Frequency Response of BJT Amplifiers

Low-Frequency Response of BJT Amplifiers:

For the **high-pass filter** circuit of Fig. 14-1a, the output and the input voltages are related by the voltage-divider rule in the following manner:

$$V_o = \frac{R V_i}{R - jX_C},$$

with the magnitude of V_o determined by

$$|V_o| = \frac{R|V_i|}{\sqrt{R^2 + X_C^2}}.$$

For special case where $X_C = R$,

$$|V_o| = \frac{1}{\sqrt{2}}|V_i|,$$

$$|A_V| = \frac{|V_o|}{|V_i|} = \frac{1}{\sqrt{2}} = 0.707 \quad at\ XC = R$$

In "**deciBel**" (dB):

$$G(dB) = 20\log_{10}|V_o| = 20\log_{10}\frac{1}{\sqrt{2}} = -3\,dB.$$

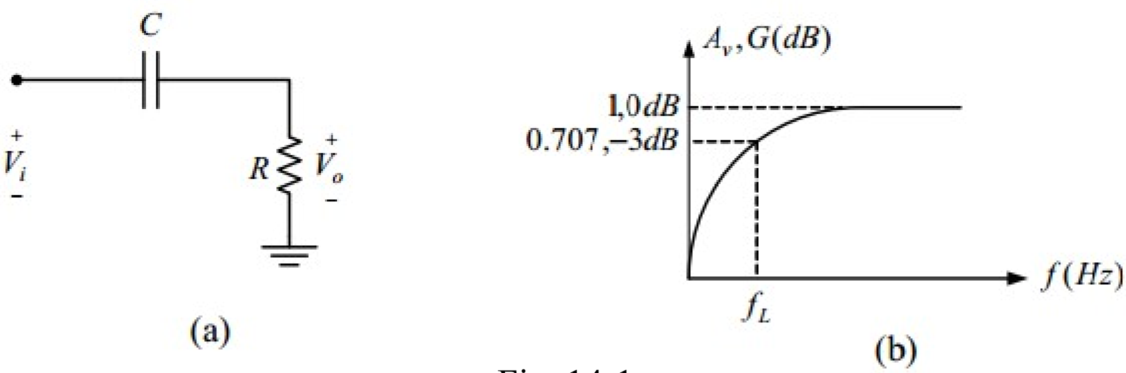

Fig. 14-1

At the frequency of witch $X_C = R$, the output will be 70.7 % of the input (a 3 dB drop in gain, see Fig. 14-1b) for the *RC* circuit. The frequency (f_L) at witch this occurs is determined from:

$$X_C = \frac{1}{\omega c} = \frac{1}{2\pi f C} = R \Rightarrow$$

$$f_L = \frac{1}{2\pi RC} \qquad f_L : the\ low - cutoff\ \ frequency$$

The Capacitors C_S, C_C, and C_E will determine the lower-cutoff frequency (f_L) of the loaded voltage divider BJT bias configuration shown in Fig. 14-2, but the results can be applied to any BJT configuration.

For the amplifier circuit of Fig. 14-2:

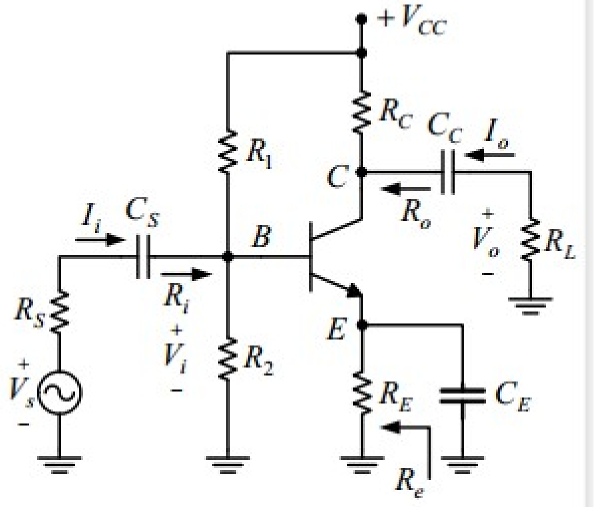

Fig. 14-2

The cutoff-frequency of C_S,

$$f_{L_s} = \frac{1}{2\pi (R_s + R_i) C_S}$$

$$\text{where } R_i = R' \parallel h_{ie},$$

$$R' = R_1 \parallel R_2$$

The cutoff-frequency of C_C,

$$f_{L_c} = \frac{1}{2\pi (R_L + R_o) C_C}$$

$$\text{where } R_o = 1/h_{oe}$$

The cutoff-frequency of C_E,

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

$$\text{where } R_e = R_E \parallel \frac{R'_s + h_{ie}}{h_{fe} + 1},$$

$$R'_s = R_s \parallel R' \quad .$$

The lower-cutoff frequency,

$$f_L = Max.\left(f_{L_s}, f_{L_c}, f_{L_E}\right)$$

Miller's Theorem and Its Dual:

For the circuit of Fig. 14-3a,

$$I_i = \frac{V_i}{Z_i},\ I_1 = \frac{V_i}{R_i},\ I_2 = \frac{V_i - V_o}{Z_F} = \frac{V_i - A_v V_i}{Z_F} = \frac{V_i}{Z_F/(1-A_v)}$$

$$I_i = I_1 + I_2 \Rightarrow \frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{V_i}{Z_F/(1-A_v)} \Rightarrow \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{Z_F/(1-A_v)}.$$

$$when\ Z_F = R_F \Rightarrow \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{R_{Mi}},\ where\ R_{Mi} = \frac{R_F}{1-A_v}.$$

As shown in Fig. 14-3b,

$$when\ Z_F = X_{C_F} \Rightarrow \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_{Mi}}},\ X_{C_{Mi}} = \frac{X_{C_F}}{1-A_v} = \frac{1}{\omega(1-A_v)C_F} \Rightarrow$$

$$C_{Mi} = (1-A_v)C_F$$

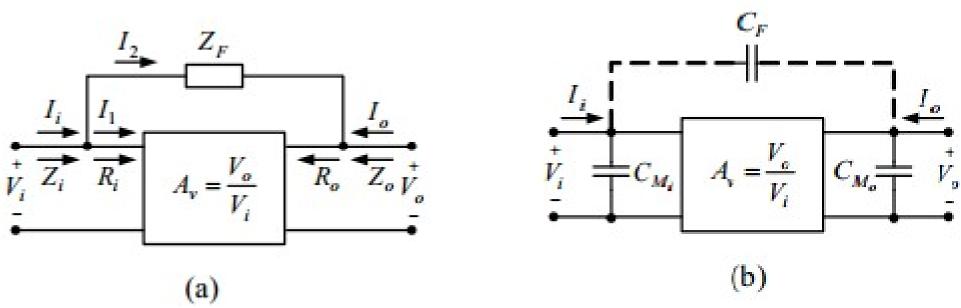

Fig. 14-3

In a similar way

$$\frac{1}{Z_o} = \frac{1}{R_o} + \frac{1}{Z_F/(1-1/A_v)} \Rightarrow R_{Mo} = \frac{R_F}{1-1/A_v},$$

$$\omega\left(1-\frac{1}{A_v}\right)C_F \Longrightarrow$$

$$X_{C_{Mo}} = \frac{X_{C_F}}{1-1/A_v} = \frac{1}{\omega\left(1-\frac{1}{A_v}\right)C_F}$$

$$C_{Mo} = \left(1-\frac{1}{A_v}\right)C_F$$

The above shows us that:

For any **inverting** amplifier (phase shift of 180o between input and output resulting in a negative value for A_v), the input and output capacitance will be increased by a Miller effect capacitance sensitive to the gain of the amplifier and the inter electrode capacitance connected between the input and output terminals of the active device.

High-Frequency Response of BJT Amplifiers:

A frequency response of the **low-pass filter** circuit of Fig. 14-4a is given by Fig. 14-4b, where the high-cutoff frequency is determined from:

$$f_H = \frac{1}{2\pi RC} \qquad f_h: \textit{the high-cutoff frequency}.$$

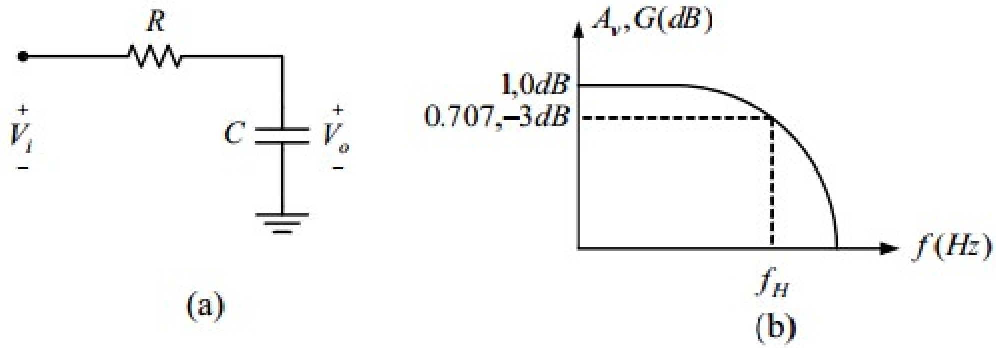

Fig. 14-4

At the high-frequency end, there are two factors that will define the -3 dB point: the circuit capacitance (parasitic and introduced) and the frequency dependence of h_{fe}

Circuit (Capacitances) Parameters:

In high-frequency region the capacitive elements of the importance are the inter-electrode (between terminals) capacitances internal to the active device and the wiring capacitance between leads of the circuit. In Fig. 14-5, the various parasitic capacitances (C_{be}, C_{bc}, and C_{ce}) of the transistor have been included with the wiring capacitances (C_{Wi}, C_{Wo}) introduced during construction.

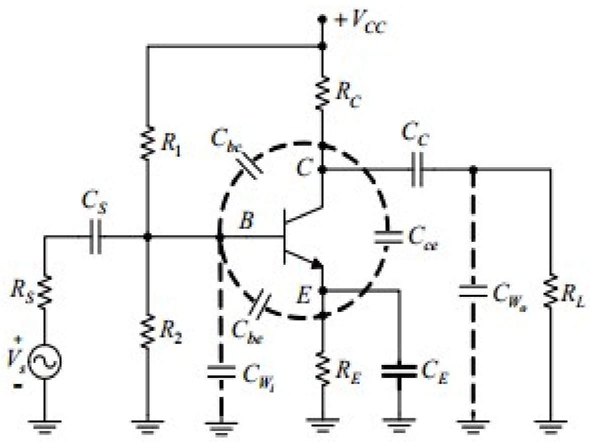

Fig. 14-5

The high-frequency equivalent model for the amplifier circuit of Fig. 14-5 appears in Fig. 14-6. Note the absence of the capacitors C_S, C_C, and C_E, which are all assumed to be in the short circuit state at these frequencies.

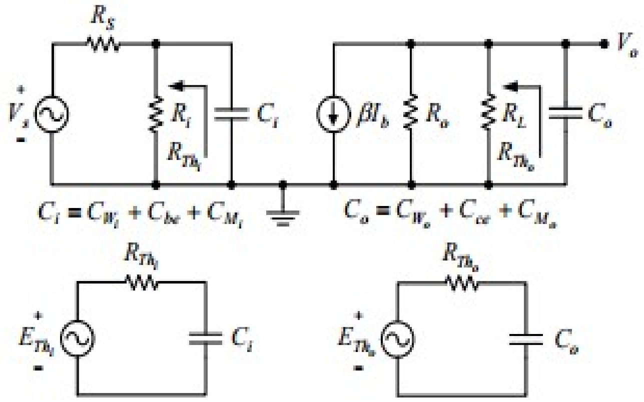

Fig. 14-6

For the circuit of Fig. 14-6:

The input high cutoff frequency,

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}$$

where $R_{Th_i} = R_S \parallel R_i$, $R_i = R_1 \parallel R_2 \parallel h_{ie}$.

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v) C_{bc}.$$

The output high cutoff frequency,

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}$$

where $R_{Th_o} = R_L \parallel R_o$, $R_o = R_C \parallel 1/h_{oe}$.

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = C_{W_o} + C_{ce} + (1 - 1/A_v) C_{bc}.$$

The higher-cutoff frequency,

$$f_H = Min.\left(f_{H_i}, f_{H_o}\right)$$

h_{fe} (β) Variation:

The beta cutoff frequency (f_β) is another important transistor cutoff frequency. The f_β, the frequency where the β of the transistor drop to 0.707 of its low-frequency value, is given by

$$f_B \cong \frac{1}{2\pi\beta r_e (C_{be} + C_{bc})}$$

If the frequency of operation is increased above the f_β of the transistor, the β will continue to decrease. Eventually, we find a frequency where the $\beta = 1$; this frequency is called the f_T of the transistor. The f_T of a transistor is much higher than the f_β. The relation between these two frequencies is

$$f_T \cong \beta . f_B \approx h_{fe} . BW \quad f_T : \text{the gain} - \text{bandwidth product frequency}.$$

Finally, in data sheet, the CB high-frequency parameters rather than CE parameters are often specified for a transistor. The following equation permits a direct conversion for determining f_β if f_α and α are specified.

$$f_B \cong f_\alpha (1 - \alpha)$$

Example 14-1:

For the BJT amplifier circuit shown in Fig. 14-7, with the following parameters:
C_{be} = 36 pF, C_{bc} = 4 pF, C_{ce} = 1 pF, = 6 pF, C = 8 pF, and $r_o = 1/h_{oe} = \infty\ \Omega$. C_{Wi} $_{Wo}$
1. Determine f_L, f_H, BW, f_β, and f_T.
2. Sketch the frequency response.

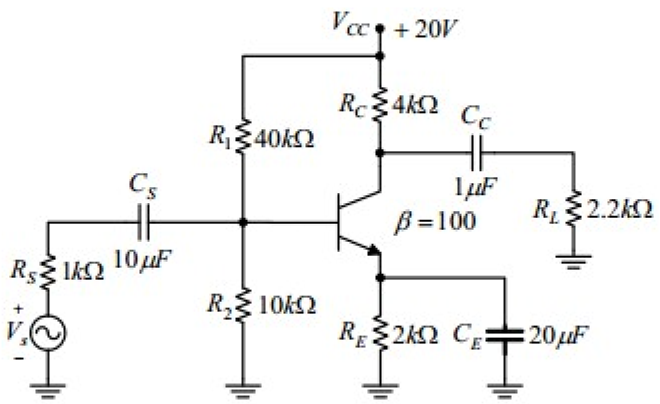

Fig. 14-7

Solution:

Testing: $\beta R_E \geq 10 R_2$, $\quad 100(2k) \geq 10(10k)$, $200\,k\Omega > 100\,k\Omega$ *Stisfied*

$$V_B = \frac{V_{cc} . R_2}{R_1 + R_2} = \frac{20(10k)}{40k + 10k} = 4V, \; I_E = \frac{V_E}{R_E} = \frac{V_B - V_{BE}}{R_E} = \frac{4 - 0.7}{2k} = 1.65\,mA,$$

$$r_e = \frac{26mV}{I_E} = \frac{26mV}{1.65m} = 15.76\,\Omega, \qquad h_{ie} = \beta r_e = 100(15.76) = 1.58\,k\Omega.$$

$$A_{v_{mid}} = \frac{-R_L \parallel R_C}{r_e} = \frac{-2.2k \parallel 4k}{15.76} = -90$$

$$R_i = R_1 \parallel R_2 \parallel h_{ie} = 40k \parallel 10k \parallel 1.58k = 1.32\,k\Omega$$

$$f_{L_s} = \frac{1}{2\pi(R_s + R_i)CS} = \frac{1}{2\pi(1k + 1.32k)(10\,\mu)} = 7\,Hz$$

$$R_o = R_C \parallel 1/h_{oe} = 4\,k\Omega,$$

$$f_{L_c} = \frac{1}{2\pi(R_L + R_o)CC} = \frac{1}{2\pi(2.2k + 4k)(1\,\mu)} = 26\,Hz$$

$$R'_S = R_S \parallel R_1 \parallel R_2 = 1k \parallel 40k \parallel 10k = 0.89\,k\Omega$$

$$R_e = R_E \parallel \left(\frac{R'_S}{\beta} + r_e\right) = 2k \parallel \left(\frac{0.89k}{100} + 15.76\right) = 24.35\,\Omega$$

$$f_{L_E} = \frac{1}{2\pi R_e CE} = \frac{1}{2\pi(24.35)(20\,\mu)} = 327\,Hz$$

The lower-cutoff frequency, $f_L = Max.\{f_{L_s}, f_{L_c}, f_{L_E}\}$

$$\approx Max.\{7, 26, 326\} = 327\,Hz$$

$$R_{Th_i} = R_S \parallel R_i = 1k \parallel 1.32k = 0.57\,k\Omega.$$

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc} = 6P + 36P + (1 + 90)(4P) = 406\,PF$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi (0.57\,k)(406\,P)} = 687.732\,Hz\,.$$

$$R_{Th_o} = R_L \parallel R_o = 2.2\,k \parallel 4\,k = 1.42\,k\Omega\,.$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = C_{W_o} + C_{ce} + \left(1 - \frac{1}{A_v}\right) C_{bc} = 8\,P + 1\,P + \left(1 + \frac{1}{90}\right)(4\,P) = 13\,PF\,.$$

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o} = \frac{1}{2\pi (1.42\,k)(13\,P)} = 8.622\,MHz\,.$$

The higher-cutoff frequency , $f_H = Min.\{f_{H_i}, f_{H_o}\}$

$$\therefore Min(687.732, 8.622\,M) = 687.732\,kHz\,.$$

The bandwidth, $BW = f_H - f_L = 687.732\,k - 327 = 687.405\,kHz\,.$

The beta cutoff frequency, $f_B = \dfrac{1}{2\pi\beta r_e (C_{be} + C_{bc})}$

$$= \frac{1}{2\pi(100)(15.76)(36P + 4P)} = 2.52\,MHz.$$

The gain-bandwidth product, $f_T = \beta f_B = 100(2.52M) = 252\,MHz.$

The frequency response for the low- and high-frequency regions, bandwidth, beta cutoff frequency, and gain-bandwidth product frequency are shown in Fig. 14-8

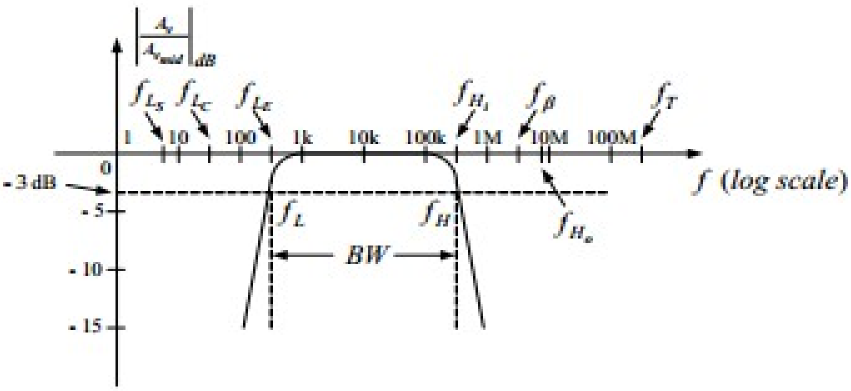

Fig. 14-8

Exercise:

For the BJT amplifier circuit of Fig. 14-9, determine the lower- and higher-cutoff frequencies.

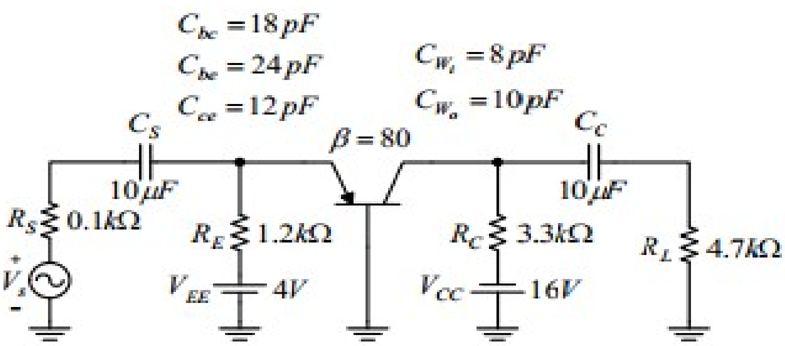

Fig. 14-9