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Electrical and Electronic Engineering Department
.Second year, Electronic ɪ, 2016-2017

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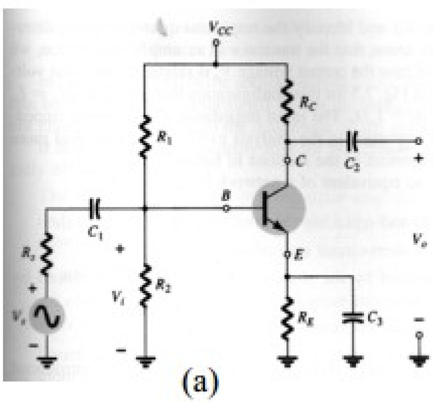
BJT Modeling and AC Equivalent Circuit

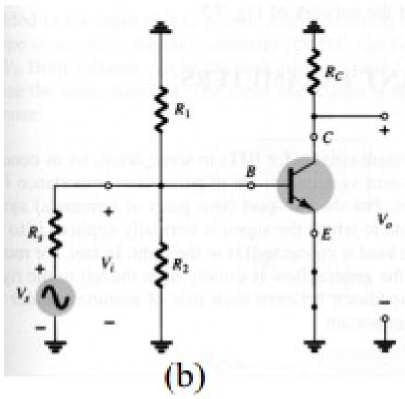
Basic Concepts:

The key to the transistor small-signal analysis is the use of ac equivalent circuits or models. A model is the combination of circuit elements, properly chosen, that best approximates the actual behavior of a semiconductor device (BJT) under specific operating conditions. Once the ac equivalent circuit has been determined, the graphical symbol of the device can be replaced in the schematic by this circuit and the basic methods of ac circuit analysis (mesh analysis, nodal analysis, and Thevenin's theorem) can be applied to determine the response of the circuit. There are two schools of thought in prominence today regarding the equivalent circuit to be substituted for the transistor: **hybrid** and **r_e model**. In summary, the ac equivalent circuit of the BJT amplifier is obtained by (see Fig. 12-1):

1. Setting all dc sources to zero and replacing them by a short-circuit equivalent.
2. Replacing all capacitors by a short-circuit equivalent.
3. Removing all elements bypassed by the short-circuit equivalents introduced by stapes 1 and 2.
4. Redrawing the circuit in a more convenient and logical form.
5. Use the **hybrid** or **r_e** equivalent circuit of the BJT to complete the equivalent circuit of the amplifier.
6. Finally, the following parameters are determined for the amplifier:

a. Input impedance (Z_i). b. Output impedance (Z_o). c. Voltage gain (A_v).
d. Current gain (A_i). e. Phase relationship (θ).


(a)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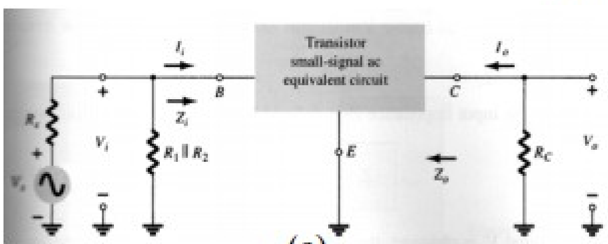


(c)

Fig. 12-1

The Hybrid (h-parameters) Equivalent Model:

For the general hybrid two-port system of Fig. 12-2:

$V_I = h_{11} I_i + h_{12} V_o$ [12.1a]

$I_o = h_{21} I_i + h_{22} V_o$ [12.1b]

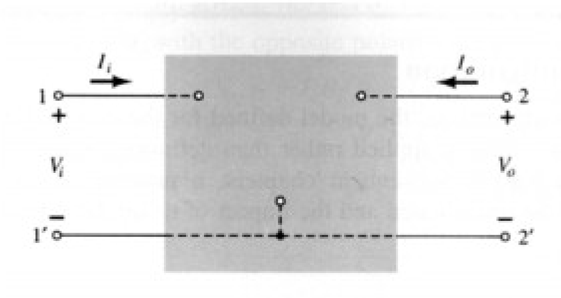


Fig. 12-2

where

$h_{11} = \left(\frac{V_I}{I_i}\right) = h_i\,(\Omega)$, at $V_o = 0$ short-circuit input impedance parameter.

$h_{12} = \left(\frac{V_I}{V_o}\right) = h_r\,(unitless)$, at $I_o = 0$ open-circuit reverse transfer voltage ratio parameter.

$h_{21} = \left(\frac{I_o}{I_i}\right) = h_f\,(unitless)$, at $V_o = 0$ short-circuit forward transfer current ratio parameter.

$h_{22} = \left(\frac{I_o}{V_o}\right) = h_o\,(S)$, at $I_i = 0$ open-circuit output admittance parameter.

From the BJT hybrid equivalent circuit of Fig. 12-3, Eqs. [12.1a] and [12.1b] becomes:

$V_I = h_i I_i + h_r V_o$ [12.2a]

$I_o = h_f I_i + h_{2o} V_o$ [12.1b]

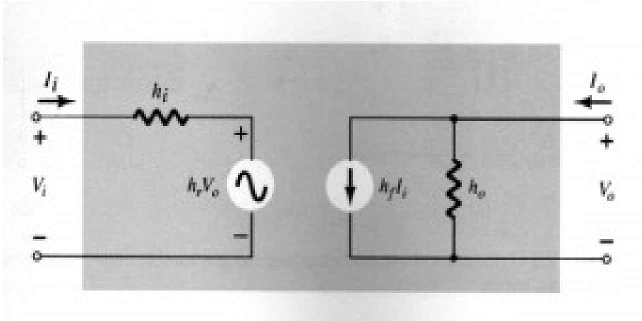


Fig. 12-3

Gain and Impedance Computation of the Complete Hybrid Equivalent Circuit:

For the circuit of Fig. 12-4,

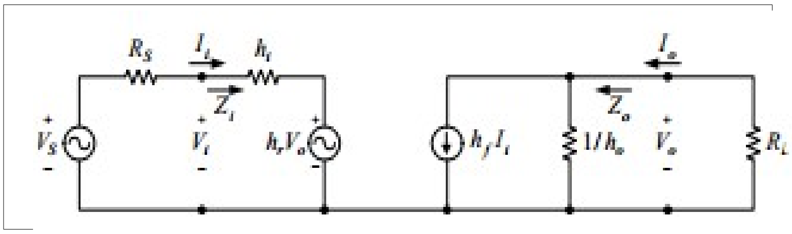


Fig. 12-4

the **voltage gain** $(A_V = V_o / V_i)$

$$I_i = \frac{V_i - h_r V_o}{h_i},\; I_o = \frac{-V_o}{R_L},\; I_o = h_f I_i + h_o V_o \Rightarrow$$

$$A_V = \frac{V_o}{V_i} = \frac{-h_f R_L}{h_i + (h_i h_o - h_f h_r) R_L} \qquad [12.3a]$$

the **current gain** $(A_i = I_o / I_i)$

$$I_o = h_f I_i \frac{1/h_o}{1/h_o + R_L} = \frac{h_f I_i}{1 + h_o R_L} \Rightarrow$$

$$A_i = \frac{I_o}{I_i} = \frac{h_f}{1 + h_o R_L} \qquad [12.3b]$$

the **input impedance** $(Z_i = V_i / I_i)$;

$$\frac{V_i}{I_i} = h_i + h_r \frac{V_o}{I_i},\; V_o = -I_o R_L \Rightarrow \frac{V_i}{I_i} = h_i - h_r R_L \frac{I_o}{I_i} = h_i - h_r R_L A_i$$

$$Z_i = \frac{V_i}{I_i} = h_i - \frac{h_f h_r R_L}{1 + h_o R_L} \qquad [12.3c]$$

the **output impedance** $\left(Z_o = V_o / I_o \right);$

$$V_S = I_i (R_s + h_i) + h_r V_o = 0 \Longrightarrow I_i = \frac{-h_r V_o}{R_s + h_i}, \quad I_o = h_f I_i + h_o V_o$$

$$I_o = h_o V_o - \frac{h_f h_r}{R_s + h_i} V_o \Longrightarrow$$

$$Z_o = \frac{V_o}{I_o} = \frac{1}{h_o - \frac{h_f h_r}{R_s + h_i}} \qquad [12.3d]$$

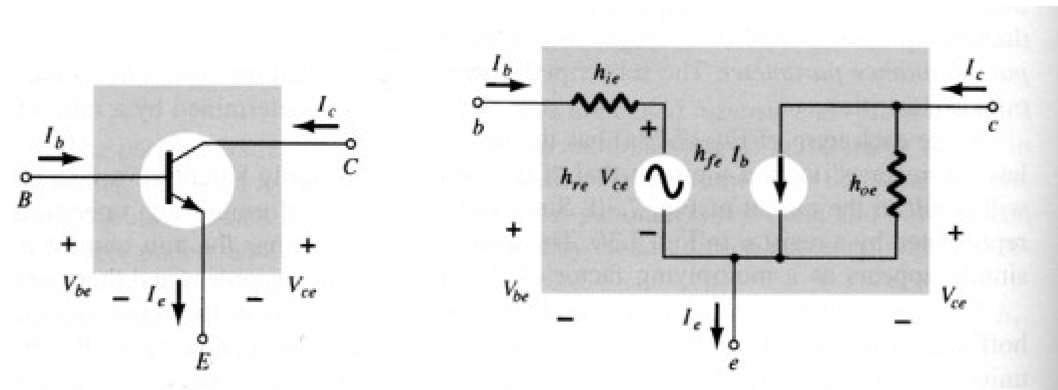
Types of Hybrid Parameters:

Since there are three types of BJT configuration (CE, CC, and CB), there are three different ways that the input and output can be defined and therefore three corresponding sets of *h*-parameters as shown in Table 12-1. If all of the *h*-parameters values in one configuration are known, then the values corresponding to any other configuration can be determined. The common-emitter values of the *h*-parameters are the ones most often given.

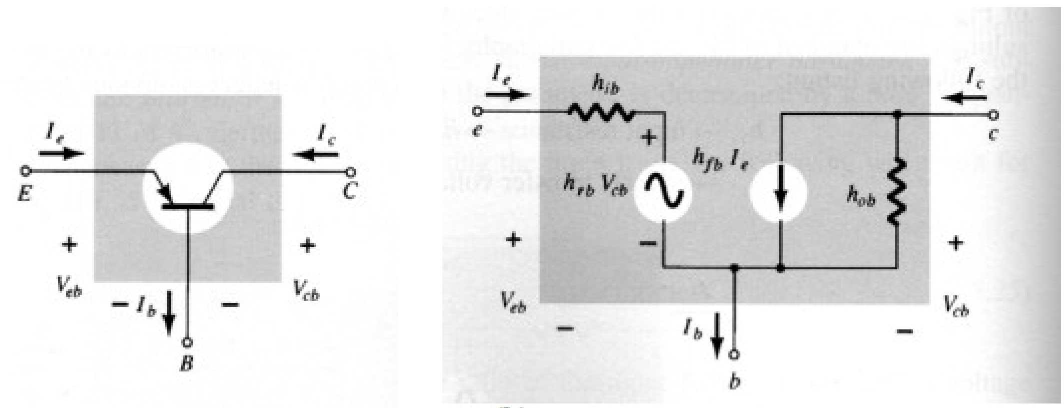
Table 12-1

BJT configuration		h-parameters sets
1-	Common-Emitter	*hie* , *hfe* , *hre* , *hoe*
2-	Common-Collector	*hic* , *hfc* , *hrc* , *hoc*
3-	Common-Base	*hib* , *hfb* , *hrb* , *hob*

The hybrid equivalent circuits of the CE and CB transistor configuration are shown in Fig. 12-5 (a) and (b) respectively.



(a)



(b)

Fig. 12-5

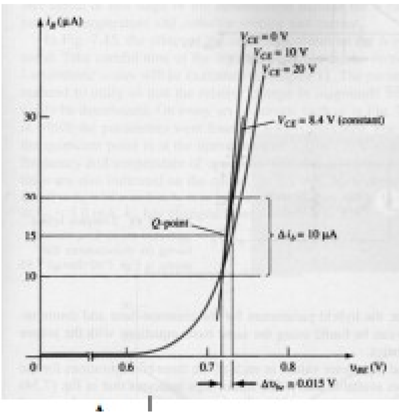
Table 12-2 lists typical parameter values in each of the three transistor configurations (CE, CC, and CB) fjor the broad range of transistors available today.

Table 12-2

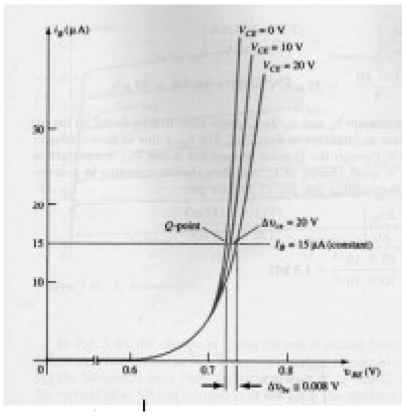
h-parameters	CE	CC	CB
h_i	1kΩ	1kΩ	20kΩ
h_r		≈ 1	
h_f	50	−50	−0.98
h_o	25 µS	25 µS	0.5 µS
$1/h_o$	40 kΩ	40 kΩ	2 MΩ

Graphical Determination of the CE Hybrid Parameters:

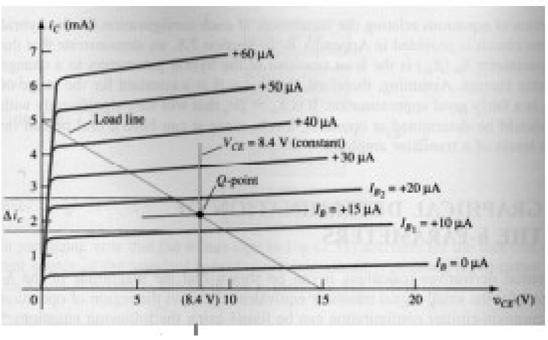
The parameters h_{ie} and h_{re} are determined from the input or base characteristics, while the parameters h_{fe} and h_{oe} are obtained from the output or collector characteristics as shown in Fig. 12-6.



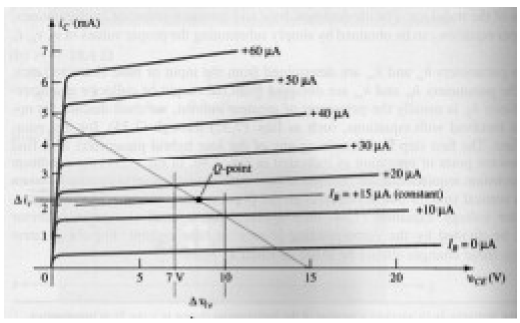
$$h_{ie} = \frac{\Delta v_{be}}{\Delta i_b} \cong 1.5\,k\,\Omega \; at \; V_{CE} = const.$$



$$h_{\Re} = \frac{\Delta v_{be}}{\Delta v_{ce}} \cong 4 \times 10^{-4} \; at \; I_B = const.$$



$$h_{fe} = \frac{\Delta i_c}{\Delta i_b} \cong 100 \; at \; V_{CE} = const$$



$$h_{oe} = \frac{\Delta i_c}{\Delta v_{ce}} \cong 33\,\mu S \; at \; I_B = const.$$

Fig. 12-5

For the transistor whose characteristics have appeared in Fig. 12-6, the resulting hybrid small-signal equivalent circuit is shown in Fig. 12-7.

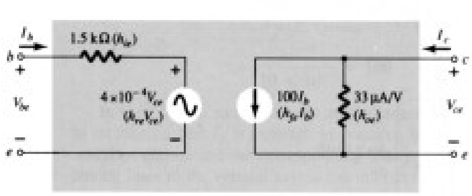


Fig. 12-7

The typical values of *h*-parameters for CE transistor configuration are shown in Table 12-3.

Table 12-3

h_{xe} parameters		Min.	Max	Unit
Input impedance	*hie*	0.5	7.5	kΩ
Voltage feedback ratio	*hre*	0.1	8.0	$\times 10^{-4}$
Small-signal current gain	*hfe*	20	250	-
Output admittance	*hoe*	1.0	30	μS

Approximate CE Hybrid Equivalent Model:

Since h_{re} is normally a relatively small quantity, its removal is approximated by $h_{re} \approx 0$ and $h_{re}V_{ce} = 0$, resulting in a short-circuit equivalent for the feedback element. The resistance determined by $1/h_{oe}$ is often large enough to be ignored in comparison to a parallel load permitting its replacement by an open-circuit equivalent for the CE model as shown in Fig. 12-8.

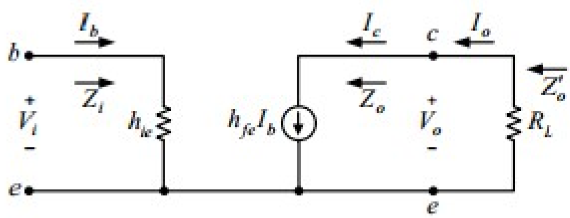


Fig. 12-8

For the circuit of Fig. 12-8

$Z_i \cong h_{ie},\ Z_o \cong \infty.$

$$A_i = \frac{I_c}{I_b} = h_{fe}, \quad A_v = \frac{V_o}{V_i} = \frac{-I_o R_L}{I_b h_{ie}} = \frac{-I_c R_L}{I_b h_{ie}} = -h_{fe}\frac{R_L}{h_{ie}} = -A_i \frac{Z_o}{Z_i}$$

The re Equivalent Model:

CB Transistor Configuration:

From Fig. 12-9, the input impedance at the emitter of CB transistor configuration (dynamic resistance of the forward diode) can determined by:

$$r_e = \frac{26mV}{I_E} \qquad [12.4]$$

the output impedance at the collector (dynamic resistance of the reverse diode) is:

$$r_o = \infty$$

Also;

$$Z_i = r_e, \quad Z_o = \infty.$$

$$V_o = -I_o R_L = -(-I_c) R_L = \alpha I_e Z_i, \quad V_i = I_e Z_i = I_e r_e \Rightarrow$$

$$A_v = \frac{V_o}{V_i} = \frac{\alpha R_L}{r_e} \approx \frac{R_L}{r_e}.$$

$$I_c = \alpha I_e, \quad A_i = \frac{I_c}{I_i} = -A_i = \frac{I_c}{I_e} \Rightarrow$$

$$A_i = -\alpha \approx -1$$

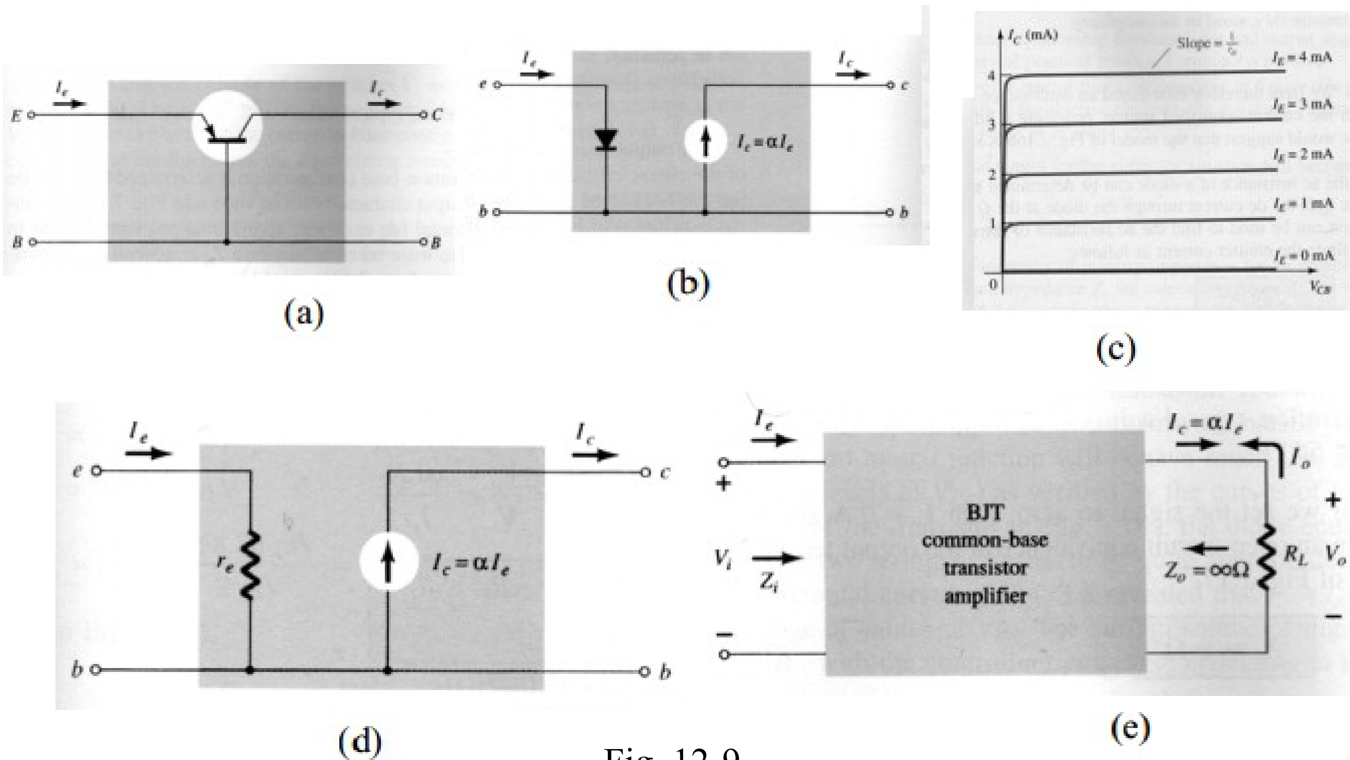


Fig. 12-9

CE Transistor Configuration:

From Fig. 12-10;

$$I_c = \beta I_b , I_e = I_c + I_b = \beta I_b + I_b = (\beta + 1) I_b \approx \beta I_b ,$$

$$V_{be} = I_e r_e \approx \beta I_b r_e \quad .$$

$$Z_i = \frac{V_i}{I_i} = \frac{V_{be}}{I_b} = \beta . r_e$$

$$Z_o = r_o \approx \infty .$$

$$V_o = - I_o R_L = - I_c R_L = - \beta I_b R_L ,$$

$$A_v = \frac{V_o}{V_i} = \frac{V_o}{V_{be}} = \frac{- \beta I_b R_L}{\beta I_b r_e} = \frac{- R_L}{r_e} .$$

$$A_i = \frac{I_o}{I_b} = \frac{I_c}{I_b} = \frac{\beta I_b}{I_b} = \beta \quad .$$

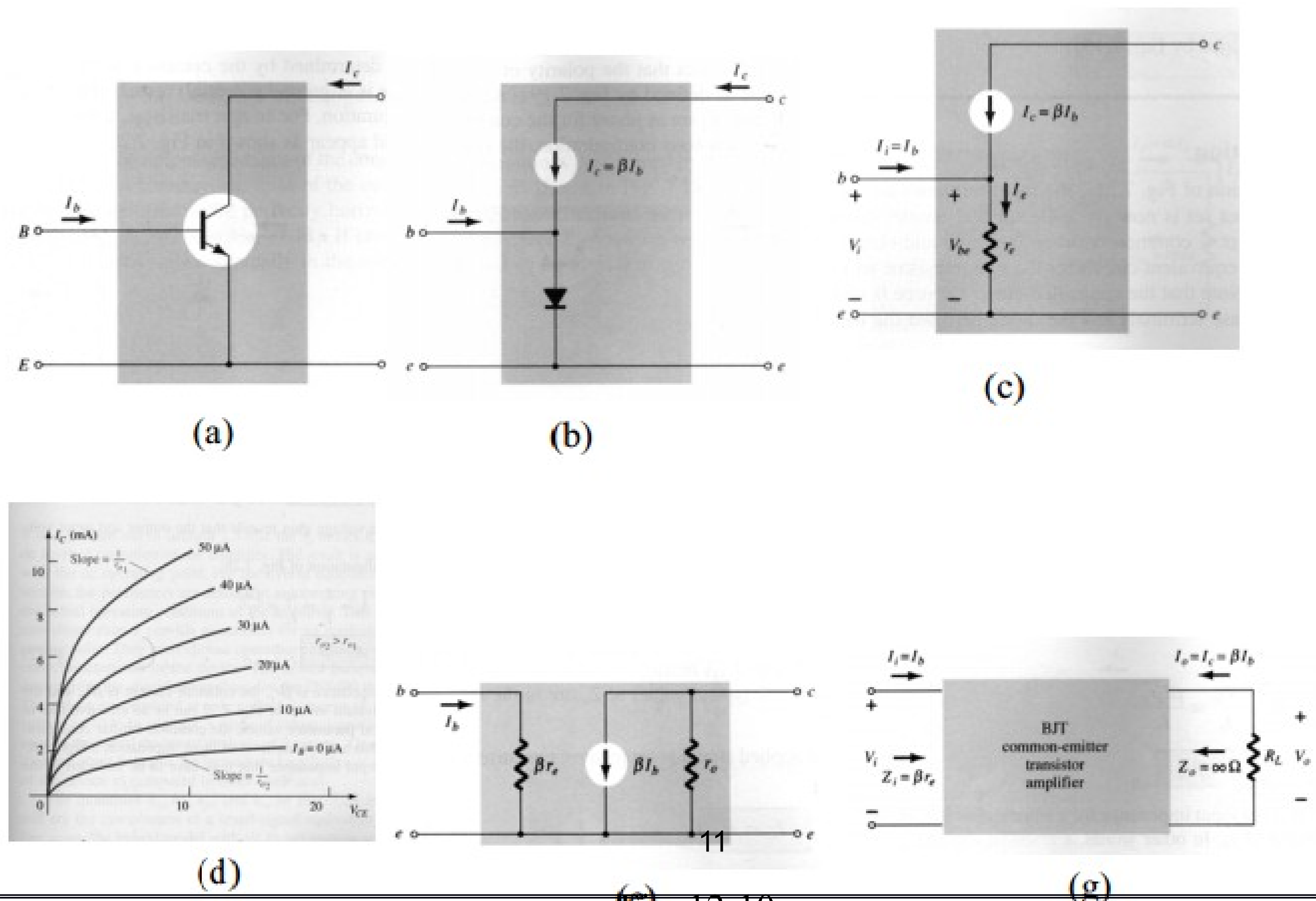
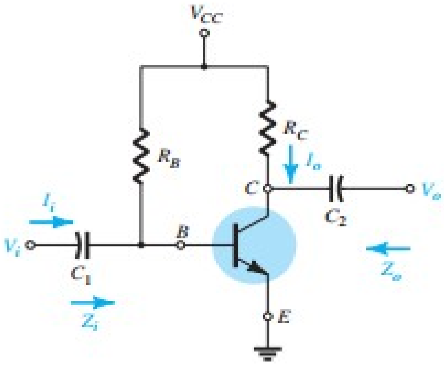
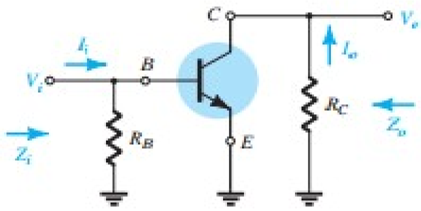


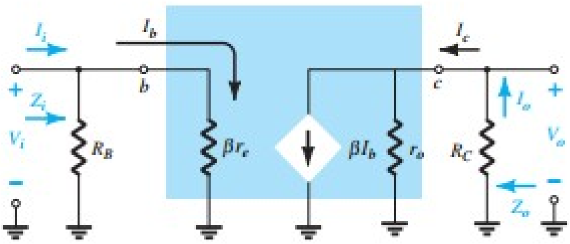
Fig. 12-10

Variations of Transistor Parameter:

1- Common-Emitter Fixed-bias Configration:







1) $Z_i = R_B \| \beta r_e \; (\Omega)$ if $R_B \gg \beta r_e$ then $Z_i \cong \beta r_e$

2) ($V_i = 0$, $I_b = 0$, $\beta I_b = 0$ open CCT.)

$Z_o = R_C \| r_o \; (\Omega)$ if $r_o \geq 10 R_C$ then $R_C \| r_o \cong R_C$, $Z_o \cong R_C$

3) A_v ……….. $V_o = -\beta I_B (R_C \| r_o)$

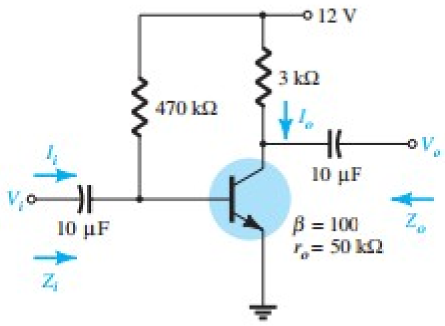
$I_b = \dfrac{V_i}{\beta r_e} \rightarrow V_o = -\beta \dfrac{V_i}{\beta r_e}(R_C \| r_o)$

$A_v = \dfrac{V_o}{V_i} = \dfrac{-R_C \| r_o}{r_e}$ if $r_o \geq 10 R_C \rightarrow A_v = \dfrac{-R_C}{r_e}$

Example: For the network Determine

a. *re* b. *Zi* (with *ro* = ∞ Ω). c. *Zo* (with *ro* = ∞ Ω). d. *Av* (with *ro* = ∞ Ω).

e. Repeat parts (c) and (d) including *ro* = 50 kΩ.



Solution:

a. DC analysis:

$$I_B = \frac{V_{CC}-V_{BE}}{R_B} = \frac{12-0.7}{470k} = 24.04\,\mu A.$$

$$I_E = (\beta + 1)\, I_B = (101)(24.04\,\mu) = 2.428\,mA.$$

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{2.428\,m} = 10.71\,\Omega$$

b. $\beta r_e = (100)(10.71) = 1.071\,K\,\Omega.$

$$Z_i = R_B \,||\, \beta r_e = 470 \,||\, 1.071\,K = 1.07\,k\,\Omega.$$

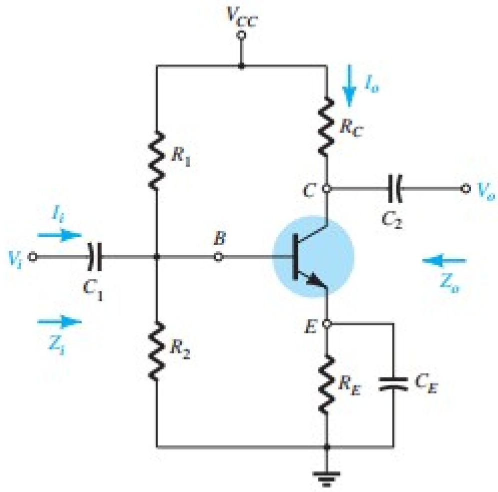
c. $Z_o = R_C = 3\,k\,\Omega.$

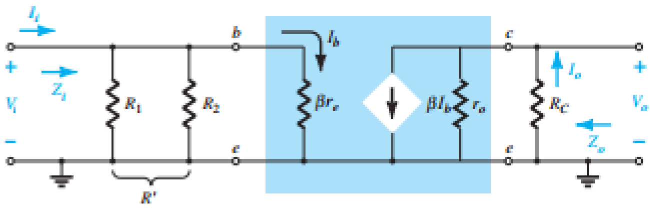
d. $A_v = \dfrac{-R_C}{r_e} = \dfrac{-3\,k}{10.71\,\Omega} = -280.11$

e. $Z_o = R_C \,||\, r_o = 3\,k \,||\, 50\,k = 2.83\,k\,\Omega.$

$$A_v = \frac{-R_C \,||\, r_o}{r_e} = \frac{-2.83\,k}{10.71} = -264.24$$

2-Voltage-Divader Bias





1) $Z_i = R' \parallel \beta r_e$ where $R' = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$

2) $V_i = 0,\ I_b = 0,\ \beta I_b = 0$ open CCT.

$Z_o = R_C \parallel r_o$ if $r_o \geq 10 R_C$ then $R_C \parallel r_o \cong R_C \Rightarrow Z_o = R_C$

3) $A_v \ldots\ldots\ V_o = -\beta I_B (R_C \parallel r_o)$

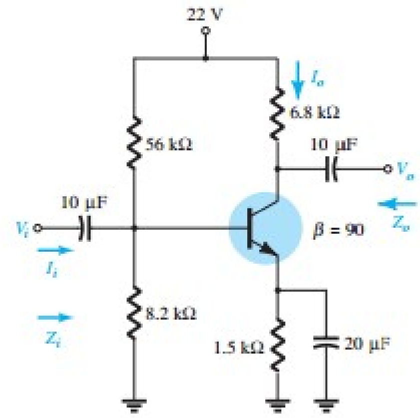
$I_b = \frac{V_i}{\beta r_e} \Rightarrow V_o = -\beta \frac{V_i}{\beta r_e}(R_C \parallel r_o)$

$A_v = \frac{V_o}{V_i} = \frac{-R_C \parallel r_o}{r_e}$ if $r_o \geq 10 R_C \Rightarrow A_v = \frac{-R_C}{r_e}$

Example: For the network Determine

a. *re* b. *Zi* (with *ro* $= \infty\Omega$). c. *Zo* (with *ro* $= \infty\Omega$). d. *Av* (with *ro* $= \infty\Omega$).

e. Repeat parts (c) and (d) including *ro* = 50 kΩ



Solution:

a. DC : Testing $\beta R_E \geq 10 R_2$

$$(90)(1.5k) \geq 10(8.2k)$$

$$135 \geq 82k$$

Using the approximate approach, we obtain

a) $$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{(8.2k)(22V)}{56k + 8.2k} = 2.81V$$

$$V_E = V_B - V_{BE} = 2.81 - 0.7 = 2.11V$$

$$I_E = \frac{V_E}{R_E} = \frac{2.11V}{1.5k} = 1.41mA$$

$$r_e = \frac{26mV}{I_E} = \frac{26mV}{21.41m} = 18.44\Omega$$

b) $$R' = R_1 \parallel R_2 = 56k \parallel 8.2k = 7.15k\Omega$$

$$Z_i = R' \parallel \beta r_e = 7.15k \parallel (90)(18.44) = 1.35k\Omega$$

c) $Z_o = R_C = 6.8k\,\Omega.$

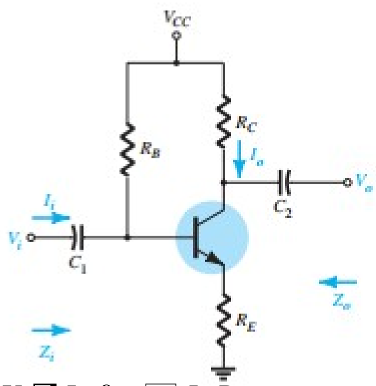
d) $A_v = \frac{-R_C}{r_e} = \frac{-6.8k}{18.44\,\Omega} = -368.76$

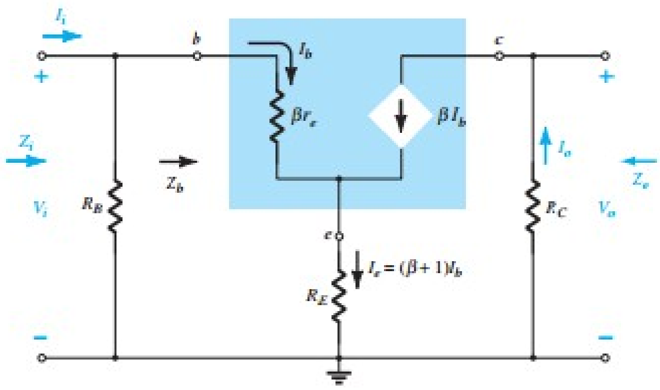
e) $Z_o = R_C \parallel r_o = 6.8k \parallel 50k = 5.98k\,\Omega.$

f) $A_v = \frac{-R_C \parallel r_o}{r_e} = \frac{-5.98k}{18.44} = -324.3$

There was a measurable difference in the results for *Zo* and *Av*, because the condition *ro* ≥ 10*RC* was *not* satisfied.

3-CE-Emitter bias configuration





1) $V_i = I_b \beta r_e + I_e R_E$

$V_i = I_b \beta r_e + (\beta + 1) I_b R_E$

$Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta + 1) R_E$ then $Z_b = \beta r_e + (\beta + 1) R_E$

since $\beta \gg 1$, $Z_b = \beta(r_e + R_E)$

$\beta(r_e + R_E)$, since $R_E \gg r_e$ then $Z_b \cong \beta R_E$

$Z_b \cong \beta(r_e + R_E)$

$Z_i = R_B \parallel Z_B$

2) $\because V_i = 0 , I_b = 0 , \beta I_b = 0 \; open\, CCT . \therefore$

$Z_o = R_C$

3) $I_b = \dfrac{V_i}{Z_b} , V_o = -I_o R_C = -\beta I_b R_C = -\,(\,) - \beta \dfrac{V_i}{Z_b} R_C$

$$A_v = \frac{V_o}{V_i} = \frac{-\beta R_C}{Z_b}$$

Effect of r_o

$$Z_i : Z_b = \beta r_e + \left(\frac{\left[\beta + 1\right] + R_C / r_o}{1 + \left[R_C + R_E\right] / r_o} \right) R_E$$

Since the ratio $\dfrac{R_C}{r_o} \ll \beta + 1$

$$Z_b = \beta r_e + \left(r_o + \frac{\left[\beta + 1\right] R_E}{1 + \left[R_C + R_E\right] / r_o} \right)$$

$For\; r_o \geq 10 \left[R_C + R_E\right]$

$Z_b = \beta r_e + \left[\beta + 1\right] R_E$

$$Z_o : Z_o = R_C \parallel \left(r_o + \frac{\beta \left[r_o + r_e\right]}{1 + \beta r_e / R_E} \right)$$

$However\; r_o \gg r_e$

$$Z_o = R_C /\!/ r_o \left(1 + \frac{\beta}{1 + \beta r_e / R_E} \right)$$

which can be written as

$$Z_o = R_C /\!/ r_o \left(1 + \frac{1}{\dfrac{1}{\beta} + \dfrac{r_e}{R_E}} \right)$$

$Z_o \cong R_C$

$$A_v : A_v = \frac{V_o}{V_i} = \frac{\frac{-\beta R_C}{Z_b}\left(1 + \frac{r_e}{r_o}\right) + \frac{R_C}{r_o}}{1 + \frac{R_C}{r_o}}$$

$$The\,ratio\; \frac{r_e}{r_o} \ll 1$$

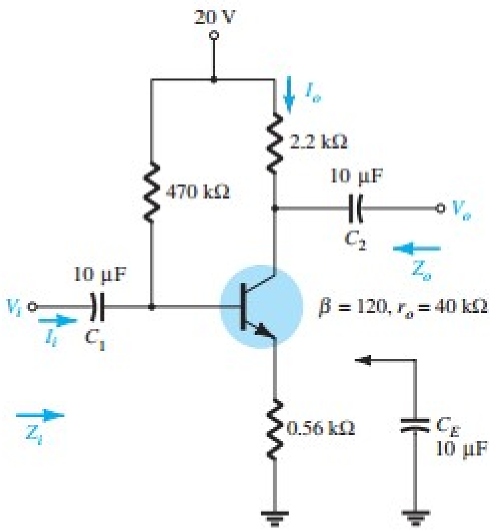
$$A_v = \frac{V_o}{V_i} = \frac{\frac{-\beta R_C}{Z_b} + \frac{R_C}{r_o}}{1 + \frac{R_C}{r_o}}$$

$$For\; r_o \geq 10\, R_C$$

$$A_v = \frac{V_o}{V_i} = \frac{-\beta R_C}{Z_b}$$

Example: For the network Determine

a. *re* b. *Zi* c. *Zo* d. *Av*



Solution:

a. DC:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1) R_E} = \frac{20 - 0.7}{470\,k + (121)(0.56\,k)} = 35.89\,\mu A.$$

$$I_E = (\beta + 1) I_B = (121)(35.89\,\mu A) = 4.34\,mA.$$

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{4.34\,m} = 5.99\,\Omega$$

b. Testing the condition $r_o \geq 10(R_C + R_E)$

$$40\,k \geq 10(2.2\,k + 0.56\,k) \Longrightarrow 40\,k \geq 27.6\,k \;Satisfied$$

Therefore,

$$Z_b \cong \beta(r_e + R_E) = 120(5.99\,\Omega + 560\,\Omega) = 67.92\,k\,\Omega.$$

$$Z_i = R_B \parallel Z_B = 470\,k \parallel 67.92\,k = 59.34\,k\,\Omega.$$

c. $Z_o = R_C = 2.2\,k\,\Omega.$

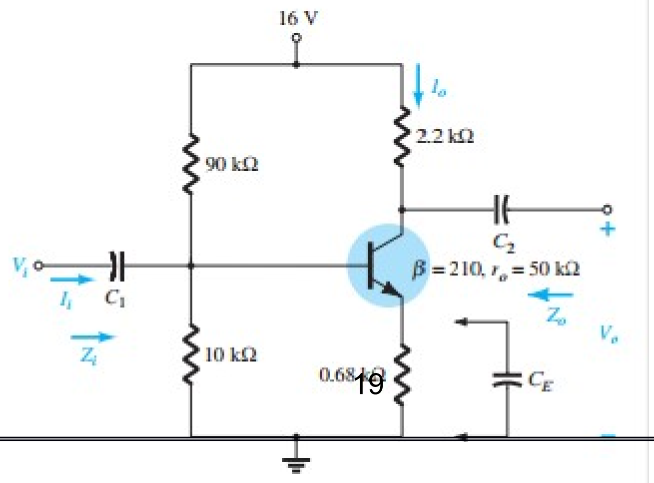
d. $r_o \geq 10 R_C$ *is satisfied*

$$A_v = \frac{V_o}{V_i} = \frac{-\beta R_C}{Z_b} = \frac{-(120)(2.2\,k)}{67.92\,k} = -3.89$$

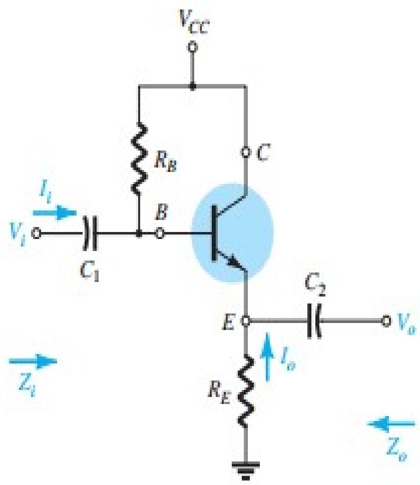
compared to -3.93 using Eq. $Av = -\frac{R_C}{R_E}$.

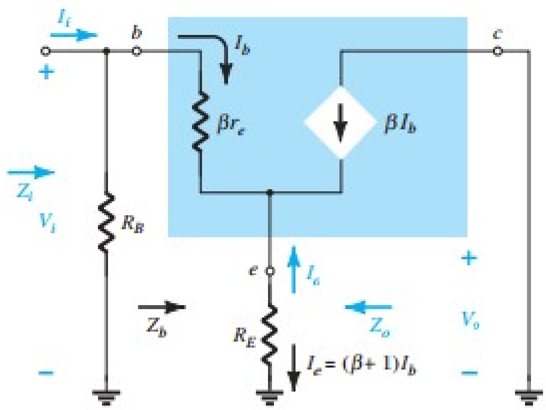
Home Work :

1- Repeat the analysis of last Example with *CE* in place.
2- For the network determine a. *re*. b. *Zi*. c. *Zo*. d. *Av* and repeat with *CE* in place



4-Emitter-Follower Configuration:





- $Z_i = R_C \parallel Z_b$

  $Z_b = \beta r_e + (\beta + 1) R_E$

  $at\ R_E \gg r_e$

  $Z_b \cong \beta(r_e + R_E) \rightarrow Z_b \cong \beta R_E$

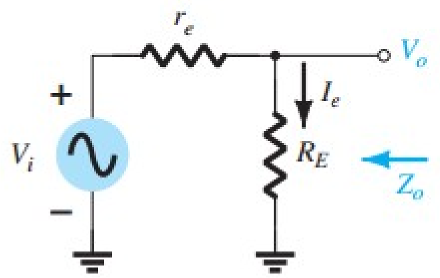
- Z_o:

$I_b = \frac{V_i}{Z_b}, I_e = (\beta + 1) I_b = (\beta + 1)\frac{V_i}{Z_b}$

$I_e = (\beta + 1)\frac{V_i}{\beta r_e + (\beta + 1) R_E}$

$I_e = \frac{V_i}{[\beta r_e / (\beta + 1)] + R_E}$ $But\ \beta + 1 \cong \frac{\beta \rightarrow \beta r_e}{\beta + 1} = r_e$

$I_e = \frac{V_i}{r_e + R_E}$



To determine Z_o , V_i is set to zero

$Z_o = R_E \parallel r_e\ then\ Z_o = r_e$

- A_v:

  $V_o = \frac{R_E V_i}{r_e + R_E} \rightarrow A_v = \frac{V_o}{V_i} = \frac{R_E}{r_e + R_E}$

  $Since\ R_E \gg r_e\ the\ r_e + R_E \cong R_E$

  $A_v = \frac{V_o}{V_i} = 1$

Effect of r_o

$Z_i : Z_b = \beta r_e + \frac{(\beta + 1) R_E}{1 + R_E / r_o}$

$if\ r_o \geq 10 R_E$

$Z_b = \beta r_e + (\beta + 1) R_E$

$Z_b \cong \beta (r_e + R_E)$

$Z_o : Z_o = r_o \parallel R_E \parallel \dfrac{\beta r_e}{\beta + 1}$

$Using\ \beta + 1 \cong \beta$

$Z_o = r_o \parallel R_E \parallel r_e,\ since\ r_o \gg r_e$

$Z_o = r_o /\!/ R_E$

$A_v : A_v = \dfrac{(\beta + 1) R_E}{1 + R_E / r_o},\ A_i = -A_v \dfrac{Z_i}{R_L}$

$If\ r_o \geq 10 R_E,\ \beta + 1 \cong \beta\ We\ find\ A_v = \dfrac{\beta R_E}{Z_b}$

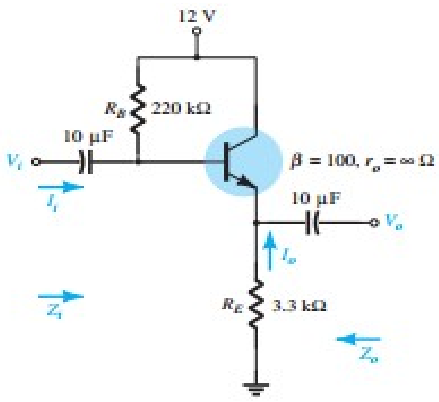
$But\ Z_b \cong \beta (r_e + R_E),\ A_v \cong \dfrac{\beta R_E}{\beta (r_e + R_E)} \Rightarrow A_v \cong \dfrac{R_E}{r_e + R_E}$

Example :

For the emitter-follower network, determine:

a. *re*. b. *Zi*. c. *Zo*. d. *Av*.

e. Repeat parts (b) through (d) with *ro* = 25 k and compare results.



Solution:

a. $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (\beta + 1) R_E} = \dfrac{12 - 0.7}{220\,k + (101)(3.3\,k)} = 20.42\ \mu A.$

$I_E = (\beta + 1) I_B = (101)(20.42\ \mu A) = 2.062\ mA.$

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{2.062\,m} = 12.61\,\Omega$$

b. $Z_b \cong \beta r_e + (\beta + 1) R_E = (100)(12.61) + (101)(3.3\,k) = 334.56\,k\,\Omega \cong \beta R_E$

$Z_i = R_B \,//\, Z_B = 220\,k \,//\, 334.56\,k = 132.72\,k\,\Omega.$

c. $Z_o = R_C \,//\, r_e = 3.3\,k \,//\, 12.61\,k = 12.56\,\Omega \cong r_e.$

d. $A_v = \frac{V_o}{V_i} \cong \frac{R_E}{r_e + R_E} = \frac{3.3\,k}{12.61 + 3.3\,k} = 0.996 \cong 1$

e. $r_o \geq 10\,R_E$ *Checking*

$25\,k\,\Omega \geq 10(3.3\,k) = 33\,k$ *Not satisfied, Therefore*

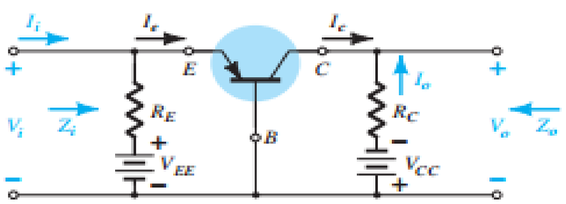
$$Z_b = \beta r_e + \frac{(\beta + 1) R_E}{1 + R_E / r_o} = 100 \times 12.61 + \frac{(100 + 1)\,3.3\,k}{1 + \frac{3.3\,k}{25\,k}} = 295.7\,k\Omega.$$

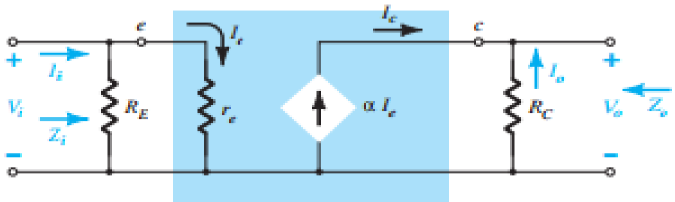
$Z_i = R_B \,//\, Z_B = 220\,k \,//\, 295.7\,k\Omega = 126.15\,k\,\Omega.$

$Z_o = R_C \,//\, r_e = 3.3\,k \,//\, 12.61\,k = 12.56\,\Omega.$

$$A_v = \frac{(\beta + 1) R_E / Z_b}{1 + R_E / r_o} = \frac{(100 + 1)\,3.3\,k / 295.7\,k}{1 + \frac{3.3\,k}{25\,k}} = 0.996 \cong 1$$

4-Common-Base Configuration:





$Z_i: Z_i = R_E \parallel r_e$

$Z_o : Z_o = R_C$

$A_v : V_o = I_o R = -(-I_C) R_C = \alpha I_C R_C$

$$I_C = \frac{V_i}{r_e}, V_o = \alpha \left(\frac{V_i}{r_e} \right) R_C$$

$$A_v = \frac{V_o}{V_i} \cong \frac{\alpha R_C}{r_e} \cong \frac{R_C}{r_e}$$

$A_i : Assuming\ R_E \gg r_e\ Yields$

$$I_C = I_i$$

$$A_i = \frac{I_o}{I_i} = -\alpha \cong -1$$

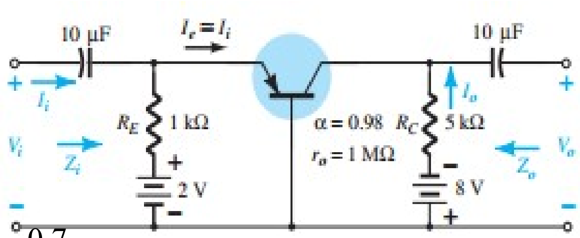
Effect of r_o

$$r_o = \frac{1}{h_{ob}} \gg R_C, Then\ r_o \parallel R_C \cong R_C$$

Example :

For the emitter-follower network, determine:

a. *re*. b. *Zi*. c. *Zo*. d. *Av*. e. A_i



Solution:

a. $I_E = \frac{V_{E-V_{BE}}}{R_E} = \frac{2.-0.7}{1\,k} = 1.3\,mA$

$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{1.3\,m} = 20\,\Omega$

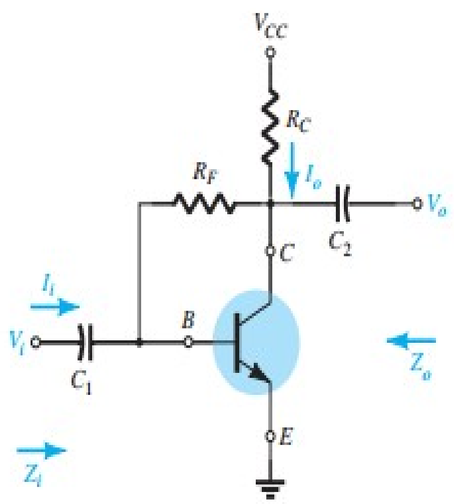
b. $Z_i = R_E \parallel r_e = 1\,k \parallel 20 = 19.61\,\Omega \cong r_e$

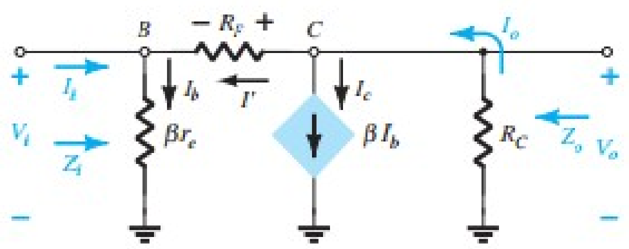
c. $Z_o = R_C = 5\,k\,\Omega$

d. $A_v \cong \frac{R_C}{r_e} = \frac{5\,k}{20} = 250$

e. $A_i = -\alpha = -0.98 \cong -1$

5-Collector Feedback Configuration:





$Z_i: I' = \frac{V_o - V_i}{R_F}$ with $V_o = -I_o R_C$, $I_o = \beta I_b + I'$

since βI_b is normally much larger than I', $I_o \cong \beta I_b$

$\therefore V_o = -(\beta I_b) R_C = -\beta I_b R_C$

but $I_b = \frac{V_i}{\beta r_e}$

$\therefore V_o = -\beta\left(\frac{V_i}{\beta r_e}\right) R_C = \frac{-R_C}{r_e} V_i$

Therefore $I' = \frac{V_o - V_i}{R_F} = \frac{V_o}{R_F} - \frac{V_i}{R_F} = \frac{-R_C V_i}{r_e R_F} - \frac{V_i}{R_F} = \frac{-1}{R_F}\left[1 + \frac{R_C}{r_e}\right] V_i$

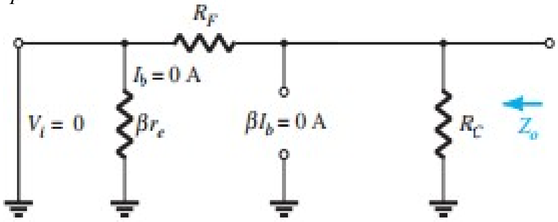
The result: $V_i = I_b \beta r_e = (I_i + I')\beta r_e = I_i \beta r_e + I' \beta r_e$

$$V_i = I_i \beta r_e - \frac{1}{R_F}\left[1 + \frac{R_C}{r_e}\right]\beta r_e V_i \Rightarrow V_i\left[1 + \frac{\beta r_e}{R_F}\left[1 + \frac{R_C}{r_e}\right]\right] = I_i \beta r_e$$

$$Z_i = \frac{V_i}{I_i} = \frac{\beta r_e}{1 + \frac{\beta r_e}{R_F}\left[1 + \frac{R_C}{r_e}\right]}, \text{ But } R_C \gg r_e, \ 1 + \frac{R_C}{r_e} \cong \frac{R_C}{r_e}$$

$$Z_i = \frac{\beta r_e}{1 + \frac{\beta R_C}{R_F}} \Rightarrow Z_i = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}}$$

Z_o: If we let $V_i = 0\,V$.



$Z_o = R_C \parallel R_F$

A_v: at node C, $I_o = \beta I_b + I'$

For typical value $\beta I_b \gg$ $I \cong I_o \cong \beta I_b$, *we have*:

$$V_o = -I_o R_C = -\beta I_b R_C \quad , \quad I_b = \frac{V_i}{\beta r_e}$$

$$V_o = -\beta\left(\frac{V_i}{\beta r_e}\right) R_C \Rightarrow A_v = \frac{V_o}{V_i} = \frac{-R_C}{r_e}$$

Phase Relationship

The negative sign indicates a 180° phase shift between *Vo* and *Vi*.

Effect of r_o

Z_i: *A complete analysis without applying approximations results*:

$$Z_i = \frac{1 + \frac{R_C \parallel r_o}{R_F}}{\frac{1}{\beta r_e} + \frac{1}{R_F} + \frac{R_C \parallel r_o}{R_F r_e}}$$

Recognizing that $\frac{1}{R_F} \cong 0$ and Applying the condition $r_o \geq 10 R_C$ we obtain

$$Z_i = \frac{1 + \frac{R_C}{R_F}}{\frac{1}{\beta r_e} + \frac{R_C}{R_F r_e}}$$ but typically $\frac{R_C}{R_F} \ll 1$ and

$$Z_i = \frac{1}{\frac{1}{\beta r_e} + \frac{R_C}{R_F r_e}}$$ or $$Z_i = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}}$$

Z_o: *Including* r_o *in parallel with* R_C

$$Z_o = R_C \parallel R_F \parallel r_o$$

For $r_o \geq 10 R_C$, $Z_o = R_C \parallel R_F$ as obtained $R_F \gg R_C$

$$Z_o \cong R_C$$

$$A_i = \frac{h_{fe} R_F}{R_F + h_{fe} R_C} \approx \frac{R_F}{R_C}$$

$$A_v = -\left(\frac{1}{R_F} + \frac{1}{r_e}\right)\left(\frac{r_o \parallel R_C}{1 + \frac{r_o \parallel R_C}{R_F}}\right)$$

A_v: $A_v =$

Since $R_F \gg r_e$

$$A_v = \frac{r_o \parallel R_C}{1 + \frac{r_o \parallel R_C}{R_F}}$$

For $r_o \geq 10\, R_C$

$$A_v = \frac{R_C}{1 + \frac{R_C}{R_F}}$$

And since $\frac{R_C}{R_F} \ll 1$

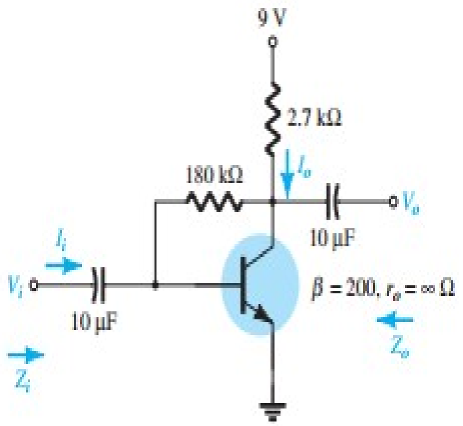
$$A_v = \frac{-R_C}{r_e}$$

Example:

For the network Determine

a. *re* b. *Zi* c. *Zo* d. *Av*

e. Repeat parts (b) through (d) with r_o = 20 k .



Solution:

a. $$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta R_E} = \frac{9 - 0.7}{180\,k + (200)(2.7\,k)} = 11.53\,\mu A.$$

$$I_E = (\beta + 1)\, I_B = (201)(11.53\,\mu A) = 2.32\,mA.$$

$$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{2.32\,m} = 11.21\,\Omega$$

b. $$Z_i = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}} = \frac{11.21}{\frac{1}{200} + \frac{2.7\,k}{180\,k}} = \frac{11.21}{0.02} = 50(11.21\,\Omega) = 560.5\,\Omega$$

c. $$Z_o = R_C \,\|\, R_F = 2.7\,k \,\|\, 180\,k = 2.66\,k\,\Omega$$

d. $$A_v = \frac{-R_C}{r_e} = \frac{-27\,k}{11.21} = -240.86$$

e. The condition $r_o \geq 10 R_C$ is *not* satisfied. Therefore,

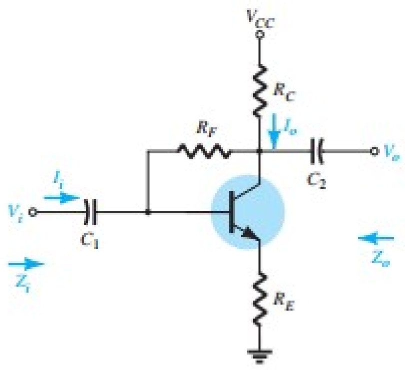
$$Z_i = \frac{1 + \frac{R_C \,\|\, r_o}{R_F}}{\frac{1}{\beta r_e} + \frac{1}{R_F} + \frac{R_C \,\|\, r_o}{R_F r_e}} = \frac{1 + \frac{2.7\,k \,\|\, 20\,k}{180\,k}}{\frac{1}{(200)(11.21)} + \frac{1}{180\,k} + \frac{2.7\,k \,\|\, 20\,k}{(11.21)(180\,k)}}$$

$$Z_o = R_C \,\|\, R_F \,\|\, r_o = 2.7\,k \,\|\, 180\,k \,\|\, 20\,k = 2.35\,k\,\Omega \;vs\; 2.66\,k$$

$$A_v = \frac{-\left(\frac{1}{R_F} + \frac{1}{r_e}\right)(r_o \,\|\, R_C)}{1 + \frac{r_o \,\|\, R_C}{R_F}} = \frac{-\left(\frac{1}{180\,k} + \frac{1}{11.21}\right)(2.38\,k)}{1 + \frac{2.38\,k}{180\,k}} = \frac{-\left(5.56\times10^{-6} - 8.92\times10^{-2}\right)}{1 + 0.013} = -209.56 \;vs\; -240.8$$

6-Collector Feedback Configuration with an emitter resistor R_E :

$$Z_i = \frac{R_E}{\frac{1}{\beta} + \frac{(R_E + R_C)}{R_F}}$$



$$Z_o = R_C \parallel R_F$$

$$A_v = \frac{-R_C}{R_E}$$

$$A_i = \frac{R_F}{R_E + R_F + \frac{R_F}{h_{fe}}}$$

Feedback CCT

Z_i :

$$I_b = I_i + \frac{V_o - V_i}{R_F}$$

$$I_b \cong I_i + \frac{V_o}{R_F} \qquad V_o \gg V_i$$

$$V_i = I_b h_{ie} \Rightarrow V_i = \left(I_i + \frac{V_o}{R_F}\right) h_{ie}$$

$$V_i = \left(h_{ie} I_i + \frac{h_{ie}}{R_F} V_o\right)$$

$$V_o = A_v V_i \, , \, A_v = \frac{V_o}{V_i}$$

$$V_i = h_{ie} I_i + \frac{h_{ie} A_v V_i}{R_F}$$

$$V_i\left(1-\frac{h_{ie}A_v}{R_F}\right)=h_{ie}I_i$$

$$\frac{V_i}{I_i}=\frac{h_{ie}}{1-\frac{h_{ie}A_v}{R_F}}=Z_i \quad X \parallel Y=\frac{X.Y}{X+Y}=\frac{Y}{1+\frac{Y}{X}}$$

$$Z_i=\frac{h_{ie}}{1-\frac{h_{ie}A_v}{R_F}} \Rightarrow Z_i=h_{ie}\parallel\frac{R_F}{A_v}$$

A_i:

$$V_i+V_{R_F}-V_o=0$$

$$I_b h_{ie}+(I_b-I_i)R_F+I_oR_C=0$$

$$I_o=h_{fe}I_b$$

$$I_bh_{ie}+I_bR_F-I_iR_F+h_{fe}I_bR_C$$

$$I_b(h_{ie}+R_F+h_{fe}R_C)\rightarrow I_iR_F$$

$$I_b=\frac{I_o}{h_{fe}}\rightarrow I_o=h_{fe}I_b$$

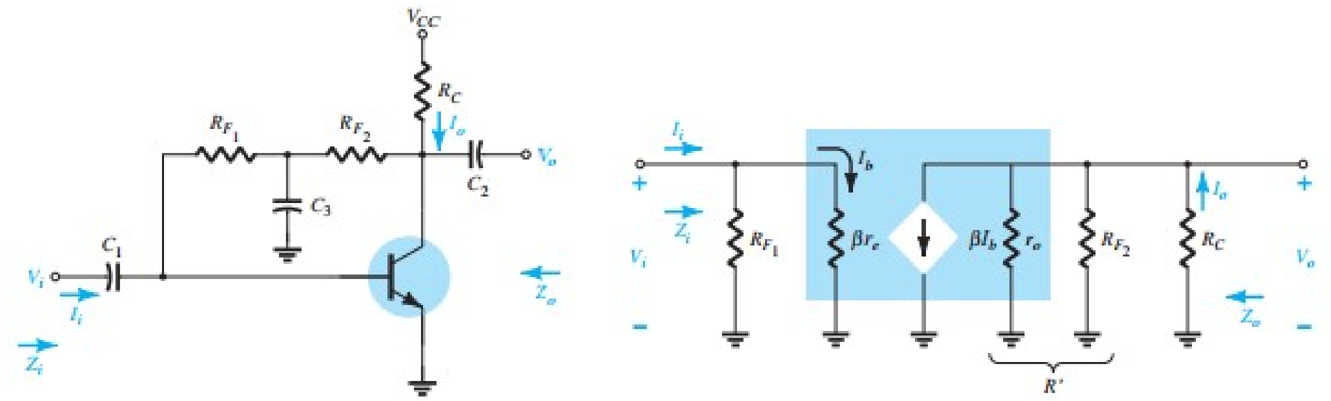
$$\frac{I_o}{h_{fe}}(h_{ie}+R_F+h_{fe}R_C)=I_iR_F$$

$$I_o=\frac{h_{fe}I_iR_F}{h_{ie}+R_F+h_{fe}R_C}$$

$$A_i=\frac{I_o}{I_i}=\frac{h_{fe}R_F}{R_F+h_{fe}R_C} \quad h_{ie}\ll$$

$$A_i \approx \frac{R_F}{R_C} \; if \; h_{fe} R_C \gg R_F$$

Collector DC Feedback Configuration:



$$Z_i = R_{F1} /\!/ \beta r_e$$

$$Z_o = R_C /\!/ R_{F2} /\!/ r_o$$

For $r_o \geq 10 R_C$ $\qquad Z_o \cong R_C /\!/ R_{F2}$

$$R' = r_o /\!/ R_C /\!/ R_{F2}$$

$$V_o = -\beta I_b R' \quad , \quad I_b = \frac{V_i}{\beta r_e} \quad , \quad V_o = -\beta \left(\frac{V_i}{\beta r_e} \right) R'$$

$$A_v = \frac{V_o}{V_i} = \frac{-r_o /\!/ R_C /\!/ R_{F2}}{r_e}$$

For $r_o \geq 10 R_C$

$$A_v = \frac{-R_C /\!/ R_{F2}}{r_e}$$

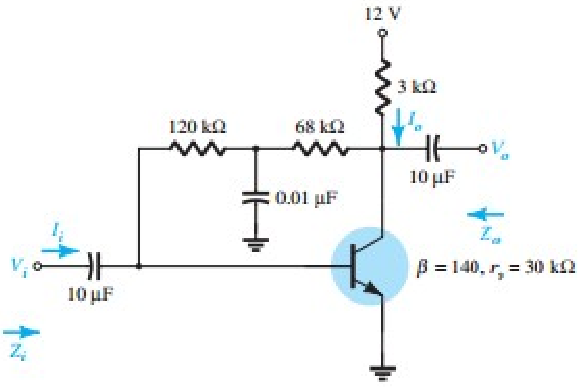
Phase Relationship :

The negative sign ,clearly reveals a 180° phase shift between input and output voltages.

Example :

For the network, determine:

a. *re*. b. *Zi*. c. *Zo*. d. *Av*.



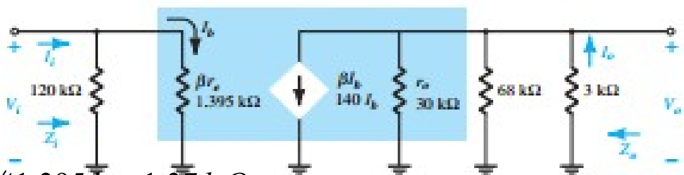
Solution:

a. $I_B = \frac{V_{CC}-V_{BE}}{R_F+\beta R_C} = \frac{12-0.7}{(120+60)+(140)(3k)} = 18.6\,\mu A.$

$I_E = (\beta+1)I_B = (141)(18.6\,\mu A) = 2.62\,mA.$

$r_e = \frac{26\,mV}{I_E} = \frac{26\,mV}{2.62\,m} = 9.92\,\Omega$

b. $\beta r_e = (140)(9.92) = 1.39\,k\Omega$



$Z_i = R_{F1} /\!/ \beta r_e = 120k /\!/ 1.395k \cong 1.37\,k\Omega$

c. Testing condition $r_o \geq 10 R_C$ we find

$30\,k\Omega \geq 10(3k)$ which is satisfied

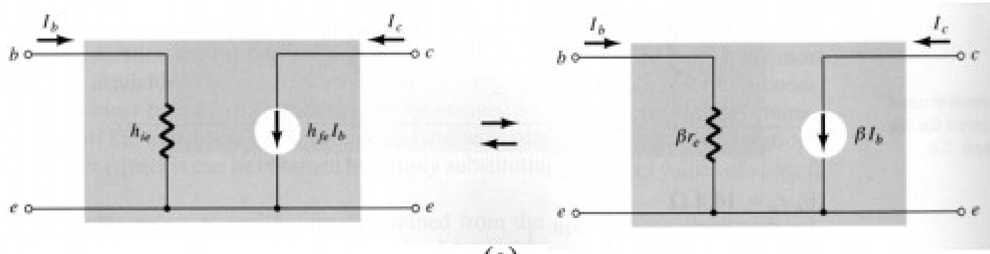
$Z_o \cong R_C /\!/ R_{F2} = 3k /\!/ 68k = 2.87\,k\Omega$

d. $r_o \geq 10 R_C$ therefore

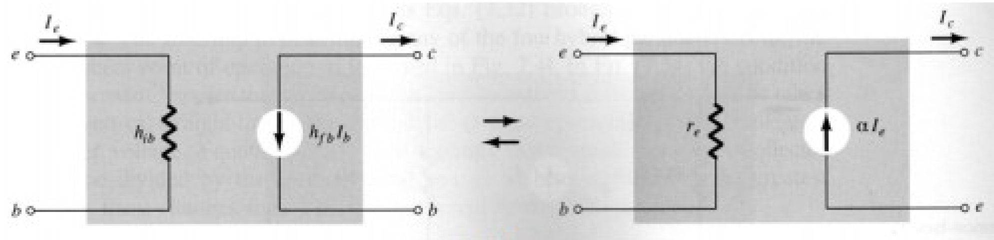
$A_v = \frac{-R_C /\!/ R_{F2}}{r_e} = \frac{-68k /\!/ 3k}{9.92} = \frac{-2.87k}{9.92} = -289.3$

Hybrid Versus r_e Model:

The hybrid versus r_e model for CE and CB transistor configurations are shown in Figs. 12-11 (a) and (b) respectively.



(a)



(b)

Fig. 12-11

Approximate Conversion Formulas for Hybrid and r_e Models:

The approximate conversion formulas for hybrid and r_e models for CB and CC configurations are listed in Table 12-4.

Table 12-4

CB Configuration	CC Configuration
1 ⎕ h_{fe} ☎⌚☽ $\cong r_e$ $h_{ib} \cong h_{ie}$ ☜⌚	$h_{ic} \cong h_{ie} \cong \beta r_e$
1 ⎕ h_{fe} ☎⌚☽ $- h_{\Re}$ $h_{rb} \cong h_{ie} h_{oe}$ ☜⌚	$h_{rc} = 1 - h_{\Re} \cong 1$
1 ⎕ h_{fe} ☎⌚☽ $\cong -\alpha$ $h_{fb} \cong h_{fe}$ ☜⌚	$h_{fc} \cong -$☎1 ⎕ h_{fe}☽ $\cong -\beta$
1 ⎕ h_{fe} ☎⌚☽ $h_{ob} \cong h_{oe}$ ☜⌚	$h_{oc} \cong h_{oe} \cong r_o$

Exercise:

Given $I_E = 1.3$ mA, $\beta = 100$, and $r_o = 40$ kΩ, sketch:

1. The CE and CB hybrid equivalent models.
2. The CE and CB r_e equivalent models.