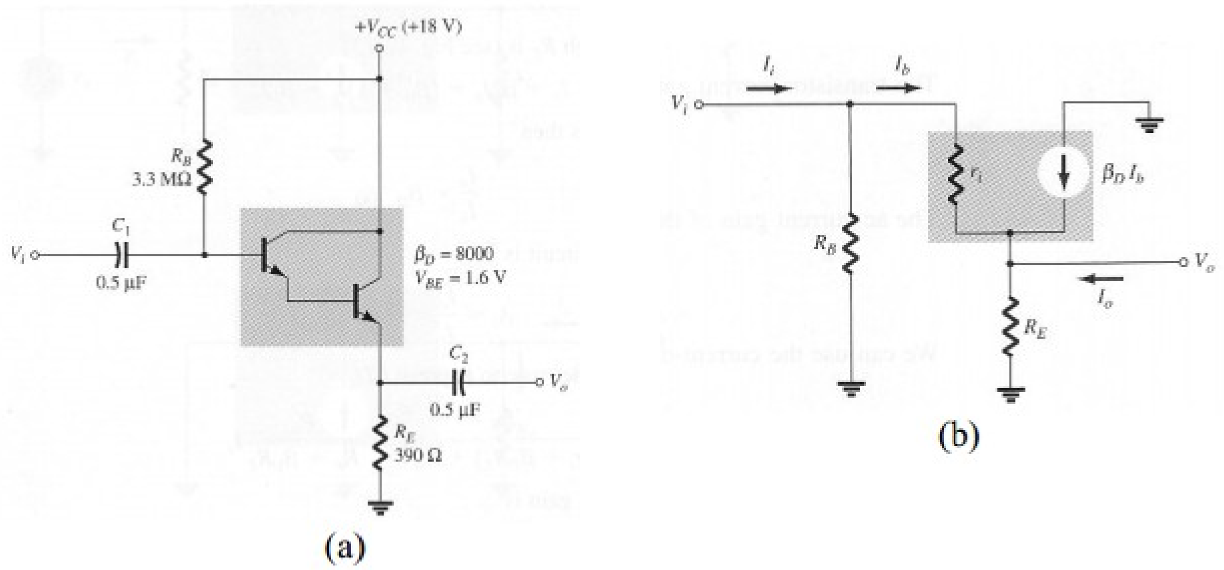


Fig. 19-6

Solution:

From dc analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E} = \frac{18 - 1.6}{3.3\,M + 8000(390)} = 2.56\,\mu A$$

$$I_E = (\beta_D + 1) I_B \approx \beta_D I_B = I_C = 8000(2.56\,\mu) = 20.48\,mA,$$

$$V_E = I_E R_E = 20.48\,m(390) = 8\,V,$$

$$V_B = V_E + V_{BE} = 8 + 1.6 = 9.6\,V,$$

$$V_C = V_{CC} = 18\,V.$$

From ac equivalent circuit of Fig. 19-6b:

$$Z_b = r_i + \beta_D R_E = 5\,k + 8000(390) = 3.125\,M\,\Omega,$$

$$Z_i = R_B \parallel Z_b = 3.3\,M \parallel 3.125\,M = 1.605\,M\,\Omega,$$

$$Z_o = R_E \parallel r_i \parallel \frac{r_i}{\beta_D} \approx \frac{r_i}{\beta_D}$$ [Derive]

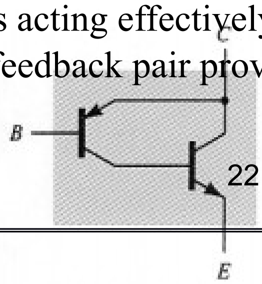
$$= \frac{5\,k}{8000} = 0.625,$$

$$A_i = \frac{\beta_D R_B}{R_B + Z_b} = \frac{8000(390)}{3.3\,M + 3.125} = 4109,$$

$$A_i = \frac{R_E + \beta_D R_E}{r_i + (R_E + \beta_D R_E)} = \frac{390 + (8000)(390)}{5\,k + [390 + (8000)(390)]} = 0.998 \approx 1.$$

Feedback Pair Amplifiers:

The feedback pair connection (see Fig. 19-7) is a two-transistor circuit that operates like the Darlington circuit. Notice that the feedback pair uses a *pnp* transistor driving an *npn* transistor, the two devices acting effectively much like one *pnp* transistor. As with a Darlington connection, the feedback pair provides very high current gain (the product



of the transistor current gains). A typical application uses a Darlington connection and a feedback pair connection to provide complementary transistor operation.

Fig. 19-7

Example 19-6:

For the feedback pair connection amplifier circuit shown in Fig. 19-8:
1. Calculate the dc bias currents and voltages to provide V_o(dc) at one-half the supply voltage.

2. Calculate the ac circuit values of Z_i, Z_o, A_i, and A_v. Assume that r_{i1} = 3 kΩ.

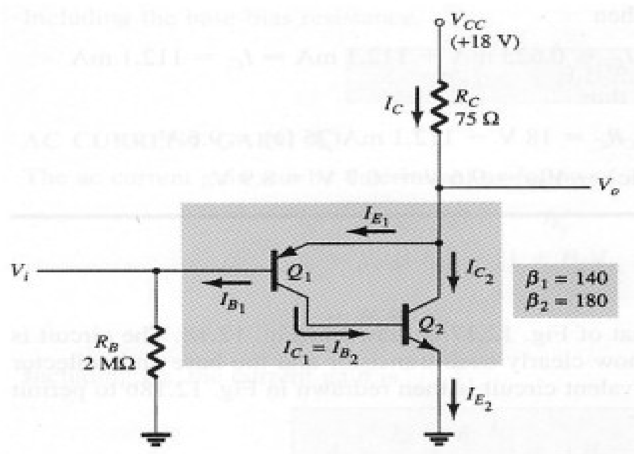


Fig. 19-8

Solution:

From the Q_1 bias-emitter loop, on obtains

$$V_{CC}-I_C R_C-V_{EB1}-I_{B1}R_B=0,\; V_{CC}-\beta_1\beta_2 I_{B1}R_C-V_{EB1}-I_{B1}R_B=0$$

$$I_{B1}=\frac{V_{CC}-V_{EB1}}{R_B-\beta_1\beta_2R_C}=\frac{18-0.7}{2\,M+(140)(180)(75)}=4.45\,\mu A.$$

The base Q_2 current is then

$$I_{B2}=I_{C1}=\beta_1 I_{B1}=140(4.45\,\mu)=0.623\,mA,$$

resulting in a Q_2 collector current of

$$I_{C2}=\beta_2 I_{B2}=180(0.623\,m)=112.1\,mA,$$

and the current through R_C is then

$$I_C = I_{E1} + I_{C2} = 0.623\,m + 112.1\,m \approx I_{C2} = 112.1\,mA.$$

The dc voltage at the output is thus

$$V_o(dc) = V_{CC} - I_C R_C = 18 - 112.1\,m(75) = 9.6\,V,$$

And $$V_i(dc) = V_o(dc) - V_{EB1} = 9.6 - 0.7 = 8.9\,V.$$

From the ac equivalent circuits of Fig. 19-9a and b:

$$Z_i \approx R_B \parallel (r_{i1} + \beta_1 \beta_2 R_C)$$

$$= 2\,M \parallel [3\,k + (140)(180)(75)] = 974\,k\,\Omega.$$

$$Z_o = R_C \parallel r_{i1} \parallel (r_{i1}/\beta_1) \parallel (r_{i1}/\beta_1\beta_2) \approx \frac{r_{i1}}{\beta_1 \beta_2}$$

$$= \frac{3\,k}{(140)(180)} = 0.12\,\Omega.$$

$$A_v = \frac{\beta_1 \beta_2 R_B}{R_B + Z_i}$$

$$= \frac{(140)(180)(2\,M)}{2\,M + 974\,k} = 16950.$$

$$A_v = \frac{\beta_1 \beta_2 R_C}{\beta_1 \beta_2 R_C + r_{i1}}$$

$$= \frac{(140)(180)(75)}{(140)(180)(75) + 3\,k} = 0.9984 \approx 1.$$

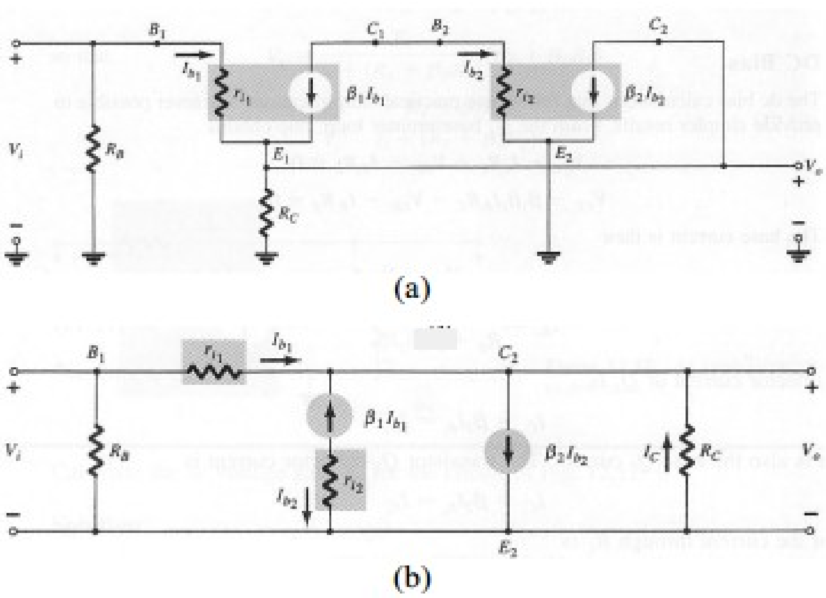


Fig. 19-9

Exercises:

1. The transistors in Fig. 19-10 have the following parameter values: Q_1: $g_{m1} = 4$ mS, $r_{d1} \approx \infty$, and Q_2: $r_{e2} = 30\ \Omega$, $r_{o2} \approx \infty$. Find A_{vs}.

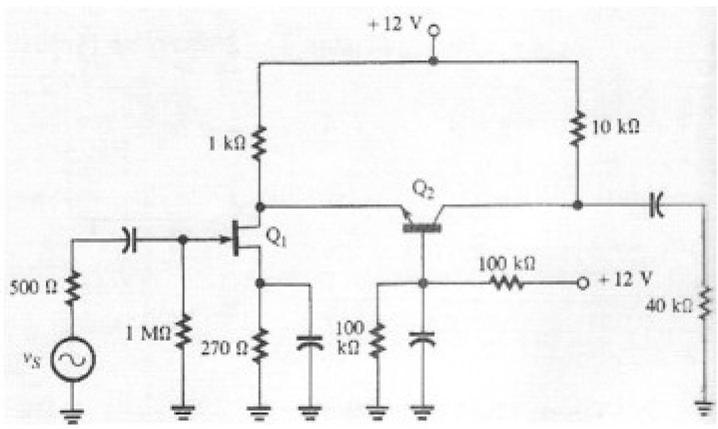


Fig.19-10

2. Fig. 19-11 shows a common-emitter stage driving a Darlington pair connected as an emitter follower. The β-values for the silicon transistors are $\beta_1 = 200$, $\beta_2 = 100$, and $\beta_3 = 100$. Find A_{vs}.

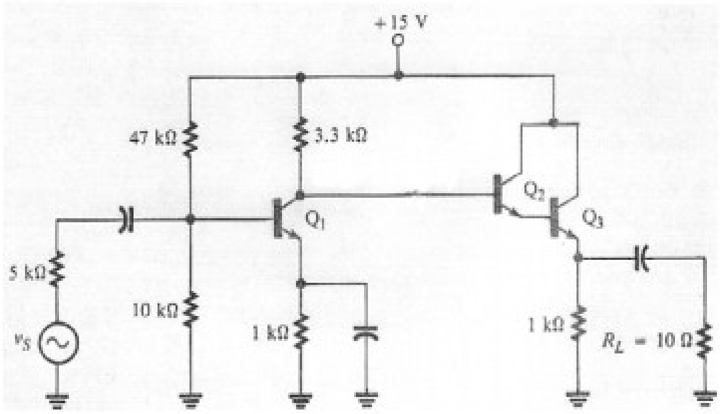


Fig. 19-11